PERFORMANCE COMPARISON BETWEEN (ADI-FDTD) AND CONVENTIONAL (FDTD)

Adamu Abubakar Isah, Mehmet Kusaf and Oyku Akaydin
Cyprus International University
albattuta88@gmail.com

Abstract

The Impetus for the work presented here arose from the need that implicit FDTD method (ADI-FDTD) and explicit FDTD method (Conventional FDTD) method have distinguished advantage which is speed and precision respectively. It was proved that ADI-FDTD method is less restrained by Courant-Friedrich Levy (CFL) stability condition, conventional FDTD on the other hand is affected by CFL condition which makes the method difficult to be applied in the solution of large electrical problems which makes the use of Perfectly Matched Layer Absorbing Boundary Conditions (PML ABC) more effective and advantageous in terms of accurate results. Numerical simulation of both ADI-FDTD and conventional FDTD method are presented. The results are compared to find the performance of each method in terms of precision and accuracy with respect to the CPU time and memory usage.

Keywords: **Conventional FDTD, ADI-FDTD, Courant-Friedrich Levy (CFL) stability condition, Absorbing boundary conditions (ABC), Perfectly matched layer (PML).**

Introduction

In solving electromagnetic problems, a computer system is needed to translate the solution into a numerical system. The inherent limitations in the manner numbers are saved in computer, some errors will virtually be present in the resulting solution. These errors will surely be few but they are an artifact about which one should be aware of. Although FDTD methods are precise and well defined, computer system technology limits the speed at which these operations can be performed. Simulation time in the order of hours, weeks or longer are common when solving large electromagnetic problems, some problems are even too large to be effectively solved due to simulation time and memory constrain.

To reduce the computational time, it is required to build clusters of computers to give a parallel processing speed up or acquire a super computer. These solutions can be expensive and frequently impractical. As a result, a new approach that increases the speed of the FDTD method in a relatively inexpensive and practical way is needed. The FDTD method is a time domain method which can handle all frequency responses during the computation of electromagnetic wave propagation. This technique was first introduced by Yee in 1966. The explicit nature of this method made it easy to understand and implement in simulation programs. Hence, FDTD method is the most popular numerical method for the solution of electromagnetic problems.

LITERATURE REVIEW

The FDTD methods are a simple and at the same time powerful numerical method used for solving Maxwell's time dependent curl equations. The method uses finite difference approximation for the differential operators both in time and space, so that the curl equations become linearized by the employment of central differencing. To perform the FDTD method, initially a computational domain which contains the physical structures to be simulated is gridded. The unbounded domain which involves conducting structures and/or dielectric and magnetic objects that can be either homogeneous or inhomogeneous can be modeled easily by the FDTD method. In order to model open region problems the computational domain must be terminated with absorbing boundary conditions (ABCs). FDTD method has a number of very good ABCs that simulates the effect of free space beyond the boundary. Applying some excitation to an object or group of objects that already grided in a three-dimensional computational domain with surrounding of boundary conditions, and calculating the fields using the coupled finite difference equations is known as FDTD method. To do the simulation the spatial and the time step size must be specified in the beginning of the simulation, these parameters are very important in terms of accuracy and efficiency of the simulation. In principle, the properties of the electromagnetic waves cannot be reproduced exactly in the discretized space, instead, the numerically obtained wavenumber deviates from its physical value and becomes a function of frequency. Other then this numerically induced dispersion; simulations further become worse by the dependence of the wavenumber on the direction of propagation in the grid, which can be described as numerical anisotropy. So the solution of Maxwell's equations using the FDTD method, introduces a numerical error in the phase velocity for a sinusoidal plane wave due to the use of finite difference (FD) operators instead of the exact analytical derivatives.

In order to increase the performance of the FDTD method, many schemes has been developed and still the contributions are going on, some of the examples are given in the following Chapters. In analyzing the performance of the FDTD schemes, there are various issues to be addressed, such as dispersion characteristics, and stability condition. The accuracy of each finite-difference time-domain scheme can be obtained from numerical dispersion relation of that particular algorithm. The dispersion characteristics are typically derived by assuming a time harmonic planewave solution of the discretized form of Maxwell's equations for an isotropic, homogeneous, linear, and lossless medium.

