Single Vendor-Buyer Integrated Inventory Model for Deteriorating Items Considering Carbon Emission

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Abstract

In this study, we present an integrated single vendor-buyer inventory model considering deterioration and carbon emission simultaneously. We focus on the emissions from warehousing, transport activity as well as emission from the disposal of the deteriorated items. A numerical example is developed to illustrate the proposed solution procedure. Computational results indicate that by incorporating carbon emissions into the integrated inventory model, we can reduce the total carbon emission and slightly reduce the total supply chain cost. We also show that the integrated inventory model has significantly reduced the total cost and total carbon emissions as compared to the result when the decision is made solely from the buyer’s perspective.

Keywords
Carbon emission reduction; single vendor-buyer; integrated inventory; deteriorating items

1. Introduction

Sustainability is a global issue and has received much attention in supply chain management. Carter & Rodgers (2008) examined the relationship among environmental, social, and economic performance in a supply chain. A sustainable supply chain is advantageous and lead to economic sustainability. Researchers have investigated many aspects of sustainable supply chain, such as sustainable supply chain network design, sustainable supply chain planning, sustainable sourcing, etc. As part of sustainable supply chain management, researchers have also studied the sustainable inventory management.

This study developed a new single vendor-buyer integrated inventory model considering deterioration and carbon emission simultaneously and examined how the integrated vendor-buyer inventory model reduces the carbon emission. We incorporated carbon emission cost into the model based on carbon emission tax mechanism. Carbon emission tax is one mechanism commonly used to promote energy efficiency and the reduction of fossil fuel combustion (Wahab et al., 2011; Bouchery et al., 2012; Benjaafar et al., 2013; Chen et al., 2013; Hovelaque & Bironneau, 2015; Hua et al., 2016). Some countries have implemented this policy and seen some benefits.
2. Literature Review

2.1. Carbon Emission in a Supply Chain

Sundarakani et al. (2008) modelled the carbon emission at each stage of a supply chain to understand and calculate the total carbon transferred from one stage to another in a closed loop supply chain. All the supply chain stages consume energy and produce wastages and carbon emission. Further, Sundarakani et al. (2010) proposed a supply chain design to study the carbon emissions across the supply chain. Carbon emissions can be minimized by reducing inventory and increasing visibility at the distribution level.

Montoya-Torres et al. (2015) used a real case study to validate a framework for measuring carbon footprint in a supply chain. They considered the breadth and depth of the scope by taking into consideration Scope 1 (directly controlled by the company), Scope 2 (indirect emission from business’ value chain) and Scope 3 (other indirect emissions) of the emission sources. The scope of the supply chain emission sources includes the electricity and natural gas at the facilities, product deliveries between facilities, emissions from business travel, waste disposal at the facilities, and other fuel usages.

Bonney & Jaber (2011) pioneered the development of an environmentally responsible inventory model by considering the emission cost from transport activity and waste disposal. Chen et al. (2013) developed an EOQ model by considering the emission from order initiated, unit held in inventory, and from unit purchased. Battini et al. (2014) investigated the emission cost from warehouse operation, waste disposal, and transportation. Hovelaque & Bironneau (2015) developed an EOQ model by considering the emission from warehouse and transport operation. Hua et al. (2016) proposed an inventory model for perishable inventory in which carbon emission comes from inventory holding, shipping, and item deterioration. Our study only considers the Scope 1 of the emission sources focusing on emissions from warehousing, transport activity and from the disposal of deteriorated items.

2.2. Sustainable Integrated Production-Inventory Model

In an integrated inventory model, supply chain partners jointly decide on the inventory variables such as delivery frequency, delivery quantity, and the related time for mutual benefits. Goyal (1977) is one of the pioneers that studied the integrated inventory policy problem. Yang & Wee (2000) relaxed the early assumptions and studied the effects of item deterioration. They found an impressive cost reduction when compared with the buyer’s independent decision. Lee & Kim (2014) extended the integrated inventory model by considering the effect of deterioration and defective items.

In recent years, researchers have considered the sustainable issue in the integrated inventory model. However, most of them focused on the economic and environmental aspects. Wahab et al. (2011) developed an EOQ model considering environmental impact and imperfect items for a coordinated two-level international supply chain. For the same percentage of defective items, the optimal shipment size would be higher when the issue of the environment considered.

Jaber et al. (2013) proposed a supply chain coordination model for one vendor and one buyer supply chain that considered emission in the manufacturing processes. Supply chain coordination minimizes the total system cost with no change in the total emissions and penalty costs. Jauhari et al. (2014) extended the model to consider unequal-sized shipment for imperfect quality items. Zanoni et al. (2014a) considered an emission trading scheme for single vendor-single buyer joint economic lot sizing (JELS) model with vendor managed inventory and consignment stock (VMI-CS). The numerical example indicates a reduction in carbon emission around 9% as well as the total cost reduction. Further, Zanoni et al. (2014b) studied the JELS model for coordinated inventory replenishment decisions considering price and environmentally sensitive demand.

Recently, Bazan et al. (2017) studied a closed-loop supply chain with remanufacturing and VMI-CS. The models also considered carbon emissions and energy cost. Bouchery et al. (2017) compared the cost and carbon emission of a non-coordinated and coordinated two-echelon supply chain. Wangsa (2017) studied carbon emission costs from the perspective of industrial and transport activities that involved Government’s penalty and incentive policies. These policies resulted in an impressive total cost saving, but the integrated decision did not guarantee a total emission saving.

