Privatization and Productive Efficiency in an International Stackelberg Mixed Duopoly

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Abstract

We consider a Stackelberg mixed market in which a state-owned welfare-maximizing (domestic) public firm competes against a profit-maximizing (foreign) private firm. We suppose that the domestic firm is less efficient than the foreign private firm. However, the domestic firm can lower its marginal costs by conducting cost-reducing R&D investment. We examine the impacts of privatization on decisions upon cost-reducing R&D investment by the domestic firm and how these affect the domestic welfare. We show that privatization lowers productive efficiency of the domestic firm, regardless of whether the domestic firm is leader or follower. Furthermore, we also show that privatization of the domestic public firm deteriorates the domestic social welfare, regardless of whether the domestic firm is leader or follower.

Keywords
Operations research; Game Theory; Stackelberg model; privatization

1. Introduction

Competition between public and private firms exists in a range of industries like telecommunications, electricity, natural gas, airlines industries, as well as services including hospitals, banking and education. Some authors studied mixed oligopolies under Cournot competition (firms move simultaneously) and some others considered Stackelberg models (firms move sequentially). Pal (1998) studied a Stackelberg mixed duopoly with homogenous goods, and Matsumura (2003) studied a Stackelberg mixed duopoly where a public firm competes with a foreign private firm.
Many studies about international mixed markets assume that all the firms have identical technologies (see, for instance, Fjell and Heywood (2002) Nishimori and Ogawa (2002), using a mixed oligopoly model, investigate the effects of deregulation on the cost-reducing incentives of a public firm. Lin and Ogawa (2005) show that while a private firm carries out the cost-reducing investment, a public firm does not have an incentive to reduce its costs as long as the market share of the private firm is sufficiently large.
There are many other papers on the field of international mixed models (see, for example, Ferreira and Ferreira (2013, 2014) and Fjell and Pal (1996) among others). Tomaru (2007) analyzed, in a Cournot model, how decision-making upon cost-reducing R&D investment by a domestic public firm is affected by privatization when competing in the domestic market with a foreign private firm. He shows that privatization of the domestic public firm lowers productive efficiency and deteriorates domestic social welfare. In this paper, we examine the same question but in a Stackelberg formulation instead of Cournot. We show that the results are qualitatively similar, if the firms take their decisions either simultaneously or sequentially.

2. Description of the model

We consider a mixed duopoly market with one domestic firm and one foreign firm. The foreign firm is a profit-maximizing firm (i.e. a private firm), and the domestic firm is social-welfare-maximizing firm (i.e. a public firm). The two firms are assumed to sell homogeneous products.

The inverse demand function for the product in the domestic market is given by $P = 1 - \left(q_d + q_f\right)$, where P is the price, q_d is the output of the domestic firm and q_f is the output of the foreign firm. Both firms have constant marginal cost technologies c_i, with $i = d, f$. For simplicity, we assume that $c_d = c \in \left(0, 1/3\right)$ and $c_f = 0$, so that the domestic public firm is less efficient than the foreign private firm. We consider that the domestic firm can lower its marginal production cost by conducting cost-reduction R&D investment. We denote by t the amount of that reduction. The marginal production cost of the firm declines with the R&D investment. The effective marginal production cost will, then, be given by $c - t$. As in (2007), we assume a cost function TC for R&D investment given by $TC(t)) = kt^2/2$, with $k > 0$.

The profit of the foreign private firm is defined by

$$\pi_f = \left(1 - q_d - q_f\right) q_f$$

while the domestic firm's profit is given by

$$\pi_d = \left(1 - q_d - q_f - \left(c - t\right)\right) q_d - \frac{1}{2} k t^2 .$$

Domestic social welfare W is the summation of the consumer surplus and the domestic firm's profit, that is,

$$W = \frac{1}{2}\left(q_d + q_f\right)^2 + \pi_d .$$

The model is a three-stage game. In the first stage, the domestic public firm chooses the amount t of cost-reducing. Then, the firms compete à la Stackelberg. We will consider two cases:

- Case I - The domestic firm is the leader;
- Case II - The foreign firm is the leader.