In 1873, James Clerk Maxwell [17] shows that electromagnetic waves can exist independently of any system of conductors and can travel through free space. Using the S.I system of units, the differential forms of Maxwell's equations are given by: [36]

$$D = \varepsilon E \quad (2.1)$$

$$B = \mu H \quad (2.2)$$

$$J = \sigma E \text{ Ohm's Law} \quad (2.3)$$

$$\frac{\partial \boldsymbol{D}}{\partial t} = \nabla \times \boldsymbol{H} - \boldsymbol{J_e} \text{ Ampere's Law} \quad (2.4)$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \boldsymbol{E} - \boldsymbol{J_m} \text{ Faraday's Law} \quad (2.5)$$

$$\nabla \cdot D = \rho \text{ Gauss' Law for the Electric Field} \quad (2.6)$$

$$\nabla \cdot B = 0 \text{ Gauss' Law for the Magnetic Field} \quad (2.7)$$

Where;

- **E** is the electric field (V/m)
- **D** is the electric flux density (C/m)
- **H** is the magnetic field (A/m)
- **B** is the magnetic flux density (Web/m^2)
- **J** is the conduction current density (A/m^2)
- **ε** is the permittivity (F/m)
- **μ** is the permeability (H/m)
- **σ** is the direct current (DC) conductivity (S/m)

The following six coupled scalar equations in the three-dimensional Cartesian coordinate system. Where Writing out the vector component of the above curl equations yields the following six coupled scalar equations in the three-dimensional Cartesian coordinate system.

$$\frac{\partial H_x}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} - \sigma^* H_x\right) \quad (2.14a)$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} - \sigma^* H_y\right) \quad (2.14b)$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} - \sigma^* H_z\right) \quad (2.14c)$$

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \sigma\, E_x\right) \quad (2.15a)$$

$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \sigma\, E_y\right) \quad (2.15b)$$

The above six coupled partial differential equations form the basis of the FDTD numerical algorithm in three-dimension. Two-dimension and one-dimension can be formulated by reducing the dimensions accordingly.

COMPARISON BETWEEN FDTD METHOD AND ADI-FDTD

CONVENTIONAL FDTD METHOD

Advantages

1. It is a flexible demonstrating strategy used to explain Maxwell's comparisons and clients can without much of a stretch see how to utilize it and what's in store from a given model.
2. FDTD is a period area procedure, utilized for the reaction of the framework over an extensive variety of frequencies can be acquired.
3. It ascertains the E and H fields all around in the computational area as they develop in time and to help guarantee that the model is working accurately.
4. It method permits the materials at all focuses inside the computational space together with direct and nonlinear dielectric and attractive materials.

Disadvantages

1. FDTD has a small electromagnetic wavelength and the smallest geometrical feature in a model, with very large computational domains.
2. There is no way to determine unique values for permittivity and permeability at a material interface.

3. Space and time steps must satisfy the CFL condition, or the integration used to solve the partial differential equation is probable to become unstable.
4. FDTDs fields directly introduces a distance that may force the computational domain to be excessively large.

IMPLICIT ADI-FDTD METHOD

Advantage of ADI-FDTD Method

1. The ADI method is an implicit method that solves a set of simple, one-dimensional, tridiagonal systems as part of the time step update.
2. It is a powerful implicit method for solving a finite-difference time-domain (FDTD) discretization of Maxwell's equations.
3. It is basically faster than the conventional ADI-FDTD methods due to its computational accuracy.
4. ADI-FDTD methods time size is determined by the computational accuracy rather than the CFL condition.

Disadvantages of ADI-FDTD Method

1. The truncation error on the time step of the spatial derivatives of the fields is found to be a major limitatıon of ADI-FDTD scheme.
2. Results between the courant limit and field data computed with the explicit Yee scheme are not obtained with ADI-FDTD.
3. It uses the backward differencing scheme, which is unconditionally stable.
4. In the ADI algorithm, the equations are implicit only in one direction, making it very weak over the conventional FDTD.

