Single Vendor-Buyer Integrated Inventory Model for Deteriorating Items Considering Carbon Emission

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Abstract

In this study, we present an integrated single vendor-buyer inventory model considering deterioration and carbon emission simultaneously. We focus on the emissions from warehousing, transport activity as well as emission from the disposal of the deteriorated items. A numerical example is developed to illustrate the proposed solution procedure. Computational results indicate that by incorporating carbon emissions into the integrated inventory model, we can reduce the total carbon emission and slightly reduce the total supply chain cost. We also show that the integrated inventory model has significantly reduced the total cost and total carbon emissions as compared to the result when the decision is made solely from the buyer's perspective.

Keywords
Carbon emission reduction**;** single vendor-buyer; integrated inventory; deteriorating items

1. Introduction

Sustainability is a global issue and has received much attention in supply chain management. Carter & Rodgers (2008) examined the relationship among environmental, social, and economic performance in a supply chain. A sustainable supply chain is advantageous and lead to economic sustainability. Researchers have investigated many aspects of sustainable supply chain, such as sustainable supply chain network design, sustainable supply chain planning, sustainable sourcing, etc. As part of sustainable supply chain management, researchers have also studied the sustainable inventory management.

This study developed a new single vendor-buyer integrated inventory model considering deterioration and carbon emission simultaneously and examined how the integrated vendor-buyer inventory model reduces the carbon emission. We incorporated carbon emission cost into the model based on carbon emission tax mechanism. Carbon emission tax is one mechanism commonly used to promote energy efficiency and the reduction of fossil fuel combustion (Wahab et al., 2011**;** Bouchery et al., 2012; Benjaafar et al., 2013; Chen et al., 2013; Hovelaque & Bironneau, 2015; Hua et al., 2016). Some countries have implemented this policy and seen some benefits.

2. Literature Review

2.1. Carbon Emission in a Supply Chain

Sundarakani et al. (2008) modelled the carbon emission at each stage of a supply chain to understand and calculate the total carbon transferred from one stage to another in a closed loop supply chain. All the supply chain stages consume energy and produce wastages and carbon emission. Further, Sundarakani et al. (2010) proposed a supply chain design to study the carbon emissions across the supply chain. Carbon emissions can be minimized by reducing inventory and increasing visibility at the distribution level.

Montoya-Torres et al. (2015) used a real case study to validate a framework for measuring carbon footprint in a supply chain. They considered the breadth and depth of the scope by taking into consideration Scope 1 (directly controlled by the company), Scope 2 (indirect emission from business' value chain) and Scope 3 (other indirect emissions) of the emission sources. The scope of the supply chain emission sources includes the electricity and natural gas at the facilities, product deliveries between facilities, emissions from business travel, waste disposal at the facilities, and other fuel usages.

Bonney & Jaber (2011) pioneered the development of an environmentally responsible inventory model by considering the emission cost from transport activity and waste disposal. Chen et al. (2013) developed an EOQ model by considering the emission from order initiated, unit held in inventory, and from unit purchased. Battini et al. (2014) investigated the emission cost from warehouse operation, waste disposal, and transportation. Hovelaque & Bironneau (2015) developed an EOQ model by considering the emission from warehouse and transport operation. Hua et al. (2016) proposed an inventory model for perishable inventory in which carbon emission comes from inventory holding, shipping, and item deterioration. Our study only considers the Scope 1 of the emission sources focusing on emissions from warehousing, transport activity and from the disposal of deteriorated items.

2.2. Sustainable Integrated Production-Inventory Model

In an integrated inventory model, supply chain partners jointly decide on the inventory variables such as delivery frequency, delivery quantity, and the related time for mutual benefits. Goyal (1977) is one of the pioneers that studied the integrated inventory policy problem. Yang & Wee (2000) relaxed the early assumptions and studied the effects of item deterioration. They found an impressive cost reduction when compared with the buyer's independent decision. Lee & Kim (2014) extended the integrated inventory model by considering the effect of deterioration and defective items.

