Estimating Utility Functions through Experimental Designs

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Abstract Utility function is an interesting tool generally used to measure the preference over a set of attributes. Calculation of the utility function is possible by substituting preferences related assumptions in predefined mathematical models. The method may be challenging while the number of preferences available are reduced. The paper, presented herein, proposes a new methodology for a more accurate estimation of the utility function. Two experimental designs tools are suggested to find the best approximation to the utility function meeting decision makers preferences. First, Latin Hypercube Sampling is employed for filling the experimental space with meaningful representative data. Second, the model response surfaces are analyzed and optimized.

Keywords Latin Hypercube, Utility function, Experimental design, Response surfaces

1. INTRODUCTION

Multi-Criteria Decision Aid (MCDA) methods are massively used as a support for decision making selection. One of the powerful techniques that have proved its efficiency is Multi-Attribute Utility Theory (MAUT). The strength of this structured method is that it can handle tradeoffs among various objectives. It works by converting all the objectives scales to one unique scale called utility unit scale. The problem solving is then converted to a modeling and maximization of only one main utility function; de Almeida (2007), Fishburn (1970). Indeed, as expressed by Bernoulli (1954), the determination of the value of an item must not be based on its proper unit but rather on the utility it represents. In fact, the unit is independent to the circumstances of the problem, whereas the utility is closely dependent to the problem's circumstances and to the decision maker behaviors.

However, one limitation of MAUT is that it should predict the utility function based on few available decision maker preferences data. This may end with wide inconsistency between what is reflected by decision makers' preferences and reality. To overcome this opportunity, this paper proposes an integration of experimental approaches to MAUT. The purpose is to construct a more reliable utility function based on decision maker preferences and reinforced by confirmed statistical methods.

Actually, design and analysis of statistical methods are an important part of any engineering assignment. Running physical experiments, calculating and minimizing uncertainties are tightly linked to their main responsibilities. However, scientific research development have led to an increase of processes complexity. As a result, data spectrums become larger making of the physical experiments time consuming and much more expensive. To overcome this, computer experiments models were developed to simulate the complex systems based on mathematical and statistical models; Sacks et al. (1989).

In this research work, we propose a methodology for estimating utilty function using two experimental design methods: Latin Hypercube Sampling (LHS) and Response Surfaces Methodology.

Several papers could be found in literature proposing a combined analysis based on LHS and MAUT. The main research topic of these papers is to analyze uncertainty among MAUT models' inputs. As an example, LHS was used in Heinrich et al. (2007) to assign uncertainty distributions to the attributes of

the proposed MAUT model. A hybrid method based on LHS and Kriging models was used to estimate the expected value of the utility in Conigliaro et al. (2009). Kriging model was introduced to be able to optimize the model with a reduced number of LHS sample.
Similarly, integration of Response Surfaces to MAUT were used by researchers to find the best approximations of utility function. Baker et al. (1984) considered in his research the impact of response time and provided services on the performance of the Emergency Medical Services (EMS). The analysis, achieved with an approximation to the utility surface, enabled a better understanding of the criterion influencing the performance of EMS. Camargo et al. (2009) suggested a framework that integrates expert judgment in the development process of a new product. The determination of utility function with approximation aimed to reflect a better interpretation of the expert preferences.
The framework proposed in this paper is different than the above. It aims to explore LHS and Response surfaces to reinforce the construction of the utility function. Indeed, the major added value of this work is that utility function is not constructed based on decision makers preferences only, but reinforced with statistical data generated from the input space.
This paper is organized as following. In section 2, we go through an overview of MAUT and some of the challenges of this method. . In section 3, we present the suggested framework for the utility function estimation. Section 4 goes through an illustrative example to illustrate the model. Last but not least, section 5 concludes the paper with a discussion and summary of the main findings.

2. Review and challenges of MAUT Method

Below an overview of the purpose and proprieties of MAUT method with an introduction to its major calculation steps. Right after, we go through a talk on some of the challenges of the method.