METHODOLOGY

ADI - IMPLICIT TIME STEPPING ALGORITHM FOR OPERATION BEYOND THE COURANT LIMIT

For conventional FDTD to be stable it requires a bounding of the time step Δt relative to the space increment Δx as shown in the equation (1) above.

This Δt allows application of FDTD methods to a wide variety of electromagnetic wave interaction problems of moderate electrical size and quality factors. Thus, such problems need 10^3 to 10^4 time steps to complete a single simulation. The stated restriction limit the use of FDTD modeling applications that fall into this difficult regions have the following characteristics.

When solving stability problem, relationships between Δt and Δx are derived to ensure that energy is bounded, which ıs referred to as the stability criterion. The stability criterion is obtained using the combination of eigenvalue analysis and Von-Neumann. The procedure performs a Fourier transformation along all spatial dimensions; thereby reducing the finite difference scheme.

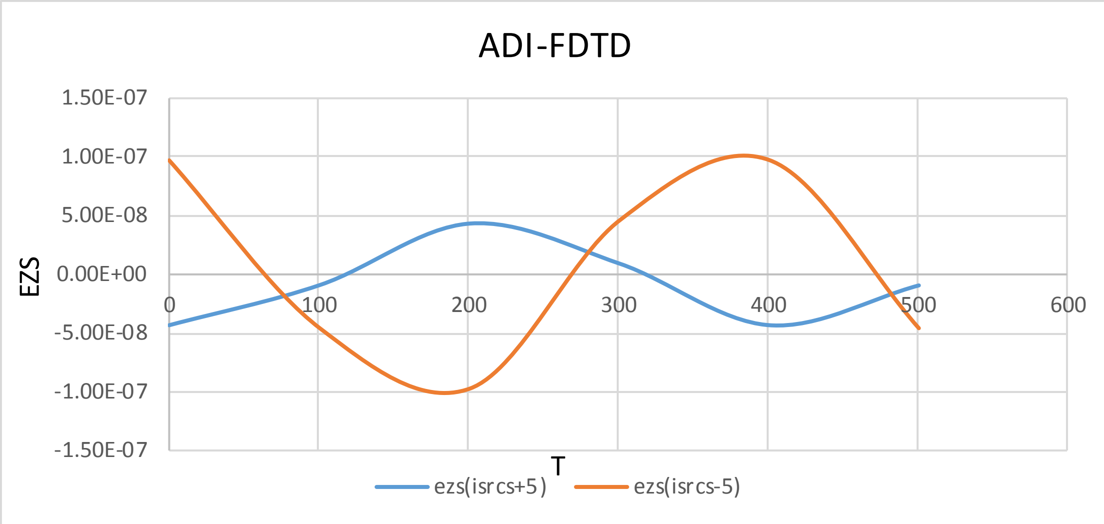
A new ADI time stepping algorithm for FDTD was reported by Zheng, Chan and Zhang; it has a theoretical unconditional numerical stability for the general 1-D one-dimensional case. However, the ZCZ algorithm utilizes the same Yee space lattice as conventional FDTD.

