Design of multiattribute EWMA chart

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Abstract

Multiattribute control charts have been widely found to be more effective process control than separate uniattribute control charts in different applications. Based on an available statistical model which was developed for designing multiattribute np (Mnp) chart, in this study we proposed a multiattribute Exponentially Weighted Moving Average (MEWMA) chart. We also compared the MEWMA chart with the optimal version of the (Mnp) chart. The comparison results indicate that the proposed MEWMA chart outperforms the Mnp chart by about 65% in terms of the average number of defectives.

Keywords
Control chart, Statistical Process Control, Multiattribute EWMA chart, Average number of defectives

1. Introduction

In most multiattribute processes, it is required to simultaneously control several attribute quality characteristics as the quality of a product depends on all of them. The practice is to maintain a separate uniattribute control chart for each characteristic. Unfortunately, this practice is neither efficient nor effective. It is more practical and economical to use a single multiattribute control chart for such applications. The multiattribute chart provides a summary measure of all characteristics and more effective process control than separate uniattribute control charts. Moreover, it reduces errors that may result from the testing procedure of the separate uniattribute charts.

In the literature, while a lot of attention has been paid to multivariate control charts for variables, little work has been reported on multiattribute control charts. Lowry and Montgomery (1995) gave a review on control charts for multivariate quality control. It is found that a multivariate control chart is more effective in monitoring a multivariate process than separate univariate control charts. Moreover, they discussed some topics such as the use of principal components and the regression adjustment of variables in multivariate quality control. Lu *et al.* (1998) developed a multiattribute np chart to deal with multiattribute processes. This chart uses the weighted sum of all the quality characteristics as the plotted statistics. Jolayemi (1999) developed a multiattribute np chart (abbreviated as the Mnp chart in this article) for monitoring independent attribute characteristics. His study revealed that the Mnp chart is more cost-effective and more efficient than the ordinary np charts. The dependency between attributes was considered in developing some other multiattribute control charts. For example, Limam and Taleb (2003) applied Hotelling's T^2 control chart to multiattribute processes. Gadre and Rattihalli (2005) proposed some multiattribute control charts based on group inspection to identify process deterioration. Niaki and Abbasi (2007) proposed a methodology to monitor high-quality multiattribute processes. Stefatos and Hamza (2008) developed a multivariate control chart for fault detection using robust statistics and principal component analysis. Niaki and Abbasi (2008) designed a neural network to monitor either the proportions of several types of product nonconformities (instead of using several np charts) or the number of different types of defects (instead of using several c charts). Chiu and Kuo (2008) constructed a control chart to monitor multivariate Poisson count data. Mukhopadhyay (2008) proposed a multiattribute control chart by expanding the concept of "Mahalanobis Distance" and compared its sensitivity with the individual p charts and overall p chart. Khoo *et al*. (2009) proposed a multivariate synthetic control chart for skewed populations based on the weighted standard deviation method. Aparisi and Luna (2009) developed the synthetic-T^2 control chart which consists of a CRL chart and a Hotelling's T^2 chart. Topalidou and Psarakis (2009) conducted a comprehensive review on the multiattribute quality covering the design, performance and applications of multiattribute control charts. Recently, Yang and Yeh (2011) proposed cause selecting control charts to monitor two dependent process stages with attributes data. Haridy *et al*. (2012) introduced a new multiattribute chi-squared chart to monitor the processes in which more than one type of defects exist on the non-conforming item.

In this article, a new multiattribute Exponentially Weighted Moving Average (MEWMA) chart is proposed to monitor the multiattribute processes in which the product has different types of defects. Furthermore, the proposed MEWMA chart is compared with the optimal version of the (Mnp) chart proposed by Jolayemi (1999). The sampling inspection is adopted in this article. When taking a sample of size n, the units are inspected one by one. A unit is classified as nonconforming as soon as one defect pertaining to any of the k attributes is detected, and the inspection on this unit is terminated (Mukhopadhyay 2008).

The rest of this article proceeds as follows. Firstly, the assumptions and operating conditions of the MEWMA is briefly presented. Then, the implementation of the proposed MEWMA chart is outlined followed by the optimization design of this chart. Next, a comparison between the MEWMA and Mnp charts is conducted.