In recent years, researchers have considered the sustainable issue in the integrated inventory model. However, most of them focused on the economic and environmental aspects. Wahab et al. (2011) developed an EOQ model considering environmental impact and imperfect items for a coordinated two-level international supply chain. For the same percentage of defective items, the optimal shipment size would be higher when the issue of the environment considered.

Jaber et al. (2013) proposed a supply chain coordination model for one vendor and one buyer supply chain that considered emission in the manufacturing processes. Supply chain coordination minimizes the total system cost with no change in the total emissions and penalty costs. Jauhari et al. (2014) extended the model to consider unequal-sized shipment for imperfect quality items. Zanoni et al. (2014a) considered an emission trading scheme for single vendor-single buyer joint economic lot sizing (JELS) model with vendor managed inventory and consignment stock (VMI-CS). The numerical example indicates a reduction in carbon emission around 9% as well as the total cost reduction. Further, Zanoni et al. (2014b) studied the JELS model for coordinated inventory replenishment decisions considering price and environmentally sensitive demand.

Recently, Bazan et al. (2017) studied a closed-loop supply chain with remanufacturing and VMI-CS. The models also considered carbon emissions and energy cost. Bouchery et al. (2017) compared the cost and carbon emission of a non-coordinated and coordinated two-echelon supply chain. Wangsa (2017) studied carbon emission costs from the perspective of industrial and transport activities that involved Government's penalty and incentive policies. These policies resulted in an impressive total cost saving, but the integrated decision did not guarantee a total emission saving.

Our model extends the previous works on sustainable integrated vendor-buyer inventory model by considering deteriorating items. Certain deteriorating items such as milk, fruit, and vegetables need a special storage handling (i.e. cold storage) that has increased carbon emissions. Further, deteriorated items may need additional handling and transport for its disposal. Therefore it is important to consider the carbon emission for deteriorating item.

3. Mathematical Model

This study deals with a single-vendor single-buyer integrated model of a single deteriorating item in which the demand rate (D) and production rate (P) are known and constant. Furthermore, our mathematical model considers the following assumptions:

1. there is only one production cycle per order with multiple equal-sized deliveries;
2. the item has a constant deterioration rate per unit time;
3. the replenishment is instantaneous;
4. deterioration occurs in vendor's and buyer's inventory and the deteriorated items have no value;
5. the carbon emission comes from transportation, warehousing and the disposal of the deteriorated item; transportation emissions depend on the delivery distance and quantity, while the warehouse emissions depend on the quantity of inventory held;
6. shortage is not allowed;
7. all the costs incorporated are known and constant;
8. the distance traveled from the supplier to the buyer is known.

Notation:

Q	delivery quantity (units);
n	number of delivery per order;
R	production quantity (units; $R = PT_{v1}$);
θ	proportion of the on-hand inventory lost due to deterioration ($0 \le \theta < 1$);
T	cycle length;
T_{v1}	production period for vendor in each cycle;
T_{v2}	non production period for vendor in each cycle;
T_b	inventory cycle length per delivery for the buyer ($T_b = T/n$);
$I_v(t)$	vendor's inventory level at time t;
$I_b(t)$	buyer's inventory level at time t;
o	buyer's ordering cost ($/order);
r_c	buyer's receiving cost ($/delivery);
h_b	buyer's holding cost ($/unit/year);
d_{cb}	buyer's deteriorating cost ($/unit), which is equal to the purchase cost;
s	vendor's setup cost ($/order);
h_v	vendor's holding cost ($/unit/year);
d_{cv}	vendor's deteriorating cost ($/unit);
t_f	fixed transportation cost per delivery for vendor ($);
t_v	variable transportation cost for vendor, which equal to fuel price ($/litre);
d	distance travelled from the supplier to the buyer (km);
tx	carbon emission tax ($/kg$CO_2$);
V	vehicle standard emission from fuel consumption (kgCO_2/litre);
c_1	vehicle fuel consumption when empty (litre/km);
c_2	additional vehicle fuel consumption per ton of payload (litre/km/ton);
e_1	carbon emission cost from vehicle ($/km);
e_2	additional carbon emission cost from transporting one unit item ($/unit/km);
E	electricity standard emission (kgCO_2/kWh);
w	average warehouse energy consumption (kWh/unit/year)
w_e	carbon emission cost from holding items in warehouse ($/unit/year);
b	product weight (kg/unit);
v_{de}	average emission for the disposal of vendor's deteriorated item (kgCO_2/unit);
b_{de}	average emission for the disposal of buyer's deteriorated item (kgCO_2/unit);
d_{ev}	carbon emission cost for the disposal of the deteriorated item for vendor ($/unit);
d_{eb}	carbon emission cost for the disposal of the deteriorated item for buyer ($/unit).