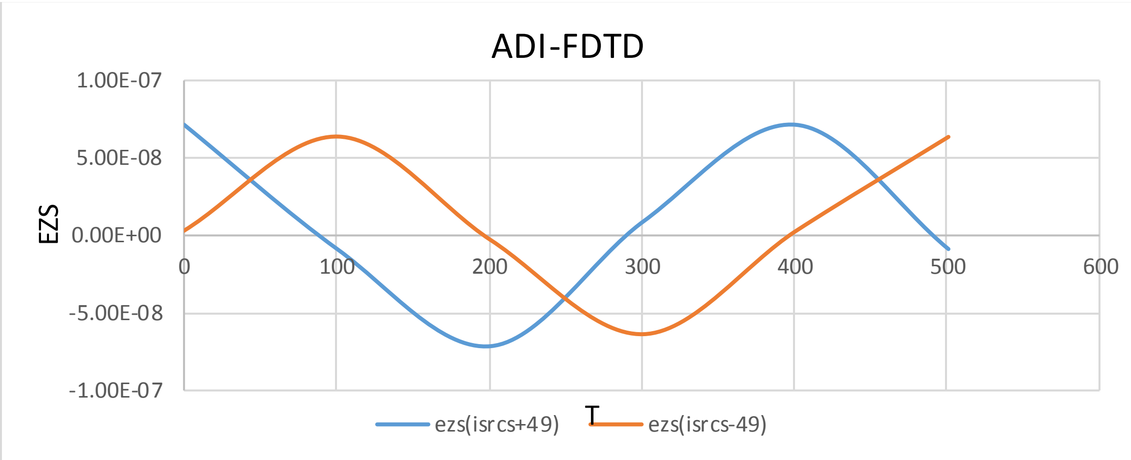
Using the **MAPLE™** software ZCZ found out that the magnitudes of all eigenvalues of a composite matrix have equal unity regardless of the time step. Therefore it was concluded that the ZCZ algorithm is unconditionally stable for all change in t and the courant stability condition is removed.