2. Assumptions and operating conditions

In multiattribute processes, in which the product has different types of defects, a single multiattribute chart can be used to monitor the number d of nonconforming units found in a sample of size n with respect to the k attributes (or defect types). Here, a unit is classified as nonconforming if any of the k types of defects is found. Jolayemi (1999) proposed the Mnp chart as an on-line monitoring technique for multiattribute processes. In this chart, the overall fraction nonconforming p can be estimated as follows:

$$p = \frac{1}{k}\sum_{j=1}^{k} p_j \tag{1}$$

where p_j represents the fraction nonconforming of the j^{th} attribute. The random number d follows a binomial distribution b(nk, p) with a sample size n and an overall fraction nonconforming p (calculated by Equation (1)). As an extension of the uniattribute np chart, the Mnp chart is more effective for detecting large p shifts. The process is considered to be in control if the number d of nonconforming units with respect to all the attributes satisfies $d \leq UCL$. Here, UCL is the upper control limit of the Mnp chart. However, if $d > UCL$, then an upward p shift is signaled.

In this article, the studies are conducted based on the statistical model developed by Jolayemi (1999) for the design of the Mnp chart. This model is established based on the following assumptions and operating conditions:

(1) If a process is monitored with respect to k independent attributes, $x_1, x_2, \ldots\ldots, x_k$ and each of them follows a binomial distribution with a same sample size n and different fraction nonconforming p_j (i.e., b(n, p_j) where j = 1, ….., k). If p_j of each independent attribute is known, then the sum (or the convolution) of the nonconforming units found in a sample of size n with respect to all k attributes is well approximated by a binomial distribution b(nk, p) where p is equal to the average of $p_1, p_2, \ldots\ldots, p_k$ (Equation (1)).
(2) Samples of size n are taken for every h hour from the process and all units in the sample are inspected with respect to all k attributes. The focus of this research is to detect increasing p shifts as attribute charts are commnly used to deterioration in quality (Lucas 1985, Reynolds and Stoumbos 1999).

In this article, it is assumed that the random number d (the number of nonconforming units found in a sample with respect to all k attributes) follows a binomial distribution with known overall in-control fraction nonconforming p_0. The value of p_0 can be obtained from the individual in-control fraction nonconforming p_{0j} of the k attributes based on Equation (1) as follows:

$$p_0 = \frac{1}{k}\sum_{j=1}^{k} p_{0j} \tag{2}$$

When a process shift occurs, the overall fraction nonconforming p will change to:

$$p = \delta \times p_0 \tag{3}$$

The index δ ($1 \leq \delta \leq \delta_{max}$) indicates the p shift in terms of p_0 where δ_{max} is the maximum shift in the overall fraction nonconforming. The process is in control when $\delta = 1$ (i.e., $p = p_0$) and out of control when ($1 < \delta \leq \delta_{max}$) with a maximum fraction nonconforming at $\delta = \delta_{max}$ (i.e., $p = p_{max} = \delta_{max} \times p_0$).

3. Implementation of the MEWMA chart

The MEWMA chart has two charting parameters: the control limit H and weighting parameter λ ($0 < \lambda \leq 1$). To detect an upward p shift, a statistic C_t is updated and plotted for the tth sample in an MEWMA chart:

$$C_0 = 0$$
$$C_t = \lambda(d_t - d_0) + (1-\lambda)C_{t-1} \tag{4}$$

where d_t is the number of nonconforming units found in the tth sample, d_0 is the in-control value of $d_t = n \times p_0$ (the product of the sample size n and the in-control fraction nonconforming p_0). An MEWMA chart is implemented as follows:

(1) Initialize the statistic C_0 in Equation (4) as zero and set $t = 1$.
(2) Take a sample of n units.
(3) Determine the number of nonconforming units, d_t, in this sample with respect to all k attributes.
(4) Calculate C_t by Equation (4)

$$C_t = \lambda(d_t - d_0) + (1-\lambda)C_{t-1} \tag{5}$$

(5) If $C_t \leq H$, this sample is a conforming one and go back to step (2) to take the next sample. Otherwise (i.e., if $C_t > H$), the current sample is nonconforming and go to step (6).
(6) Stop the process immediately for further investigation.