Our model extends the previous works on sustainable integrated vendor-buyer inventory model by considering deteriorating items. Certain deteriorating items such as milk, fruit, and vegetables need a special storage handling (i.e. cold storage) that has increased carbon emissions. Further, deteriorated items may need additional handling and transport for its disposal. Therefore it is important to consider the carbon emission for deteriorating item.
3. Mathematical Model

This study deals with a single-vendor single-buyer integrated model of a single deteriorating item in which the demand rate \( D \) and production rate \( P \) are known and constant. Furthermore, our mathematical model considers the following assumptions:

1. there is only one production cycle per order with multiple equal-sized deliveries;
2. the item has a constant deterioration rate per unit time;
3. the replenishment is instantaneous;
4. deterioration occurs in vendor’s and buyer’s inventory and the deteriorated items have no value;
5. the carbon emission comes from transportation, warehousing and the disposal of the deteriorated item;
   transportation emissions depend on the delivery distance and quantity, while the warehouse emissions depend on the quantity of inventory held;
6. shortage is not allowed;
7. all the costs incorporated are known and constant;
8. the distance traveled from the supplier to the buyer is known.

Notation:
\[ Q \] delivery quantity (units);
\[ n \] number of delivery per order;
\[ R \] production quantity (units; \( R = PT_{v1} \));
\[ \theta \] proportion of the on-hand inventory lost due to deterioration \( (0 \leq \theta < 1) \);
\[ T \] cycle length;
\[ T_{v1} \] production period for vendor in each cycle;
\[ T_{v2} \] non production period for vendor in each cycle;
\[ T_b \] inventory cycle length per delivery for the buyer \( (T_b = T/n) \);
\[ I_v(t) \] vendor’s inventory level at time \( t \);
\[ I_b(t) \] buyer’s inventory level at time \( t \);
\[ o \] buyer’s ordering cost \($/order\);
\[ r_c \] buyer’s receiving cost \($/delivery\);
\[ h_b \] buyer’s holding cost \($/unit/year\);
\[ d_{cb} \] buyer’s deteriorating cost \($/unit\), which is equal to the purchase cost;
\[ s \] vendor’s setup cost \($/order\);
\[ h_v \] vendor’s holding cost \($/unit/year\);
\[ d_v \] vendor’s deteriorating cost \($/unit\);
\[ t_f \] fixed transportation cost per delivery for vendor \($\);
\[ t_v \] variable transportation cost for vendor, which equal to fuel price \($/litre\);
\[ d \] distance travelled from the supplier to the buyer \((km)\);
\[ t_x \] carbon emission tax \($/kgCO_2\);
\[ V \] vehicle standard emission from fuel consumption \((kgCO_2/litre)\);
\[ c_1 \] vehicle fuel consumption when empty \((litre/km)\);
\[ c_2 \] additional vehicle fuel consumption per ton of payload \((litre/km/ton)\);
\[ e_1 \] carbon emission cost from vehicle \($/km\);
\[ e_2 \] additional carbon emission cost from transporting one unit item \($/unit/km\);
\[ E \] electricity standard emission \((kgCO_2/kWh)\);
\[ w \] average warehouse energy consumption \((kWh/unit/year)\);
\[ w_e \] carbon emission cost from holding items in warehouse \($/unit/year\);
\[ b \] product weight \((kg/unit)\);
\[ v_{de} \] average emission for the disposal of vendor’s deteriorated item \((kgCO_2/unit)\);
\[ b_{de} \] average emission for the disposal of buyer’s deteriorated item \((kgCO_2/unit)\);
\[ d_{ev} \] carbon emission cost for the disposal of the deteriorated item for vendor \($/unit\);
\[ d_{eb} \] carbon emission cost for the disposal of the deteriorated item for buyer \($/unit\).

We can see the inventory model of the deteriorating items for both the vendor and the buyer in Fig. 1.
3.1. Buyer’s Total Cost Function

Eq. (1) describes the buyer’s total cost per unit time \((TC_b)\). It is a function of the buyer’s ordering cost \((C_o)\), receiving cost \((C_{rc})\), holding cost \((C_{HEb})\) and deteriorating cost \((C_{DEb})\) per unit time:

\[
TC_b = C_o + C_{rc} + C_{HEb} + C_{DEb} \tag{1}
\]

\[
C_o = \frac{o}{T} \tag{2}
\]

\[
C_{rc} = r_{rc} \frac{n}{T} \tag{3}
\]

The buyer’s inventory model is identical to Yang & Wee (2000). The inventory level at any time \(t\) over period \([0, T/n]\) is being described as follow:

\[
dI_b(t) = -DT dt - I_b(t)\theta dt, \text{ therefore } I_b(t) = \frac{D}{\theta} \left(e^{\frac{\theta t}{n}} - 1\right), \text{ for } 0 \leq t \leq \frac{T}{n} \tag{4}
\]

The maximum buyer’s inventory \(Q = I_b(0) = \frac{D}{\theta} \left(e^{\frac{\theta}{n}} - 1\right)\) \tag{5}

Therefore, the buyer’s holding cost per unit time considering both traditional inventory carrying cost and carbon emission cost generated by warehousing is:

\[
C_{HEb} = \left(h_b + w_c\right) \frac{n}{T} \left[\int_0^{\frac{T}{n}} I_b(t) dt\right] = \left(h_b + w_c\right) \frac{n}{T} \left[\frac{D}{\theta}\left(e^{\frac{\theta t}{n}} - 1\right) dt\right] \tag{6}
\]