2.1. Overview of MAUT Method

In the multi-criteria problems, decision maker should select the most appropriate decision, taking into account several objectives that should be achieved simultaneously. MAUT works by affecting to each one of these objectives (called attributes) a utility function. Each utility function reflects decision makers preferences over those attributes. In fact, these preferences may be estimated depending on decision makers behaviors and willingness to take risk. Three behaviors can be distinguished: Risk Aversion (1), Risk Neutral (2) or Risk Prone (3). Each one of these behaviors is expressed with a specific utility function form as stated below, Kim and Song (2009):

$$u_i(x) = a - bexp(cx) \tag{1}$$
$$u_i(x) = a + bx \tag{2}$$
$$u_i(x) = a + bexp(cx) \tag{3}$$

where $u_i(x)$ is utility function for the attribute *i*.
a, *b* and *c* are constants included in the interval [0,1] and achieving for two values x_{1i} and x_{2i}:

$$u(x_1 i) = 0 \tag{4}$$

$$u(x_2 i) = 1 \tag{5}$$

In order to define the utility functions expressed in *(1)*, *(2)* and *(3)*, interviews should be led with decision makers to define his or her *j* preferences x_{ji} achieving:

$$u(x_j i) = k \tag{6}$$

where k is a constant included in [0,1].
Once utility functions of all the attributes are defined, they are then aggregated into one main utility function as following:

$$U(X) = F(U_1(X_1), ..., U_n(X_n)) \tag{7}$$

achieving for two values *x'* and *x"*:

$$U(x') = 0, U(x") = 1 \tag{8}$$

The model is solved then by maximizing the main utility function *U(X)*.

2.2. Overview of MAUT Challenges

Utility function have been subject to many challenging discussions. Several researchers have analyzed different aspects of the theory and challenged its capability to predict the trend of decision makers preferences. The perspective explained in Friedman and Savage (1948), stated that the expected utility theory can not completely reflect how decision makers interact in the real world. It is based on the fact that all, as individual beings, may have two or more behaviors adopted simultaneously. This implies that utility function can not have a unique curvature. Markowitz (1952) argued in his paper that utility function is tightly dependent to the reference level and not the absolute level. This means that, if we consider as an example the utility of individuals gambling, small gains would reflect increasing utility curves, while big gains would provide decreasing utility trends. Many other perspectives are discussed in literature based on the fact that the preferences assumptions are not reasonable or that the empirical evidences are not often conform to the predictions. A number of paradoxes were developed in this direction i.e Allais paradox or Ellsberg paradox, Samuelson (1937).
However, even if utility function have been subject to a number of arguments and counterarguments for its efficiency, it is still one of the breakthrough MCDA methods that have an interesting and wide area of application.

This paper proposes a model that addresses one of the challenges of the utility function. The framework estimates the utility function in the case of risk prone or risk aversion models, considering the case where decision makers can only provide his or her best and worst preferences equivalent respectively to a utility value equal to 1 and 0. In this specific case, utility function can not be calculated since the available constraints are formulated in a mathematical system of two equations and three unknowns. The paper herein proposes a framework that enables achieving an estimation to the utility function and providing the optimal approximation with the help of some confirmed experimental designs.

3. Proposed Model for Estimating Utility Function With LHS and Response Surfaces Optimization

The model proposed herein depicts an estimation of utility functions while having limited decision makers' preferences data. The framework is built upon three main steps, as explained below.

3.1. Define model's attributes

Decision maker has to define the important objectives that should be included in the decision making process. An identification of each attribute include a definition of its scale, maximal tolerated value, minimal tolerated value and decision maker's risk behavior. Utility of the maximal value would be equal to 1 whereas utility of minimal value is null. Risk behavior must be either risk prone, risk neutral or risk aversion. Utility function form is related to the risk behavior as stated earlier in the paper.

3.2. Generate Latin Hypercube Sampling

Once utility function form is known, the unknown parameters of this function (named previously a, b and c) should be calculated in order to identify the utility function related to each attribute. LHS is applied then to generate and fill in the input space with a sampling of these unknown parameters. Latin Hypercube Sampling is a statistical method that was first introduced by Mckay in 1979; McKay et al. (2000). It uses a stratified sampling scheme to generate samples or collections of points from multidimensional

distributions. It aims to improve the coverage of the input space by spreading out the design points to the entire space; Iman (2008). The methodology is generally used in computer experiments to emulate computer simulations. LHS is as an efficient method for estimating mean values and standard deviations in stochastic structural analysis, small probabilities and even reliability analysis, Olsson et al. (2003). The strength of LHS is that by sampling over the entire input space, each variable has the opportunity to show up as important, if it is indeed important. At the opposite, if we consider a simple sampling, useful information will gathered from the dominating variables only, Iman (2008).
A simple and basic explanation of LHS can be the following. In two dimensional space (square) samples should be spread in a way there is only one sample in each row and each column. A Latin hypercube sampling is the generalization of this rule to a multi-dimensional space. Each sample is unique in each axis-aligned hyperplane containing it.
Refer to McKay et al. (2000), Stein (1987) for more detailed explanation and calculation steps of the LHS methodology.