We solve the games by backward induction. Finally, we assume that $k > 4/3$, which makes optimization problems in the first stage sensible.

3. Case I - The domestic firm is the leader

First, in this section, we examine the game where the domestic firm is the leader. In the competition of quantity levels, the domestic firm chooses its output volume q_d, and the foreign firm chooses q_f after having observed the quantity level q_d.

3.1 Mixed duopoly

Here, we suppose that the domestic firm is a public firm. Given the domestic public firm's level t of cost-reduction and the output level q_d, the foreign firm maximizes its own profit π_f, which yields

$$q_f = \frac{1 - q_d}{2} .$$

The domestic public firm chooses the output level q_d that maximizes the social welfare W, which gives

$$q_d\left(t\right) = \frac{3 - 4c + 4t}{3} .$$

So,

$$q_f\left(t\right) = \frac{2\left(c - t\right)}{3} .$$

Then, in the first stage, taking into account this result, the domestic public firm maximizes social welfare:

$$\max_t W(t) = \frac{3-6(c-t)-3kt^2+4(c-t)^2}{6}, \text{ s.t. } c \geq t .$$

Solving this problem, we obtain

$$t_I^M = \begin{cases} c, & \text{if } k < \frac{1}{c} \\ \frac{3-4c}{3k-4}, & \text{otherwise} \end{cases}.$$

Then, the quantity produced by the domestic public firm is given by

$$q_{d,I}^M = \begin{cases} 1, & \text{if } k < \frac{1}{c} \\ \frac{k(3-4c)}{3k-4}, & \text{otherwise} \end{cases},$$

the quantity produced by the foreign firm is given by

$$q_{f,I}^M = \begin{cases} 0, & \text{if } k < \frac{1}{c} \\ \frac{2(kc-1)}{3k-4}, & \text{otherwise} \end{cases},$$

and the aggregate quantity is given by

$$Q_I^M = q_{d,I}^M + q_{f,I}^M = \begin{cases} 1, & \text{if } k < \frac{1}{c} \\ \frac{k(3-2c)-2}{3k-4}, & \text{otherwise} \end{cases}.$$

Thus, social welfare is given by

$$W_I^M = \begin{cases} \frac{1-kc^2}{2}, & \text{if } k < \frac{1}{c} \\ \frac{k(4c^2-6c+4)-1}{2(3k-4)}, & \text{otherwise} \end{cases}.$$

3.2 Privatized duopoly

Now, we consider the case where both domestic and foreign firms are private firms. Given the domestic firm's level t of cost-reduction and the output level q_d, the foreign firm maximizes its own profit π_f, which yields

$$q_f = \frac{1-q_d}{2}.$$

The domestic firm chooses the output level q_d that maximizes its own profit π_d, which gives

$$q_d(t) = \frac{1+2c-2t}{2}.$$

So,

$$q_f(t) = \frac{1+2c-2t}{4}.$$

In the first stage, the domestic firm solves the following maximization problem

$$\max_t \pi_d(t) = \frac{1-4(c-t)-4kt^2+4(c-t)^2}{8}, \text{ s.t. } c \geq t .$$

Solving the problem, we obtain

$$t_I^P = \begin{cases} c, & \text{if } k < \dfrac{1}{2c} \\ \dfrac{1-2c}{2(k-1)}, & \text{otherwise} \end{cases}.$$

Then, the quantity produced by the domestic firm is given by

$$q_{d,I}^P = \begin{cases} \dfrac{1}{2}, & \text{if } k < \dfrac{1}{2c} \\ \dfrac{k(1-2c)}{2(k-1)}, & \text{otherwise} \end{cases},$$

the quantity produced by the foreign firm is given by

$$q_{f,I}^P = \begin{cases} \dfrac{1}{2}, & \text{if } k < \dfrac{1}{2c} \\ \dfrac{k(1+2c)-2}{4(k-1)}, & \text{otherwise} \end{cases},$$

and the aggregate quantity is given by

$$Q_I^P = q_{d,I}^P + q_{f,I}^P = \begin{cases} \dfrac{3}{4}, & \text{if } k < \dfrac{1}{2c} \\ \dfrac{k(3-2c)-2}{4(k-1)}, & \text{otherwise} \end{cases}.$$