In this study, buyer’s deteriorating cost per unit time considers both traditional deteriorating cost and carbon emission cost generated by the deteriorated items. Hua et al. (2016) assumed that deteriorated item will emit carbon dioxide, and Bonney & Jaber (2011) considered the cost to dispose waste to the environment.

\[
C_{DEb} = \left(d_{cb} + d_{eb}\right) \frac{n}{T} \left[\int_0^{\frac{T}{n}} I_b(t) dt\right] = \left(d_{cb} + d_{eb}\right) \frac{n}{T} \left[\frac{D}{\theta}\left(e^{\frac{\theta t}{n}} - 1\right) dt\right] \tag{7}
\]

By substituting (2), (3), (6), (7) to (1) we gain:

\[
TC_b = \frac{o}{T} + \frac{r_{rc} n}{T} + \left(h_b + w_c\right) \frac{n}{T} \left[\int_0^{\frac{T}{n}} \frac{D}{\theta}\left(e^{\frac{\theta t}{n}} - 1\right) dt\right] + \left(d_{cb} + d_{eb}\right) \frac{n}{T} \left[\frac{D}{\theta}\left(e^{\frac{\theta t}{n}} - 1\right) - \frac{T}{n}\right] \tag{8}
\]
Considering $\theta \ll 1$, 
\[ TC_v = \frac{o}{T} + r_c n + \left( h_b + w_c \right) \frac{DT}{2n} \left( 1 + \frac{1}{3} \frac{\theta T}{n} + \frac{1}{12} \frac{\theta^2 T^2}{n^2} \right) + \left( d_{cb} + d_{eb} \right) \frac{1}{2} \frac{D \theta T}{n} + \frac{1}{6} \frac{D \theta^2 T^2}{n^2} + \frac{1}{24} \frac{D \theta^3 T^3}{n^3} \]

and by neglecting $\theta^2 T^2$ and more, 
\[ TC_v = \frac{o}{T} + r_c n + \left( h_b + w_c \right) \frac{DT}{2n} \left( 1 + \frac{1}{3} \frac{\theta T}{n} \right) + \left( d_{cb} + d_{eb} \right) \frac{D \theta T}{2n} \]  
(9)

### 3.2. Vendor’s Total Cost Function

Eq. (10) describes the vendor’s total cost per unit time ($TC_v$). It is the function of the vendor’s setup cost ($C_s$), transport cost ($C_{TE}$), holding cost ($C_{HEV}$) and deteriorating cost ($C_{DEV}$) per unit time:
\[ TC_v = C_s + C_{TE} + C_{HEV} + C_{DEV} \]  
(10)

\[ C_s = \frac{s}{T} \]  
(11)

The vendor’s transportation cost consists of a fix transport cost, variable transport cost, and carbon emission cost. Bonney & Jaber (2011) considered the vehicle delivery time to calculate the transport emission. In this study, the emission cost depends on the delivery quantity ($Q$) or the vehicle payload.
\[ C_{TE} = \frac{n}{T} \left( t_f + (2dt_c1 + dt_c2bQ) + (2de_1 + de_2Q) \right) = \frac{n}{T} \left( t_f + (2dt_c1 + dt_c2bI_b(0)) + (2de_1 + de_2I_b(0)) \right) \]  
(12)

By substituting (5) to (12) we gain:
\[ C_{TE} = \frac{n}{T} \left( t_f + \left( 2dt_c1 + dt_c2b \frac{D}{\theta} (e^n - 1) \right) + \left( 2de_1 + de_2 \frac{D}{\theta} (e^n - 1) \right) \right) \]  
(13)

The vendor’s total inventory model is identical to that of Yang & Wee (2000) and Lee & Kim (2014). Eq. (14) consists of total inventory during the production period plus the total inventory during the non-production period minus the inventory of the transported goods at the buyer’s side.
\[ I_v(t) = \int_{0}^{T_1} I_{v1}(t_1)dt_1 + \int_{0}^{T_2} I_{v2}(t_2)dt_2 - n \int_{0}^{T_1} I_b(t)dt \]  
(14)

During the production period $T_{v1}$, 
\[ dI_{v1}(t_1) = P dt_1 - D dt_1 - I_{v1}(t_1) \theta dt_1, \]  
therefore vendor’s inventory level at any time $t$ is
\[ I_{v1}(t_1) = \frac{P-D}{\theta} \left( 1 - e^{-\theta t_1} \right) \quad 0 \leq t_1 \leq T_{v1} \]  
(15)

And during the non-production period $T_{v2}$, 
\[ dI_{v2}(t_2) = -D dt_2 - I_{v2}(t_2) \theta dt_2, \]  
therefore
\[ I_{v2}(t_2) = \frac{D}{\theta} \left( e^{\theta(T_1-t_2)} - 1 \right) \quad 0 \leq t_2 \leq T_{v2} \]  
(16)

Therefore, the vendor’s holding cost per unit time considering both traditional inventory carrying cost and carbon emission cost generated by warehousing is:
\[ C_{HEV} = \frac{(h_b + w_c)}{T} \left( \int_{0}^{T_1} \frac{P-D}{\theta} \left( 1 - e^{-\theta t_1} \right) dt_1 + \int_{0}^{T_2} \frac{D}{\theta} \left( e^{\theta(T_1-t_2)} - 1 \right) dt_2 - n \int_{0}^{T_1} I_b(t)dt \right) \]  
(17)