3.3. Build and Optimize Response Surfaces

In this step, Response Surfaces are built for each utility function based on the outcome of LHS. Response surfaces are then optimized in order to determine the combination of the unknown parameters (a, b and c) that achieve the best approximation to the studied utility function with a low noise level.
Indeed, Response surface methodology (RSM) is a collection of mathematical and statistical techniques used to explore and analyze the relationships linking the models inputs and response variables. It aims to improve and optimize the design response by the usage of a sequence of design experiments. In fact, RSM generates a mathematical model, in general linear or square polynomial functions, to approach and describe the process, Bezerra et al. (2008), Montgomery (2008), Bass and Boyac (2007). RSM is represented and visualized graphically. The relationship linking the response and the input is described by RSM as following:

$$y = f(x_1, x_2, ..., x_i) + \epsilon \tag{9}$$

where:
- y represents the response variable;
- x_i are the input variables;
- i is the number of variables;
- ϵ represents the noise observed in the response y
- $f(x_1,x_2,...,x_i)$ is called the expected response $E(y)$.

The surface represented by the function $f(x_1,x_2,...,x_i)$ is actually the Response Surface. It can be denoted

$$E(y) = f(x_1, x_2, ..., x_i) = \eta \tag{10}$$

Optimization of the response surfaces of the model are achieved via the maximization of the desirability function. Desirability function is an objective function that converts the response variables y_i to a transformed response value d_i that varies over the range [0,1]. When the response variable is meeting the target, then d_i=*1*. When the response variable is outside the limits, then d_i=*0*. The optimization is working by the maximization of one main desirability function *D(x)* which is a general mean of all the transformed responses, expressed as following, Myers et al. (2016):

$$D = (\prod d_i)^{1/n} \tag{11}$$

Under the context of the current paper, the response variables represent the utility function of each one of the attributes analyzed. Response Surface function *E(y)* is built after analyzing data generated with LHS. Optimization of the response surfaces help afterwards to find the best approximation to the utility function.

4. ILLUSTRATIVE EXAMPLE

To illustrate our framework, we apply the proposed model to estimate the utility function of the case study presented by Laghrabli et al. (2016). The paper proposed a hybrid MCDA method based on MAUT and AHP to select the best Transportation Supplier. The research led helped in defining eight main attributes required for selecting transportation' suppliers. A didactic example is presented as an application of the proposed model to the case of Moroccan Ministry of Health. It aimed to select the best transportation supplier to ensure the availability of the pharmaceutical products at 3000 different demand points all across the country.

We consider for the current illustrative example the utility function of one attribute only, namely "Cost". The purpose of this example is to, first, illustrate the model we are proposing and, then compare the utility functions generated from the two methods.

Laghrabli et al. (2016) derived the utility functions using a didactic example to illustrate their methodology. Only maximal and minimal cost values were considered, respectively, 10 MM MAD and 15 MM MAD. A mid value x_3 achieving

$$u(x_3) = 1/2 \tag{12}$$

was estimated and added to the model to be able to solve the system of two equations below.

$$u(10) = a + bexp(10 * c) = 1 \tag{13}$$

$$u(15) = a + bexp(15 * c) = 0 \tag{14}$$

In the following, we apply the model we are proposing in this paper to estimate the utility function described in (12) and (13). So instead of estimating the value x_3, we apply LHS at this level of the framework to emulate the constants of the system above, a, b and c. To be noted again that a, b and c are included in the interval [0,1].

The model is described by two responses R_1 and R_2, where *u(10)*=R_1, *u(15)*=R_2. We start our model by generating a, b and c on 20 runs and we augment our model till we obtain meaningful data. For the current application, we agree on a 30 run model since it enables having good response surfaces analysis and optimization data. Table 1 summarizes the results of the LHS application to the model. It summarizes, for each run, the emulated a, b and c data and the value of the calculated variable responses R_1 and R_2. The space filling design generated with LHS is then used afterwards to build response surfaces for R_1 and R_2.

Results interpretation

The analysis of the two model responses suggests quadratic and cubic functions as an approximation to R_1 and R_2. We choose for our analysis the cubic approximation since it is providing better statistical indicators. The R-squared value is estimated for R_1 to 97.6% ($\epsilon \simeq$2.4%) whereas for R_2 it is estimated to 95.8% ($\epsilon \simeq$4.2%). The parameter *a* is neglected in both suggested functions (quadratic and cubic) and for both variable responses R_1 and R_2. As a result we dismiss all the combinations based on the parameter *a* in the approximation we are building for our model. Analysis outcome details for the two responses are displayed in Table II and III.