We can see the inventory model of the deteriorating items for both the vendor and the buyer in Fig. 1.

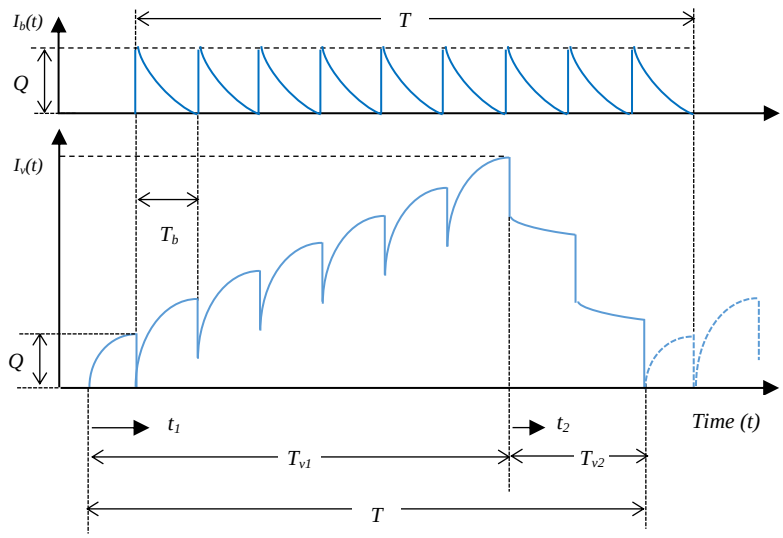

Fig. 1 Inventory level of vendor and buyer in an integrated inventory model

3.1. Buyer's Total Cost Function

Eq. (1) describes the buyer's total cost per unit time (TC_b). It is a function of the buyer's ordering cost (C_o), receiving cost (C_{rc}), holding cost (C_{HEb}) and deteriorating cost (C_{DEb}) per unit time:

$$TC_b = C_O + C_{rc} + C_{HEb} + C_{DEb} \tag{1}$$

$$C_O = \frac{o}{T} \tag{2}$$

$$C_{rc} = r_c \frac{n}{T} \tag{3}$$

The buyer's inventory model is identical to Yang & Wee (2000). The inventory level at any time *t* over period [0, T/*n*] is being described as follow:

$dI_b(t) = -D\,dt - I_b(t)\theta\,dt$, therefore

$$I_b(t) = \frac{D}{\theta}(e^{\theta(\frac{T}{n}-t)} - 1), \quad \text{for } 0 \le t \le \frac{T}{n} \tag{4}$$

The maximum buyer's inventory $Q = I_b(0) = \frac{D}{\theta}(e^{\frac{\theta T}{n}} - 1)$ (5)

Therefore, the buyer's holding cost per unit time considering both traditional inventory carrying cost and carbon emission cost generated by warehousing is:

$$C_{HEb} = (h_b + w_e)\frac{n}{T}\left[\int_0^{T/n} I_b(t)dt\right] = (h_b + w_e)\frac{n}{T}\left[\int_0^{T/n} \frac{D}{\theta}(e^{\theta(\frac{T}{n}-t)} - 1)dt\right] \tag{6}$$

In this study, buyer's deteriorating cost per unit time considers both traditional deteriorating cost and carbon emission cost generated by the deteriorated items. Hua et al. (2016) assumed that deteriorated item will emit carbon dioxide, and Bonney & Jaber (2011) considered the cost to dispose waste to the environment.