Similar to Yang & Wee (2000) and Lee & Kim (2014), as $\theta \ll 1$ we use Misra (1975) approximation
\[ T_{v1} \approx \frac{D}{P-D}T_{v2} \left( 1 + \frac{1}{2} \theta T_{v2} \right), \]  
(18)

and as $T = T_{v1} + T_{v2}$, we can derive
\[ T \approx \frac{T_{c1}}{P-D} \left( P + \frac{1}{2} DT_{c2} \right) \]  

(19)

The vendor’s deteriorating cost per unit time considering both traditional deteriorating cost and carbon emission cost generated by deteriorated items is:

\[ C_{CDev} = \frac{d_{v} + d_{w}}{T} \left( PT_{v1} - DT - n \left( I_{b}(0) - D \frac{T_{n}}{n} \right) \right) \]  

(20)

By substituting (4) to (20) we gain:

\[ C_{CDev} = \frac{d_{v} + d_{w}}{T} \left( PT_{v1} - DT - n \left( \frac{D}{\theta} \left( e^{\frac{\theta T_{n}}{n}} - 1 \right) - D \frac{T_{n}}{n} \right) \right) \]  

(21)

By substituting (11), (13), (17), (21) to (10) we gain:

\[ TC_{v} = \frac{s}{T} + \frac{n}{T} \left( T_{f} + \left( 2d_{c}c_{1} + dt_{c2}b \frac{D}{\theta} \left( e^{\frac{\theta T_{n}}{n}} - 1 \right) + \left( 2de_{1} + de_{2} \frac{D}{\theta} \left( e^{\frac{\theta T_{n}}{n}} - 1 \right) \right) + \frac{(h_{v} + w_{e})}{T} \left[ \frac{P-D}{\theta} - \frac{P-D}{\theta^{2}} \left( e^{-\theta T_{v1}} - 1 \right) - \frac{DT_{c2}}{\theta^{2}} - \frac{D}{\theta^{2}} \left( 1 - e^{-\theta T_{c2}} \right) - \frac{DT^{2}}{2n} \left( 1 + \frac{1}{3} \frac{\theta T_{n}}{n} \right) \right] \right) \]  

(22)

\[ \left( \frac{d_{v} + d_{w}}{T} \right) \left( PT_{v1} - DT - n \left( \frac{D}{\theta} \left( e^{\frac{\theta T_{n}}{n}} - 1 \right) - D \frac{T_{n}}{n} \right) \right) \]  

From the analysis of buyer’s inventory on Eq. (6) and (9),

\[ n \int_{0}^{T_{f}n} I_{b}(t) dt = n \left[ \frac{DT^{2}}{2n} \left( 1 + \frac{1}{3} \frac{\theta T_{n}}{n} \right) \right] \]  

(23)

Therefore,

\[ TC_{v} = \frac{s}{T} + \frac{n}{T} \left( T_{f} + \left( 2d_{c}c_{1} + dt_{c2}b \frac{D}{\theta} \left( e^{\frac{\theta T_{n}}{n}} - 1 \right) + \left( 2de_{1} + de_{2} \frac{D}{\theta} \left( e^{\frac{\theta T_{n}}{n}} - 1 \right) \right) + \frac{(h_{v} + w_{e})}{T} \left[ \frac{P-D}{\theta} - \frac{P-D}{\theta^{2}} \left( e^{-\theta T_{v1}} - 1 \right) - \frac{DT_{c2}}{\theta^{2}} - \frac{D}{\theta^{2}} \left( 1 - e^{-\theta T_{c2}} \right) - \frac{DT^{2}}{2n} \left( 1 + \frac{1}{3} \frac{\theta T_{n}}{n} \right) \right] \right) \]  

(24)

\[ \left( \frac{d_{v} + d_{w}}{T} \right) \left( PT_{v1} - DT - n \left( \frac{D}{\theta} \left( e^{\frac{\theta T_{n}}{n}} - 1 \right) - D \frac{T_{n}}{n} \right) \right) \]  

For \( \theta \ll 1 \), and neglecting \( \theta^{2}T^{2} \) and more terms, one has:
The integrated total cost function for the vendor and the buyer with carbon emission consideration $TC_E$ is the sum of $TC_v$ and $TC_b$:

$$
TC_v = \frac{s}{T} + \frac{n}{T} \left( t_f + 2dt_c1 + dt_c2 \left( \frac{DT}{n} + \frac{1}{2} \frac{D\theta T^2}{n^2} \right) + 2de_1 + de_2 \left( \frac{DT}{n} + \frac{1}{2} \frac{D\theta T^2}{n^2} \right) \right) + \\
\left( \frac{h_v + w_e}{T} \right) \left( \frac{(P-D)Tv_1^2}{2} \left( 1 - \frac{\theta T_{v1}}{3} \right) + \frac{DTv_2^2}{2} \left( 1 + \frac{\theta T_{v2}}{3} \right) - \frac{DT^2}{2n} \left( 1 + \frac{\theta T}{3 n} \right) \right)
$$