As a result, we keep the following cubic functions E_1 and E_2as, respectively, the best approximation for R_1 and R_2 :

$$\begin{aligned} E_1 = &- 534.699 + 659.532 * a + 7098.598 * c - 7253.799 * bc - 19565.44 * c^2 \\ &+ 12587.611 * bc^2 + 14288.687 * c^3 \end{aligned} \tag{15}$$

$$\begin{aligned} E_2 = &- 39977.47 + 45571.918 * b + 5.32E + 005 * c - 4.656E + 005 * bc \\ &- 1.492E + 006 * c^2 + 7.479E + 005 * bc^2 + 1.12E + 006 * c^3 \end{aligned} \tag{16}$$

TABLE I
LATIN HYPERCUBE SAMPLING RESULTS

Run	a	b	c	R_1	R_2
1	0.793103448	0.137931035	0.172413793	1.566577205	2.624731886
2	0.724137931	0.931034483	0.586206897	327.9367018	6134.982633
3	0.931034483	0.310344828	0.724137931	434.1780575	16188.48112
4	1	0.344827586	0.379310345	16.30826243	102.9971055
5	0.068965517	0.586206897	0.620689655	290.9208257	6478.685975
6	0.482758621	0.965517241	0	1.448275862	1.448275862
7	0.896551724	0.620689655	0.137931035	3.362031128	5.810321614
8	0.275862069	0.034482759	0.862068966	191.4899777	14239.25727
9	0.965517241	0.689655172	0.689655172	682.9372214	21446.33507
10	0	0.275862069	0.931034483	3048.760946	320508.1403
11	0.137931035	0.862068966	0.413793103	54.16639558	427.8612544
12	0.448275862	0.068965517	0.24137931	1.219053352	3.025057014
13	0.551724138	0.206896552	0.551724138	52.05806987	813.2221992
14	0.689655172	0.551724138	0.482758621	69.60501615	770.9065854
15	0.655172414	0.103448276	0.896551724	810.4927451	71653.79977
16	0.034482759	0.482758621	0.275862069	7.651479699	30.29042293
17	0.413793103	0.75862069	0.655172414	531.7902675	14063.84014
18	0.620689655	0.517241379	0.827586207	2032.296878	127331.9009
19	0.172413793	0.172413793	0.448275862	15.42731289	143.6645185
20	0.206896552	0.24137931	0.034482759	0.547662947	0.611784751
21	0.862068966	0	0.517241379	0.862068966	0.862068966
22	0.310344828	0.379310345	0.75862069	747.8641925	33187.18677
23	0.24137931	0.724137931	0.103448276	2.278850379	3.659020991
24	0.827586207	1	0.206896552	8.744215469	23.10221427
25	0.344827586	0.655172414	1	14431.47759	2141770.348
26	0.103448276	0.896551724	0.793103448	2494.582504	131577.7511
27	0.379310345	0.448275862	0.344827586	14.47585052	79.42826348
28	0.586206897	0.413793103	0.068965517	1.410908205	1.750476078
29	0.75862069	0.827586207	0.965517241	12912.99135	1612856.021
30	0.517241379	0.793103448	0.310344828	18.18332571	83.89410031

TABLE II
MODEL SUMMARY STATISTICS FOR R_1

Source	Std.Dev.	R-Squared	Adjusted R-Squared	Prediceted R-Squared	Press	
Linear	461.18	0.5044	0.4398	0.2515	7.388E+006	
2FI	441.99	0.6042	0.4854	0.1005	8.878E+006	
Quadratic	313.78	0.8304	0.7407	0.4248	5.677E+006	Suggested
Cubic	182.94	0.9763	0.9118	0.4198	5.727E+006	Suggested

The second step of the model analysis is to build the response surfaces for the two functions R_1 and R_2 and to run an opptimization for the model in order to define the best approximation to the utility function.

TABLE III
MODEL SUMMARY STATISTICS FOR R_2

Source	Std.Dev.	R-quared	Adjusted R-Suared	Prediceted R-Squared	Press	
Linear	29120.92	0.4202	0.3445	0.1484	2.864E010+	
2FI	29731.40	0.4744	0.3168	-0.1457	3.854E+010	
Quadratic	22263.21	0.7495	0.6169	0.2092	2.66E+010	Suggested
Cubic	14130.46	0.9584	0.8457	0.0114	3.326E+010	Suggested