Thus, social welfare is given by

$$W_I^P = \begin{cases} \dfrac{13-16kc^2}{32}, & \text{if } k < \dfrac{1}{2c} \\ \dfrac{k^2\left(20c^2+24c+13\right)-4k\left(4c^2+7c+4\right)+4}{32(k-1)^2}, & \text{otherwise} \end{cases}.$$

4. Case II - The foreign firm is the leader

Now, consider the case where the foreign firm is the leader. In the competition of quantity levels, the foreign firm chooses its output volume q_f, and the domestic firm chooses q_d after having observed the quantity level q_f.

4.1 Mixed duopoly

Suppose that the domestic firm is a public firm. Given the domestic public firm's level *t* of cost-reduction and the output level q_f, the domestic firm maximizes the social welfare *W*, which yields

$$q_d(t) = 1 - c + t.$$

The foreign private firm chooses the output level q_f that maximizes its own profit π_f, which gives

$$q_f(t) = \frac{c-t}{2}.$$

Then, in the first stage, taking into account this result, the domestic public firm maximizes social welfare:

$$\max_t W(t) = \frac{4-8(c-t)-4kt^2+5(c-t)^2}{8}, \text{ s.t. } c \geq t.$$

Solving this problem, we obtain

$$t_{II}^{M} = \begin{cases} c, & \text{if } k < \dfrac{1}{c} \\ \dfrac{4-4c}{4k-5}, & \text{otherwise} \end{cases}$$

Then, the quantity produced by the domestic public firm is given by

$$q_{d,II}^{M} = \begin{cases} 1, & \text{if } k < \dfrac{1}{c} \\ \dfrac{4k(1-c)-1}{4k-5}, & \text{otherwise} \end{cases},$$

the quantity produced by the foreign firm is given by

$$q_{f,II}^{M} = \begin{cases} 0, & \text{if } k < \dfrac{1}{c} \\ \dfrac{2(kc-1)}{4k-5}, & \text{otherwise} \end{cases},$$

and the aggregate quantity is given by

$$Q_{II}^{M} = q_{d,II}^{M} + q_{f,II}^{M} = \begin{cases} 1, & \text{if } k < \dfrac{1}{c} \\ \dfrac{k(4-2c)-3}{4k-5}, & \text{otherwise} \end{cases}.$$

Thus, social welfare is given by

$$W_{II}^{M} = \begin{cases} \dfrac{1-kc^2}{2}, & \text{if } k < \dfrac{1}{c} \\ \dfrac{k(5c^2-8c+4)-1}{2(4k-5)}, & \text{otherwise} \end{cases}.$$

4.2 Privatized duopoly

Here, we consider the case where both domestic and foreign firms are private firms. Given the domestic firm's level t of cost-reduction and the output level q_f, the domestic firm maximizes its own profit π_d, which yields

$$q_d = \frac{1-c+t-q_f}{2}.$$

The foreign firm chooses the output level q_f that maximizes its own profit π_f, which gives

$$q_f(t) = \frac{1+c-t}{2}.$$

So,

$$q_d(t) = \frac{1-3c+3t}{4}.$$

In the first stage, the domestic firm solves the following maximization problem

$$\max_t \pi_d(t) = \frac{1-6(c-t)-8kt^2+9(c-t)^2}{16}, \text{ s.t. } c \geq t$$

Solving the problem, we obtain

$$t_{II}^{P} = \begin{cases} c, & \text{if } k < \dfrac{3}{8c} \\ \dfrac{3(1-3c)}{8k-9}, & \text{otherwise} \end{cases}.$$