$$
TC_b = \frac{s}{T} + \frac{n}{T} \left( t_f + 2dt_c1 + dt_c2 \left( \frac{DT}{n} + \frac{1}{2} \frac{D\theta T^2}{n^2} \right) + 2de_1 + de_2 \left( \frac{DT}{n} + \frac{1}{2} \frac{D\theta T^2}{n^2} \right) \right) + \\
\left( \frac{h_b + w_e}{T} \right) \left( \frac{(P-D)Tv_1^2}{2} \left( 1 - \frac{\theta T_{v1}}{3} \right) + \frac{DTv_2^2}{2} \left( 1 + \frac{\theta T_{v2}}{3} \right) - \frac{DT^2}{2n} \left( 1 + \frac{\theta T}{3 n} \right) \right)
$$

$$
TC_E = \frac{s}{T} + \frac{n}{T} \left( t_f + 2dt_c1 + dt_c2 \left( \frac{DT}{n} + \frac{1}{2} \frac{D\theta T^2}{n^2} \right) + 2de_1 + de_2 \left( \frac{DT}{n} + \frac{1}{2} \frac{D\theta T^2}{n^2} \right) \right) + \\
\left( \frac{h_v + w_e}{T} \right) \left( \frac{(P-D)Tv_1^2}{2} \left( 1 - \frac{\theta T_{v1}}{3} \right) + \frac{DTv_2^2}{2} \left( 1 + \frac{\theta T_{v2}}{3} \right) - \frac{DT^2}{2n} \left( 1 + \frac{\theta T}{3 n} \right) \right) + \\
\left( \frac{d_{cv} + d_{ev}}{T} \right) \left( Pt_{v1} - DT - \frac{1}{2} \frac{D\theta T^2}{n} \right) + \frac{h_v + w_e}{T} \left( \frac{dt_c1}{T} - \theta \frac{T_{v1}}{T} \right) \left( \frac{dt_c2}{T} + \frac{d_{cv} + d_{ev}}{T} \right) \frac{D\theta T}{2n} \left( 1 + \frac{\theta T}{3 n} \right) + \frac{d_{cv} + d_{ev}}{T} \frac{D\theta T}{2n} \left( 1 + \frac{\theta T}{3 n} \right)
$$

3.3. Solution Procedure

The objective of this model is to minimize the expected total cost function with carbon emission consideration $TC_E$. For this purpose we need to determine the optimal number of deliveries ($n^*$). Because $n$ is a discrete variable, we can derive $n$ as follows:

Step 1. Substitute Eq. (18) and (19) to (26).
Step 2. Input all the system parameters.
Step 3. Determine the partial derivative of $TC_E$ with respect to $T_{v1}$ and set it to zero.
Step 4. For a range of $n$-value, denote the resulting minimum value of $T_{v1}$. For each $n$ and $T_{v1}$ find the corresponding value of $T_{v2}$ and $T$ from Eq. (18) and (19).
Step 5. Derive the optimal number of deliveries ($n^*$) such that satisfy $TC_E(n^* - I) \geq TC_E(n^*) \leq TC_E(n^* + I)$.
Step 6. Find the optimal delivery quantity, $Q$, from equation (5).
Step 7. Find the optimal vendor’s production quantity, $R$.

4. Numerical Example

To illustrate the performance of the developed model, we adopt the numerical example from Yang & Wee (2000) and Lee & Kim (2014) with some modification. The parameters are as follows:

- $P = 2,000,000$ units/year,
- $D = 500,000$ units/year,
- $C = 2,000$ units/order,
- $S = 100,000$ setup,
- $\theta = 0.1$,
- $r_c = 500$ delivery,
- $h_b = 60$ units/year,
- $h_v = 40$ units/year,
- $d_{cv} = 600$ units,
- $d_{ev} = 400$ units.

To incorporate the carbon emission, we use several data and assumption as follow:

- $V = 2.6$ kgCO$_2$/litre diesel fuel (US. EPA)
- $E = 500$ grCO$_2$/kWh (CCC – UK Parliament)
- $c_1 = 30$ litre/100 km (Volvo truck report for Long haul)
- $c_2 = 0.45$ litre/100 km/ton payload (Volvo truck report for Long haul)
- $tx = 61.8$ tonCO$_2$ (Taiwan government’s plan; Chan, 2009)
- $w = 100$ kWh/ton/year
- $b = 4$ kg/unit
- $v_{de} = 4$ kgCO$_2$/unit
- $b_{de} = 5$ kgCO$_2$/unit
Therefore,

\[ e_j = \$ 0.048/km, \quad e_2 = \$ 2.89 \times 10^{-6}/\text{unit/km}, \quad w_c = \$ 3.09/\text{unit/year}, \]

\[ d_{eb} = \$ 0.31/\text{unit}, \quad d_{ev} = \$ 0.25/\text{unit}. \]

Further, to solve our developed model we need several additional parameters as follow:

\[ d = 100 \text{ km}, \quad t_f = \$ 500/\text{trip}, \quad t_v = \$ 0.75/\text{litre}. \]

By implementing the proposed solution procedure, we found the optimal value of \( n \) that minimize the integrated total cost as \( n^* = 8 \) with the \( T_{C_e}^* = \$3,246,283 \). The corresponding time parameter and total cost are given in Table 1. From Eq. (5), the order delivery size \( Q^* \) is 5,372 units. Further, the optimal vendor’s production quantity (\( R \)) is 43,052 units. From Eq. (6) and (14), the buyer’s and vendor’s inventories are 2,685 units and 13,444 units respectively. Fig. 2 shows the graphical representation of \( T_{C_e} \) concavity when \( n^* = 8 \).