$$C_{DEb} = (d_{cb} + d_{eb})\frac{n}{T}\left(I_b(0) - D\frac{T}{n}\right) = (d_{cb} + d_{eb})\frac{n}{T}\left(\frac{D}{\theta}(e^{\frac{\theta T}{n}} - 1) - D\frac{T}{n}\right) \tag{7}$$

By substituting (2), (3), (6), (7) to (1) we gain:

$$TC_b = \frac{o}{T} + r_c\frac{n}{T} + (h_b + w_e)\frac{n}{T}\left[\int_0^{T/n} \frac{D}{\theta}(e^{\theta(\frac{T}{n}-t)} - 1)dt\right] + \left((d_{cb} + d_{eb})\frac{n}{T}\left(\frac{D}{\theta}(e^{\frac{\theta T}{n}} - 1) - D\frac{T}{n}\right)\right) \tag{8}$$

Considering $\theta << 1$,

$$TC_b = \frac{o}{T} + r_c \frac{n}{T} + \left((h_b + w_e) \frac{DT}{2n} \left(1 + \frac{1}{3}\frac{\theta T}{n} + \frac{1}{12}\frac{\theta^2 T^2}{n^2} \right) \right) + \left((d_{cb} + d_{eb}) \left(\frac{1}{2}\frac{D\theta T}{n} + \frac{1}{6}\frac{D\theta^2 T^2}{n^2} + \frac{1}{24}\frac{D\theta^3 T^3}{n^3} \right) \right)$$

and by neglecting $\theta^2 T^2$ and more,

$$TC_b = \frac{o}{T} + r_c \frac{n}{T} + \left((h_b + w_e) \frac{DT}{2n} \left(1 + \frac{1}{3}\frac{\theta T}{n} \right) \right) + \left((d_{cb} + d_{eb}) \frac{D\theta T}{2n} \right) \tag{9}$$

3.2. Vendor's Total Cost Function

Eq. (10) describes the vendor's total cost per unit time (*TCv*). It is the function of the vendor's setup cost (C_S), transport cost (C_{TE}), holding cost (C_{HEv}) and deteriorating cost (C_{DEv}) per unit time:

$$TC_v = C_s + C_{TE} + C_{HEv} + C_{DEv} \tag{10}$$

$$C_s = \frac{s}{T} \tag{11}$$

The vendor's transportation cost consists of a fix transport cost, variable transport cost, and carbon emission cost. Bonney & Jaber (2011) considered the vehicle delivery time to calculate the transport emission. In this study, the emission cost depends on the delivery quantity (Q) or the vehicle payload.

$$C_{TE} = \frac{n}{T}\left(t_f + \left(2dt_v c_1 + dt_v c_2 bQ\right) + \left(2de_1 + de_2 Q\right)\right) = \frac{n}{T}\left(t_f + \left(2dt_v c_1 + dt_v c_2 bI_b(0)\right) + \left(2de_1 + de_2 I_b(0)\right)\right) \tag{12}$$

By substituting (5) to (12) we gain:

$$C_{TE} = \frac{n}{T}\left(t_f + \left(2dt_v c_1 + dt_v c_2 b \frac{D}{\theta}(e^{\frac{\theta T}{n}} - 1) \right) + \left(2de_1 + de_2 \frac{D}{\theta}(e^{\frac{\theta T}{n}} - 1) \right) \right) \tag{13}$$

The vendor's total inventory model is identical to that of Yang & Wee (2000) and Lee & Kim (2014). Eq. (14) consists of total inventory during the production period plus the total inventory during the non-production period minus the inventory of the transported goods at the buyer's side.

$$I_v(t) = \int_0^{T_{v1}} I_{v1}(t_1)dt_1 + \int_0^{T_{v2}} I_{v2}(t_2)dt_2 - n\int_0^{T/n} I_b(t)dt \tag{14}$$