Then, the quantity produced by the domestic firm is given by

$$q_{d,II}^{P}=\begin{cases}\dfrac{1}{4}, & \text{if } k<\dfrac{3}{8c}\\ \dfrac{2k(1-3c)}{8k-9}, & \text{otherwise}\end{cases},$$

the quantity produced by the foreign firm is given by

$$q_{f,II}^{P}=\begin{cases}\dfrac{1}{2}, & \text{if } k<\dfrac{3}{8c}\\ \dfrac{4k(1+c)-3}{8k-9}, & \text{otherwise}\end{cases},$$

and the aggregate quantity is given by

$$Q_{II}^{P}=q_{d,II}^{P}+q_{f,II}^{P}=\begin{cases}\dfrac{3}{4}, & \text{if } k<\dfrac{3}{8c}\\ \dfrac{2k(3-c)-6}{8k-9}, & \text{otherwise}\end{cases}.$$

Thus, social welfare is given by

$$W_{II}^{P}=\begin{cases}\dfrac{11-16kc^{2}}{32}, & \text{if } k<\dfrac{3}{8c}\\ \dfrac{4k^{2}(19c^{2}-18c+11)-3k(27c^{2}-26c+27)+36}{2(8k-9)^{2}}, & \text{otherwise}\end{cases}.$$

5. Comparisons

In this section, we are going to compare the levels of cost-reducing R&D investment and the domestic social welfare in the different models previously analysed. In particular, we focus our attention on the effects of privatization of a domestic firm in the international Stackelberg models considered. Here, for our analysis, we assume that $k>1/c$. This assumption guarantees that the foreign firm is active.

Theorem 1. For any $c\in(0,1/3)$ and $k>\max\left\{\dfrac{1}{c},\dfrac{4}{3}\right\}$, we have that

$$t_{I}^{M}>t_{I}^{P} \quad \text{and} \quad t_{II}^{M}>t_{II}^{P}.$$

Theorem 1 states that, in the international Stackelberg competition considered, privatization lowers productive efficiency of the domestic firm, regardless of whether the domestic firm is leader or follower.

Theorem 2. For any $c\in(0,1/3)$ and $k>\max\left\{\dfrac{1}{c},\dfrac{4}{3}\right\}$, we have that

$$W_{I}^{M}>W_{I}^{P} \quad \text{and} \quad W_{II}^{M}>W_{II}^{P}.$$

Theorem 2 states that, in the international Stackelberg competition considered, privatization of the domestic public firm deteriorates the domestic social welfare, regardless of whether the domestic firm is leader or follower.

6. Conclusions

We have analysed the effects of trade with a foreign firm and privatization of the domestic public firm on an incentive for the domestic firm to reduce costs by undertaking R&D investment, when the firms move sequentially (Stackelberg model). We stated that the domestic privatized firm lose its incentive to raise the level of R&D investment, regardless of whether the domestic firm is leader or follower. Furthermore, this privatization leads to deterioration of the domestic social welfare, regardless of whether the domestic firm is leader or follower.

We will pursue further research on similar models but on price competition.

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Biographies

Fernanda A. Ferreira is Adjunct Professor in the Department of Mathematics at the School of Hospitality and Tourism of Polytechnic Institute of Porto, Portugal. She holds a BS in Mathematics and a PhD in Applied Mathematics from the University of Porto. She obtained also a Diploma of Advanced Studies in Statistics and Operations Research from Vigo University. A member and Coordinator of the Applied Management Research Unit (UNIAG), her publications, mostly journal and conference papers, cover the research interest areas of industrial organization, game theory and tourism (ORCID ID: orcid.org/0000-0002-1335-7821). Co-author of two books published with arbitration scientific in the Mathematics area. Supervised many dissertations in the areas of Game Theory and Management. She also organizes Symposia on "Operational Research and Applications" in several International Conferences and has collaborated as reviewer with several journals.

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