As \( T = 0.08590 \), the total demand during the order cycle time (\( DT \)) is 42,950 units. Considering the loss due to deterioration at the vendor and buyer inventory, the vendor can fulfill the demand by producing 43,052 units per order cycle time. In one unit of time (e.g. one year), the total vendor’s production quantity is 501,206 units. Comparing this result to the demand (\( D \)), we can see a significant amount of loss due to item deterioration.

From Table 1, we found that when the decision is made solely from the buyer’s perspective, the optimal value of \( n \) is \( n^* = 24 \) and the minimum \( T_{C_e} \) is \$269,238. In this situation, the total cost \( T_{C_e} \) both for vendor and buyer is \$3,357,490 which is 3.31\% higher than the integrated total cost with joint decision. This result shows the cost reduction due to the supply chain joint decision. From Eq. (5), the order delivery size \( Q^* \) is 1978 units. Further, the optimal vendor’s production quantity is 47,584 units. From Eq. (6) and (14), the buyer’s and the vendor’s inventories are 989 units and 16,836 units respectively.

The total carbon emission per year calculation for \( n^* \) are as follow:

- Fuel consumption = 1 year/0.08590 year/order cycle x 8 delivery/order cycle x [(2 x 100 km/delivery x 30 litre/100 km) + (100 km/delivery x 0.45 litre/100 km/unit x 4 kg/unit x 1.10\(^{-3}\) ton/kg x 5,372 unit)] = 6488.56 litre
- Fuel emission = 6488.56 litre x 2.6 kgCO\(_2\)/litre x 1.10\(^{-3}\) ton/kg = 16.87 ton CO\(_2\)
- Vendor’s warehouse emission = 156,506.64 unit x 100 kwh/unit x 500 grCO\(_2\)/kwh x 1.10\(^{-6}\) ton/gr = 7.825.33 ton CO\(_2\)
- Vendor’s deteriorated item emission = 937.31 unit x 4 kgCO\(_2\)/unit = 3.75 ton CO\(_2\)
- Buyer’s warehouse emission = 250,089.48 unit x 100 kwh/unit x 500 grCO\(_2\)/kwh x 1.10\(^{-6}\) ton/gr = 12,504.47 ton CO\(_2\)
- Buyer’s deteriorated item emission = 268.68 unit x 5 kgCO\(_2\)/unit = 1.34 ton CO\(_2\)

Therefore, the total carbon emission (\( T_E \)) from the vendor and the buyer is 20,351.77 ton of CO\(_2\) per year. A similar carbon emission calculation was performed for \( n^* = 24 \) and the result is 21,417.75 ton of CO\(_2\)/year. It is 4.98\% higher than the total carbon emission for \( n^* \).

<table>
<thead>
<tr>
<th>( n )</th>
<th>( T_{V2}(10^3) )</th>
<th>( T_{V1}(10^3) )</th>
<th>( T(10^3) )</th>
<th>( T_{C_e}(10^3) )</th>
<th>( T_{C_e}(10^3) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4935</td>
<td>1649</td>
<td>6585</td>
<td>2067.001</td>
<td>1737.468</td>
</tr>
<tr>
<td>2</td>
<td>5587</td>
<td>1867</td>
<td>7454</td>
<td>1188.198</td>
<td>2279.829</td>
</tr>
<tr>
<td>3</td>
<td>5884</td>
<td>1967</td>
<td>7851</td>
<td>850.482</td>
<td>2503.280</td>
</tr>
<tr>
<td>4</td>
<td>6064</td>
<td>2027</td>
<td>8092</td>
<td>672.301</td>
<td>2627.992</td>
</tr>
<tr>
<td>5</td>
<td>6191</td>
<td>2070</td>
<td>8260</td>
<td>563.141</td>
<td>2709.006</td>
</tr>
<tr>
<td>6</td>
<td>6288</td>
<td>2103</td>
<td>8391</td>
<td>490.145</td>
<td>2766.806</td>
</tr>
<tr>
<td>7</td>
<td>6368</td>
<td>2129</td>
<td>8498</td>
<td>438.466</td>
<td>2810.789</td>
</tr>
<tr>
<td>8*</td>
<td>6437</td>
<td>2153</td>
<td>8590</td>
<td>400.404</td>
<td>2845.879</td>
</tr>
<tr>
<td>9</td>
<td>6498</td>
<td>2173</td>
<td>8671</td>
<td>371.565</td>
<td>2874.905</td>
</tr>
<tr>
<td>10</td>
<td>6553</td>
<td>2192</td>
<td>8745</td>
<td>349.259</td>
<td>2899.609</td>
</tr>
<tr>
<td>20</td>
<td>6973</td>
<td>2332</td>
<td>9305</td>
<td>272.180</td>
<td>3048.286</td>
</tr>
<tr>
<td>21</td>
<td>7009</td>
<td>2344</td>
<td>9353</td>
<td>270.747</td>
<td>3058.794</td>
</tr>
<tr>
<td>22</td>
<td>7044</td>
<td>2356</td>
<td>9400</td>
<td>269.823</td>
<td>3068.928</td>
</tr>
<tr>
<td>23</td>
<td>7078</td>
<td>2368</td>
<td>9446</td>
<td>269.339</td>
<td>3078.734</td>
</tr>
<tr>
<td>24*</td>
<td>7112</td>
<td>2379</td>
<td>9491</td>
<td>269.238*</td>
<td>3088.252</td>
</tr>
<tr>
<td>25</td>
<td>7146</td>
<td>2390</td>
<td>9536</td>
<td>269.470</td>
<td>3097.513</td>
</tr>
</tbody>
</table>
When all the carbon emission costs are set to 0, we found the optimal value of \( n \) that minimizes the integrated total cost \( TC'' \) is \( n'' = 9 \). The total cost ($3,194,209) is lower because the emission cost is excluded. The corresponding time parameter and total cost are given in Table 2. By substituting the result into Eq. (26), one has \( TC_E = $3,246,970 \). It is 0.021\% higher than the total carbon emission from \( n^* \). From Eq. (5), the order delivery size \( Q'' \) is 4916 units. Further, the optimal vendor’s production quantity is 44329 units. From Eq. (6) and (14), the buyer’s and vendor’s inventories are 2457 units and 14149 units respectively. A similar carbon emission calculation was performed for \( n'' = 9 \) and the result is 20,526.97 ton CO\(_2\)/year. It is 0.854\% higher than the total carbon emission for \( n^* \).

