EXPONENTIAL INEQUALITIES IN Functional NONPARAMETRICS REGRESSION FOR MIXING PROCESS

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ABSTRACT

This paper establishes exponential inequalities for the probability of the distance between kernel estimator and its means in nonparametric regression problem with mixing variables. We consider an operator equation taking the following form $Y=A\theta(Z)+\varepsilon$, where A is a compact operator.

The goal is to estimate the functional θ when the variable Z is contaminated by measurements errors.

Keywords: convolution linear compact operator**;** kernel estimator; mixing process; non parametric regression.

Introduction

The nonparametric estimation of function is an important tool for analyzing data, as well as inferential statistics in graphical visualizations. To this end, confidence interval and uniform confidence bands are often used in statistics. Starting with the work of Bickel and Resenblatt [1], who built confidence bands for the kernel estimator of a density function of independent and identically distributed observations. Since then, their methods were much developed in the context of estimation of density function and regression function.

In the context of inverse problems, such as calibration (inverse regression) estimator deconvolution kernel and Nadaraya-Wantson are often used.

In this work, we consider a convolution operator equation of the form

$$\mathbf{Y=A\theta(Z)+ \varepsilon}$$

The goal is to estimate the functional when the variable Z is contaminated by measurement errors. A: H→H is a convolution linear compact operator, given by

$$\mathbf{A(s)=\int \Psi(s-t)\theta(t)dt} \qquad \textbf{(1)}$$

Where is a known density function.

2. Notations and preliminaries

We consider a sample of size n, $(Y_1, Z_1),\cdots,(Y_n, Z_n)$ of variable (Y,Z) satisfying the equation (1), assumed α-mixing. Assume that the sequence of regression errors $(\varepsilon_k)_{k\geq 1}$ are identically distributed random variables with zero mean and finite variance ^{2}. We assume that for all $k=1,\cdots,n$, Z_k is given by $X_k=Z_k+\delta_k$ which δ_k is a mistake to contamination.

We denote by φ_X the characteristic function of the variable X or the Fourier transform of the density function and ψ is the density function of $-\delta$.

The sequence of random variables $(Y_k)_{k\geq 1}$ is assumed to be mixing. This amounts to assume that the sequence of random variables $(\varepsilon_k)_k$ is mixing.

We propose a kernel type estimator of θ, we assume that the Fourier transform of the density function ψ is such that $\varphi_\psi(\omega)\neq 0$ for all $\omega\in\mathbb{R}$ and the Fourier transform of the kernel k has compact support, the deconvolution kernel estimator of is given by

$$\boldsymbol{\theta_n(x)=(1/2)\int_R \exp(-i\omega x)\varphi_k(h\omega)\varphi_g(\omega))/\varphi_\Psi(\omega)d\omega} \qquad \textbf{(3)}$$

3. RESULTS AND DISCUSSION

· the following lemma gives the asymptotic expression of the variance $\theta_n(x)$, in the case of highly mixing variables.

Lemma : Under some assumptions, we have,

$$\sum\sum cov(Y_k,Y_l)K(((x-z_k)/h),h)K(((x-z_l/h),h))=o(\sum Var(Y_kK(((x-z_k)/h),h)))$$

· $|E(\theta_n(x))-\theta(x)|=o(h^j)$.

· Exponential inequality of the probability of the distance between the estimator θ (x) given by (3) of θ(x) and $E(\theta_n(x))$.

We put

$$Z_n(x)=((\sqrt{n}h^{1+\beta}a_n)/\sigma)[\theta_n(x)-E(\theta_n(x))]$$

Theorem : Under some assumptions, for all >0, $\forall(n\geq 4)$ $\forall k\in\{1,\cdots,[(n/2)-1]\}$ $\forall\in]0,\cdots,(4kMe)^{-1}[$ we have

$$P(|Z_n(x)|>\varepsilon)\leq 2\exp(-n\eta[(\sigma\varepsilon/(h^\beta\sqrt{n}))-6\eta e(V+8M^2\sum\alpha_l)-((2\sqrt{e})/k\eta)\alpha_k^{2e/3n}])$$

Corollary : Under assumptions of the theorem, and if in addition, the mixing coefficients α_k satisfy $\alpha_k\leq a^k$ for a>0 and 0<ρ<1, then, for all $\gamma\in]0,1[$, there exists η_γ such that for all $n\geq\eta_\beta$, we have

$$P((1/n)|Z_n(x)|>\varepsilon)\leq 2\exp(-n^{1-\gamma/2}(\sigma\varepsilon)/(h^\beta 5de)))$$

4. CONCLUSION

Exponential inequality helps build confidence bands of the deconvolution kernel estimator $\theta_n(x)$.

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