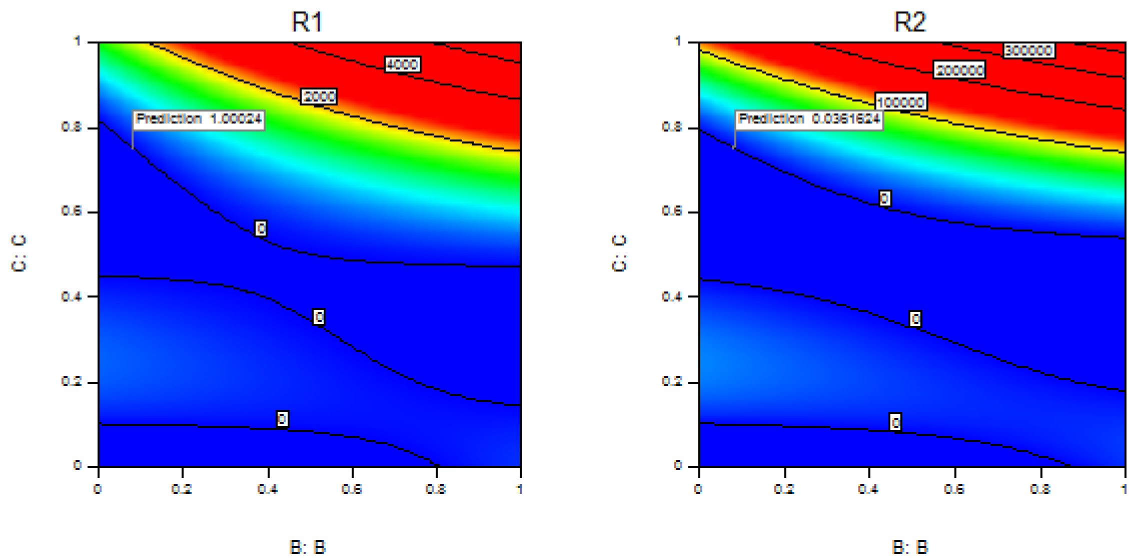

Fig. 1. Optimization of the Response Surfaces

Figure 1 & Figure 2 provide visual display for the response surfaces for the response variables R_1 and R_2. We perfomed several analysis to estimate the best approximation. The analysis included residual analysis and the generation of several diagnostic tools i.e a normal plot, a Residual vs Predicted plot, a Residual vs Run plot or Residual vs Actual plot. The model didn't show any major violation of the provided assumptions. Data analyzed were significant for all the diagnostics run.

Optimization of the response surfaces R_1 and R_2 helped in determining a, b and c values achieving $R_1 \simeq 0$ and $R_2 \simeq 1$. Four combinations of (a, b, c) were achieving a Desirability equal to 1: (0.50, 0.56, 0.31) ; (0.50, 0.08, 0.74); (0.50, 0.02, 0.10); (0.5, 0.68, 0.05). The second combination (0.50, 0.08, 0.74) provides the closest approximation to R_1 and R_2, with R_1=*1.003* and R_2=*0.022)*, Figue 1 & 2.
The best estimation u_e for utility function is then expressed as following:

$$u_e = 0.5 + 0.08exp(0.74x) \tag{17}$$

5. CONCLUSION

The paper, presented herein, proposed a framework for estimating utility function. The estimation was achieved with the help of experimental designs namely Latin Hypercube Sampling and Response Surfaces

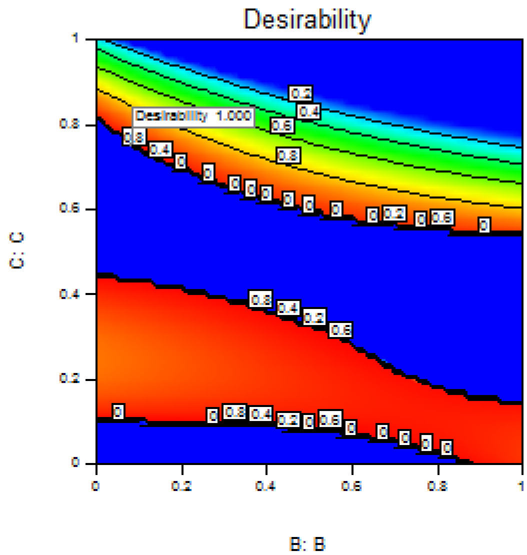

Fig. 2. Desirability

Optimization. An application of the model was shared at the last part of the research work to illustrate the proposed model efficiency. Results were satisfactory. R-square values for the two response variables were interesting and noise level values were low. The distribution of the samples through the experimental space for the two response surfaces was meaningful.
Future work will be conducted to estimate the main utility function for a multi-criteria decision aid problem. The complexity will be more challenging since the model will need to cover each attributes utility function separatly and then estimate the aggregation of these models.

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