Sensitivity analyses were carried out for several parameters by 20\% increase or decrease. Table 4 - 8 shows the result of the sensitivity analysis. The main conclusion drawn from the sensitivity analyses are as follows:

1. The range of percentage of cost reduction (PoCR) and percentage of emission reduction (PoER) is from 0.006\% to 0.043\% and from 0\% to 0.856\% respectively.
2. The values of \( n^* \), PoCR and PoER are most sensitive to the parameter \( \theta \).
3. The higher the value of the parameter \( t_x, c_1 & c_2, w \) and \( d \), the smaller the values of \( n^* \).
4. The higher the value of the parameter \( \theta \), the higher the value of \( n^* \).

<table>
<thead>
<tr>
<th>( n )</th>
<th>( TV_1 (10^5) )</th>
<th>( TV_2 (10^5) )</th>
<th>( T (10^5) )</th>
<th>( TC_b (10^5) )</th>
<th>( TC_v (10^5) )</th>
<th>( TC_E (10^5) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>6409</td>
<td>2143</td>
<td>8553</td>
<td>486.197</td>
<td>2720.463</td>
<td>3206.660</td>
</tr>
<tr>
<td>7</td>
<td>6493</td>
<td>2171</td>
<td>8664</td>
<td>434.876</td>
<td>2763.350</td>
<td>3198.226</td>
</tr>
<tr>
<td>8</td>
<td>6564</td>
<td>2195</td>
<td>8759</td>
<td>397.039</td>
<td>2797.567</td>
<td>3194.606</td>
</tr>
<tr>
<td>9&quot;</td>
<td>6627</td>
<td>2216</td>
<td>8844</td>
<td>368.341</td>
<td>2825.868</td>
<td>3194.209&quot;</td>
</tr>
<tr>
<td>10</td>
<td>6685</td>
<td>2236</td>
<td>8920</td>
<td>346.120</td>
<td>2849.949</td>
<td>3196.069</td>
</tr>
</tbody>
</table>

Table 3. Summary of cost and emission saving

<table>
<thead>
<tr>
<th>( n )</th>
<th>Buyer’s individual decision</th>
<th>Integrated decision considering emission</th>
<th>Saving</th>
<th>Integrated decision</th>
<th>Saving</th>
</tr>
</thead>
<tbody>
<tr>
<td>( TC_b ) ($/year)</td>
<td>3,357,490</td>
<td>3,246,283</td>
<td>3.31%</td>
<td>3,246,970</td>
<td>0.021%</td>
</tr>
<tr>
<td>( TE ) (tonCO(_2)/year)</td>
<td>21,417.75</td>
<td>20,351.77</td>
<td>4.98 %</td>
<td>20,526.97</td>
<td>0.854%</td>
</tr>
</tbody>
</table>
The optimal integrated decision of the PoER and PoCR are achieved when the total cost $T_e$ is minimized. The percentage of cost reduction $PoCR$ and emission reduction $PoER$ are calculated as follows:

$PoCR = \frac{|T_e(n) - T_e(n^*)|}{T_e(n^*)}$

$PoER = \frac{|E(n) - E(n^*)|}{E(n^*)}$

where $n^*$ represents the optimal decision without considering carbon emission.

### Table 4. Sensitivity analysis when $\tau_x$ is changed by 20%

<table>
<thead>
<tr>
<th>$\tau_x$ (S/tonCO₂)</th>
<th>37.08</th>
<th>49.44</th>
<th>74.16</th>
<th>86.52</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n^*$</td>
<td>9</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>$TC_e(n^*)$</td>
<td>3225684</td>
<td>3236027</td>
<td>3246283</td>
<td>3256504</td>
</tr>
<tr>
<td>$TE(n^*)$</td>
<td>20526.89</td>
<td>20351.80</td>
<td>20351.77</td>
<td>20351.71</td>
</tr>
<tr>
<td>$PoCR$</td>
<td>0.006%</td>
<td>0.012%</td>
<td>0.021%</td>
<td>0.031%</td>
</tr>
<tr>
<td>$PoER$</td>
<td>0.0004%</td>
<td>0.853%</td>
<td>0.854%</td>
<td>0.854%</td>
</tr>
</tbody>
</table>

* The optimal integrated decision of $n$ that minimizes $TC_e$ by considering carbon emission