During the production period T_{v1},

$dI_{v1}(t_1) = P\,dt_1 - D\,dt_1 - I_{v1}(t_1)\theta\,dt_1$, therefore vendor's inventory level at any time t is

$$I_{v1}(t_1) = \frac{P-D}{\theta}\left(1 - e^{-\theta t_1}\right), \quad 0 \le t_1 \le T_{v1} \tag{15}$$

And during the non-production period T_{v2},

$dI_{v2}(t_2) = -D\,dt_2 - I_{v2}(t_2)\theta\,dt_2$, therefore

$$I_{v2}(t_2) = \frac{D}{\theta}\left(e^{\theta(T_{v2}-t_2)} - 1\right), \quad 0 \le t_2 \le T_{v2} \tag{16}$$

Therefore, the vendor's holding cost per unit time considering both traditional inventory carrying cost and carbon emission cost generated by warehousing is:

$$C_{HEv} = \frac{(h_v + w_e)}{T}\left[\int_0^{T_{v1}} \frac{P-D}{\theta}\left(1 - e^{-\theta t_1}\right)dt_1 + \int_0^{T_{v2}} \frac{D}{\theta}\left(e^{\theta(T_{v2}-t_2)} - 1\right)dt_2 - n\int_0^{T/n} I_b(t)dt \right] \tag{17}$$

Similar to Yang & Wee (2000) and Lee & Kim (2014), as $\theta << 1$ we use Misra (1975) approximation

$$T_{v1} \approx \frac{D}{P-D} T_{v2}\left(1 + \frac{1}{2}\theta T_{v2}\right), \tag{18}$$

and as $T = T_{v1} + T_{v2}$, we can derive

$$T \approx \frac{T_{v2}}{P-D}\left(P+\frac{1}{2}\theta T_{v2}\right) \tag{19}$$

The vendor's deteriorating cost per unit time considering both traditional deteriorating cost and carbon emission cost generated by deteriorated items is:

$$C_{DEv} = \frac{(d_{cv}+d_{ev})}{T}\left(PT_{v1} - DT - n\left(I_b(0) - D\frac{T}{n}\right)\right) \tag{20}$$

By substituting (4) to (20) we gain:

$$C_{DEv} = \frac{(d_{cv}+d_{ev})}{T}\left(PT_{v1} - DT - n\left(\frac{D}{\theta}(e^{\frac{\theta T}{n}}-1) - D\frac{T}{n}\right)\right) \tag{21}$$

By substituting (11), (13), (17), (21) to (10) we gain:

$$\begin{aligned} TC_v = {} & \frac{s}{T} + \frac{n}{T}\left(t_f + \left(2dt_v c_1 + dt_v c_2 b\frac{D}{\theta}(e^{\frac{\theta T}{n}}-1)\right) + \left(2de_1 + de_2\frac{D}{\theta}(e^{\frac{\theta T}{n}}-1)\right)\right) + \\ & \frac{(h_v+w_e)}{T}\left[\int_0^{T_{v1}} \frac{P-D}{\theta}\left(1-e^{-\theta t_1}\right)dt_1 + \int_0^{T_{v2}} \frac{D}{\theta}\left(e^{\theta(T_{v2}-t_2)}-1\right)dt_2 - n\int_0^{T/n} I_b(t)dt\right] + \\ & \frac{(d_{cv}+d_{ev})}{T}\left(PT_{v1} - DT - n\left(\frac{D}{\theta}(e^{\frac{\theta T}{n}}-1) - D\frac{T}{n}\right)\right) \end{aligned} \tag{22}$$

From the analysis of buyer's inventory on Eq. (6) and (9),

$$n\int_0^{T/n} I_b(t)dt = n\left[\frac{DT^2}{2n^2}\left(1+\frac{1}{3}\frac{\theta T}{n}\right)\right] \tag{23}$$