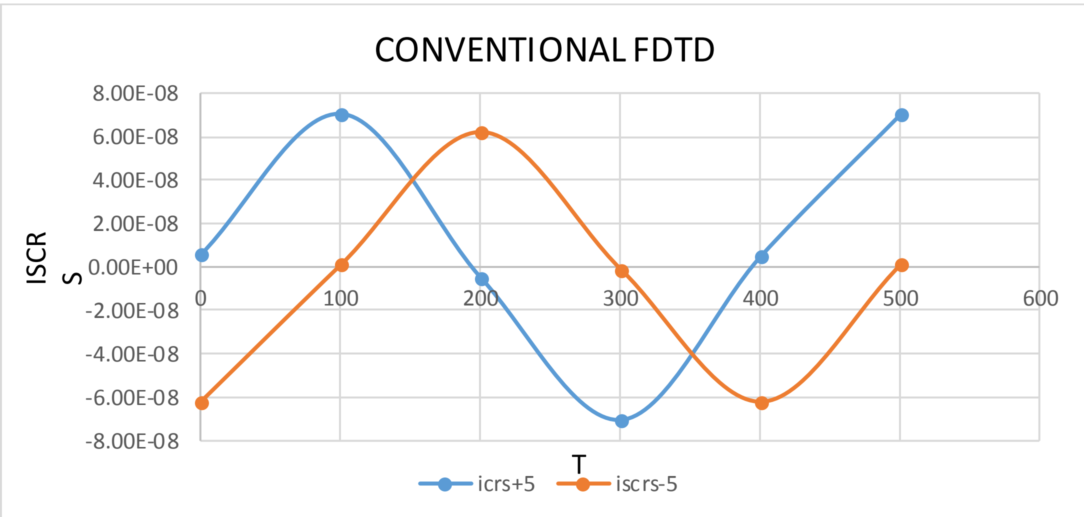
NUMERICAL RESULTS AND DISCUSSION

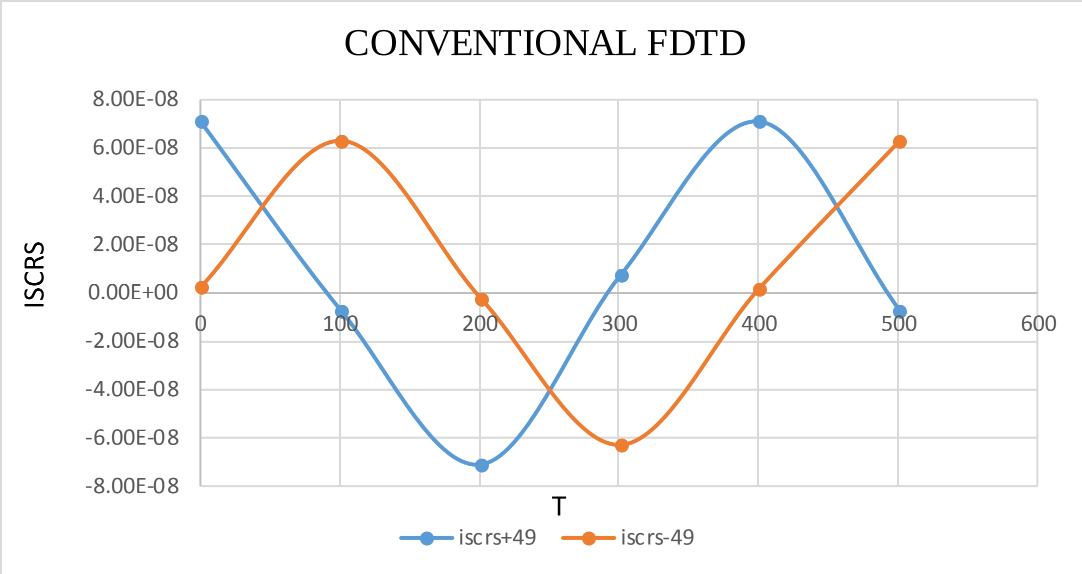
RESULT AND DISCUSSION

Overall summary of the simulation from FORTRAN compiler shows that **ADI-FDTD CPU EXIT TIME** = 0.0307310E-11 Seconds, and that of **CONVENTIONAL FDTD CPU EXIT TIME** = 0.1252500E-07 Seconds, at time step 501 respectively. The Alternating Direction Implicit (ADI-FDTD) technique has pulled in much comfort ability for electromagnetic field scenarios since it is free from the Courant stability properties. The ADI-FDTD technique can be acquired from the Crank-Nicolson FDTD procedures. It further creates a part that is relative to the time step size and to the greatness of spatial domains. ADI-FDTD produces two numerical instances viz spurious charges and a typical mode (i.e., negative gathering speed modes with positive speeds). We note that different variations of the Crank-Nicolson FDTD plan are additionally enhanced for the improvement of proficient stable and unstable steady systems.


ADI-FDTD
1.50E-07
1.00E-07
5.00E-08
0.00E+00
-5.00E-08
-1.00E-07
-1.50E-07
EZS
0
100
200
300
400
500
600
T
ezs(isrcs+5)
ezs(isrcs-5)


ADI-FDTD
1.00E-07
5.00E-08
0.00E+00
-5.00E-08
-1.00E-07
EZS
0
100
200
300
400
500
600
T
ezs(isrcs+49)
ezs(isrcs-49)


CONVENTIONAL FDTD
8.00E-08
6.00E-08
4.00E-08
2.00E-08
0.00E+00
-2.00E-08
-4.00E-08
-6.00E-08
-8.00E-08
ISCR
S
0
100
200
300
400
500
600
T
icrs+5
iscrs-5



SCHEME	**ΔT**	**TOTAL TIME**	**CPU TIME (Secs)**	**MEMORY (MB)**
FDTD	**$25*10^{-12}$**	500	19	2.06
		1000	24	16.5
		1500	35	4.0
ADI-FDTD	**$25*10^{-12}$**	500	26	4.0
		1000	33	4.0
		1500	37	2.3
ADI-FDTD	**$50*10^{-12}$**	500	24	2.8
		1000	33	2.9
		1500	36	2.5
ADI-FDTD	**$100*10^{-12}$**	500	11	3.5
		1000	10	2.6
		1500	17	2.7

CONCLUSION AND RECOMMENDATION

In this paper, we display an improved ADI-FDTD method which allows the displaying of lossy materials, for example, semiconductors and metals that are required for a typical perfectly matched layer (PML) with retaining limit conditions. The results of the simulations summed up a new and improved definition which are exhibited to show the precision and degree to which the computational capacity is decreased as well as the ADI property. The (FDTD) system for settling the full-wave Maxwell's comparisons has been as of late stretched out to give exact and numerically steady operation for time steps surpassing each other as far as possible. One such class of issues is the investigation of fast execution time which interconnects the techniques that are frequently required for the exact determination of a typical electromagnetic wave phenomena.

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