### Table 5. Sensitivity analysis when $c_1$ & $c_2$ are changed by 20%

<table>
<thead>
<tr>
<th>$c_1$ (litre/km)</th>
<th>18/100</th>
<th>24/100</th>
<th>30/100</th>
<th>36/100</th>
<th>42/100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_2$ (litre/km/ton)</td>
<td>0.27/100</td>
<td>0.36/100</td>
<td>0.45/100</td>
<td>0.54/100</td>
<td>0.63/100</td>
</tr>
<tr>
<td>$n^*$</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>$TC_e(n^*)$</td>
<td>2974016</td>
<td>3110168</td>
<td>3246283</td>
<td>3382405</td>
<td>3519423</td>
</tr>
<tr>
<td>$TE(n^*)$</td>
<td>20519.42</td>
<td>20348.39</td>
<td>20351.77</td>
<td>20355.14</td>
<td>20358.52</td>
</tr>
<tr>
<td>$PoCR$</td>
<td>0.018%</td>
<td>0.019%</td>
<td>0.021%</td>
<td>0.023%</td>
<td>0.026%</td>
</tr>
<tr>
<td>$PoER$</td>
<td>0.0012%</td>
<td>0.852%</td>
<td>0.854%</td>
<td>0.855%</td>
<td>0.856%</td>
</tr>
</tbody>
</table>

### Table 6. Sensitivity analysis when $w$ is changed by 20%

<table>
<thead>
<tr>
<th>$w$ (kWH)</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>140</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n^*$</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>$TC_e(n^*)$</td>
<td>3226266</td>
<td>3236296</td>
<td>3246283</td>
<td>3256231</td>
<td>3266141</td>
</tr>
<tr>
<td>$TE(n^*)$</td>
<td>12325.56</td>
<td>16285.82</td>
<td>20351.77</td>
<td>24417.69</td>
<td>28483.58</td>
</tr>
<tr>
<td>$PoCR$</td>
<td>0.006%</td>
<td>0.013%</td>
<td>0.021%</td>
<td>0.031%</td>
<td>0.041%</td>
</tr>
<tr>
<td>$PoER$</td>
<td>0.0000%</td>
<td>0.855%</td>
<td>0.854%</td>
<td>0.853%</td>
<td>0.852%</td>
</tr>
</tbody>
</table>

### Table 7. Sensitivity analysis when $\theta$ is changed by 20%

<table>
<thead>
<tr>
<th>$\theta$ (kWH)</th>
<th>0.06</th>
<th>0.08</th>
<th>0.12</th>
<th>0.14</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n^*$</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>$TC_e(n^*)$</td>
<td>3021552</td>
<td>3136514</td>
<td>3246283</td>
<td>3351501</td>
</tr>
<tr>
<td>$TE(n^*)$</td>
<td>20342.83</td>
<td>20347.47</td>
<td>20351.77</td>
<td>20530.89</td>
</tr>
<tr>
<td>$PoCR$</td>
<td>0.022%</td>
<td>0.031%</td>
<td>0.021%</td>
<td>0.013%</td>
</tr>
<tr>
<td>$PoER$</td>
<td>0.0002%</td>
<td>0.853%</td>
<td>0.854%</td>
<td>0.0008%</td>
</tr>
</tbody>
</table>
Table 8. Sensitivity analysis when $d$ is changed by 20%

<table>
<thead>
<tr>
<th>$d$ (km)</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>140</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n^*$</td>
<td>9</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>$TC_0(n^*)$</td>
<td>2974014</td>
<td>3110165</td>
<td>3246283</td>
<td>3382401</td>
<td>3518519</td>
</tr>
<tr>
<td>$TE(n^*)$</td>
<td>20519.42</td>
<td>20348.39</td>
<td>20351.77</td>
<td>20355.14</td>
<td>20358.52</td>
</tr>
<tr>
<td>$n^{&quot;}$</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>$TC_0(n^{&quot;})$</td>
<td>2974522</td>
<td>3110746</td>
<td>3246970</td>
<td>3383194</td>
<td>3519417</td>
</tr>
<tr>
<td>$TE(n^{&quot;})$</td>
<td>20519.67</td>
<td>20523.32</td>
<td>20526.97</td>
<td>20530.61</td>
<td>20534.25</td>
</tr>
<tr>
<td>$PoCR$</td>
<td>0.017%</td>
<td>0.019%</td>
<td>0.021%</td>
<td>0.023%</td>
<td>0.026%</td>
</tr>
<tr>
<td>$PoER$</td>
<td>0.0012%</td>
<td>0.0019%</td>
<td>0.0021%</td>
<td>0.0023%</td>
<td>0.0026%</td>
</tr>
</tbody>
</table>

5. Conclusion

In this study, we extend the single vendor-buyer integrated inventory model to consider deterioration and carbon emission cost. The model extends previous studies by considering carbon emission from warehousing as well as transport activities and from the disposal of deteriorated items. The numerical example shows that in an integrated supply chain decision, the delivery frequency ($n$) is smaller and the delivery quantity ($Q$) is bigger when compared with the decision made solely from the buyer’s perspective. The integrated inventory model has significantly reduced the total cost and the total carbon emissions. It is also shown that by incorporating emission cost into the inventory model decreases the delivery frequency and increases the quantity. Incorporating carbon emission into the integrated inventory model will reduce the total carbon emissions and slightly reduce the total supply chain cost. Further research can be considered for different backordering situation and different emission control policies.

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References


Biographies

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