4. Design of the MEWMA chart

The design of an MEWMA control chart requires the following five specifications:
(1) The number k of the attribute characteristics,
(2) The allowable minimum value τ of ATS_0,
(3) The individual in-control fraction nonconforming p_{0j} of the k attributes (where j = 1, …., k), (4) The maximum shift δ_{max} in overall fraction nonconforming (see Equation (3)), and
(5) The sample size n.

The value of τ is determined with regards to the tolerable false alarm rate. The value of p_{0j} for each attribute is usually estimated from the data observed in pilot runs. The maximum shift δ_{max} is required for the calculation of the Average Number of Defectives (*AND*) which will be discussed shortly. The value of δ_{max} may be chosen based on the knowledge about a process (e.g., the maximum possible p shift in a process) or taken as the shift range the users are interested in. The sample size n is usually determined according to the available resources such as manpower and measurement instruments.

In this article, the *AND* is used to measure the overall performance of a control chart (Haridy *et al.* 2013).

$$AND = \frac{N}{\delta_{max} - 1} \sum_{\delta=2}^{\delta_{max}} p_\delta ATS(p_\delta) \tag{6}$$

In Equation (6), N may be removed because it is a constant and has no effect on the performance comparison and the optimal solution. The index *AND* directly relates the chart performance to the economic outcome. That is to say, a chart that produces smaller *AND* (less average number of nonconforming units) for different out-of-control p values is thought to be more economical. Meanwhile, *AND* can be considered as a weighted average of *ATS* that uses p_δ as the weight. If *AND* is used as the objective function to be minimized, then the larger the p_δ, the smaller the corresponding $ATS(p_\delta)$ will be resulted from the optimization design.

In this article, *AND* will be used as the objective function for the design of the MEWMA chart using the following optimization model:

Objective: Minimize *AND*
Constraint: $ATS_0 \approx \tau$ (7)
Design variables: H, λ.

The objective of the optimization design is to identify the optimal values of H and λ that minimize *AND* over a shift range of ($1 < \delta \leq \delta_{max}$) and meanwhile ensure that $ATS_0 \geq \tau$. The minimization of *AND* in turn results in a smaller out-of-control *ATS* over the entire range of p shifts. The optimization design is carried out as follows:

(1) Specify parameters τ, δ_{max}, n and the in-control p_{0j} of the individual k attributes (j = 1, ….., k).
(2) Initialize a variable AND_{min} as a very large number, say 10^7 (AND_{min} is used to store the minimum value of *AND*).
(3) Search λ with a step size within the range of ($0 < \lambda < 1$). For a given value of λ,
  (3.1) determine the control limit H that satisfies the constraint of ($ATS_0 \approx \tau$).

(3.2) When the values of the two charting parameters, λ and H, are preliminarily determined, the objective function AND is calculated by Equation (6).

(3.3) If the calculated AND is smaller than the current AND_{min}, set $AND_{min} = AND$ and the current values of λ and H are stored as a temporary optimal solution.

(4) At the end of the search, the optimal MEWMA chart that produces the minimum AND and satisfies the constraint ($ATS_0 \approx \tau$) is identified. The corresponding optimal values of λ and H are also identified.

5. Example

This illustrative example concerns the fraction of nonconforming data for four different types of defects in printed circuit boards (PCB). The defects are outgassing, joint contamination, solder balling and mask discoloration (k = 4). Based on the historical records, the in-control fractions nonconforming of the outgassing, joint contamination, solder balling and mask discoloration are estimated as 0.003, 0.007, 0.004 and 0.006, respectively. Hence, p_0 can be determined as follows (Equations (2)):

$$p_0 = \frac{1}{k}\sum_{j=1}^{k} p_{0j} = \frac{0.003+0.007+0.004+0.006}{4} = 0.005 \tag{8}$$

The quality engineer is interested in detecting a maximum shift δ_{max} is set to 8. The allowable minimum τ is set to 800. A sample size n of 100 units and a sampling interval h of 1 hour are selected based on the available manpower and working shift. The specifications are summarized as follows:

p_{01} = 0.003, p_{02} = 0.007, p_{03} = 0.004 and p_{04} = 0.006, individual in-control fraction nonconforming of the four defects.
δ_{max} = 8, maximum shift in fraction nonconforming.
τ = 800, allowable minimum in-control average time to signal.
n = 100, sample size.
h = 1, sampling interval.

A computer program has been developed to carry out the designs of the Mnp and MEWMA charts. The charting parameters of the two charts as well as their AND and AND/AND_{MEWMA} values are listed below:

Mnp chart: UCL = 7, AND = 0.0497, AND_{Mnp} / AND_{MEWMA} = 1.65.
MEWMA chart: H = 3.91, λ = 0.38, AND = 0.0301.

Table 1: Comparison of the Two Charts in the Example

δ	*ATS*	
	Mnp chart	MEWMA chart
1	944.84870	799.86920
2	19.09940	7.17780
3	3.42560	2.18810
4	1.33020	1.23160
5	0.77770	0.84040
6	0.58920	0.66360
7	0.53120	0.56740
8	0.50980	0.52610
9	0.50250	0.50890
10	0.50060	0.50240

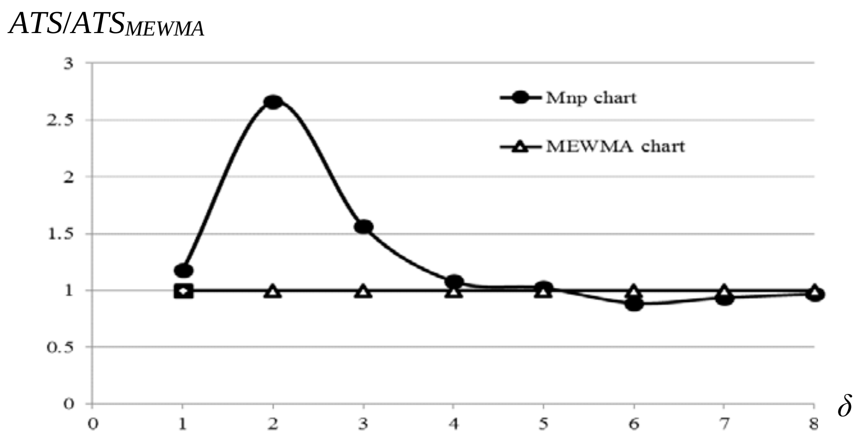

Figure 1: Normalized *ATS* of the two Charts in the Example

In this example, in terms of *AND*, the MEWMA chart reduces the average number of defectives by 65% compared with the Mnp chart. The values of the in-control ATS_0 (where $\delta = 1$) and out-of-control *ATS* (where $1 < \delta \leq 8$) of the two charts are shown in Table 1. The curves of the normalized *ATS* (i.e., ATS/ATS_{MEWMA}) of the two charts are illustrated in Figure 1. It can be observed that the MEWMA chart significantly outperforms the Mnp chart when $\delta \leq 5$, while the later slightly outdoes the former for detecting large shifts (when $\delta > 5$).

6. Conclusion

Due to the fast advancement in technology and high competition in industry, monitoring only one quality characteristic is no longer practical. It is required to simultaneously control multiple quality characteristics. There is currently an urgent need for the multattribute control charts to provide a summary measure of all quality characteristics in a multiattribute process and achieve more effective process control than the separate uniattribute charts.

This article presents a new MEWMA chart for monitoring the multiattribute processes. The MEWMA chart is found to outperform the Mnp chart proposed by Jolayemi (1999) in terms of *AND*. It is more effective than the Mnp chart by 65%. The implementation of the MEWMA chart is simple and its optimal charting parameters can be straightforwardly determined using a computer program.

In this article, the studies are conducted based on some assumptions, such as the independency of the attribute defects, and the known in-control fraction nonconforming. It is interesting to make a further study in the future to see how the MEWMA chart will perform when the attribute characteristics are dependent and there is a correlation between them. It is also worthwhile to evaluate the performance of the MEWMA chart when p_0 is estimated and d follows other distributions rather than the binomial distribution.

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