Therefore,

$$\begin{aligned} TC_v = {} & \frac{s}{T} + \frac{n}{T}\left(t_f + \left(2dt_v c_1 + dt_v c_2 b\frac{D}{\theta}(e^{\frac{\theta T}{n}}-1)\right) + \left(2de_1 + de_2\frac{D}{\theta}(e^{\frac{\theta T}{n}}-1)\right)\right) + \\ & \frac{(h_v+w_e)}{T}\left[\frac{P-D}{\theta}T_{v1} + \frac{P-D}{\theta^2}\left(e^{-\theta T_{v1}}-1\right) - \frac{DT_{v2}}{\theta} - \frac{D}{\theta^2}(1-e^{\theta T_{v2}}) - \left(\frac{DT^2}{2n}\left(1+\frac{1}{3}\frac{\theta T}{n}\right)\right)\right] + \\ & \frac{(d_{cv}+d_{ev})}{T}\left(PT_{v1} - DT - n\left(\frac{D}{\theta}(e^{\frac{\theta T}{n}}-1) - D\frac{T}{n}\right)\right) \end{aligned} \tag{24}$$

$$\begin{aligned} TC_v = {} & \frac{s}{T} + \frac{n}{T}\left(\begin{array}{l} t_f + \left(2dt_v c_1 + dt_v c_2 b\left(\frac{DT}{n} + \frac{1}{2}\frac{D\theta T^2}{n^2} + \frac{1}{6}\frac{D\theta^2 T^3}{n^3} + \frac{1}{24}\frac{D\theta^3 T^4}{n^4}\right)\right) + \\ \left(2de_1 + de_2\left(\frac{DT}{n} + \frac{1}{2}\frac{D\theta T^2}{n^2} + \frac{1}{6}\frac{D\theta^2 T^3}{n^3} + \frac{1}{24}\frac{D\theta^3 T^4}{n^4}\right)\right) \end{array}\right) + \\ & \frac{(h_v+w_e)}{T}\left[\frac{(P-D)T_{v1}^2}{2}\left(1-\frac{\theta T_{v1}}{3}\right) + \frac{DT_{v2}^2}{2}\left(1+\frac{\theta T_{v2}}{3}\right) - \left(\frac{DT^2}{2n}\left(1+\frac{1}{3}\frac{\theta T}{n}\right)\right)\right] + \\ & \frac{(d_{cv}+d_{ev})}{T}\left(PT_{v1} - DT - n\left(\frac{1}{2}\frac{D\theta T^2}{n^2} + \frac{1}{6}\frac{D\theta^2 T^3}{n^3} + \frac{1}{24}\frac{D\theta^3 T^4}{n^4}\right)\right) \end{aligned}$$

For $\theta << 1$, and neglecting $\theta^2 T^2$ and more terms, one has:

$$TC_v = \frac{s}{T} + \frac{n}{T}\left(t_f + 2dt_v c_1 + dt_v c_2 b\left(\frac{DT}{n} + \frac{1}{2}\frac{D\theta T^2}{n^2}\right) + 2de_1 + de_2\left(\frac{DT}{n} + \frac{1}{2}\frac{D\theta T^2}{n^2}\right)\right) +$$
$$\frac{(h_v + w_e)}{T}\left[\frac{(P-D)T_{v1}^2}{2}\left(1 - \frac{\theta T_{v1}}{3}\right) + \frac{DT_{v2}^2}{2}\left(1 + \frac{\theta T_{v2}}{3}\right) - \left(\frac{DT^2}{2n}\left(1 + \frac{1}{3}\frac{\theta T}{n}\right)\right)\right] + \tag{25}$$
$$\frac{(d_{cv} + d_{ev})}{T}\left(PT_{v1} - DT - \frac{1}{2}\frac{D\theta T^2}{n}\right)$$

The integrated total cost function for the vendor and the buyer with carbon emission consideration TC_E is the sum of TC_v and TC_b:

$$TC_E = \frac{s}{T} + \frac{n}{T}\left(t_f + 2dt_v c_1 + dt_v c_2 b\left(\frac{DT}{n} + \frac{1}{2}\frac{D\theta T^2}{n^2}\right) + 2de_1 + de_2\left(\frac{DT}{n} + \frac{1}{2}\frac{D\theta T^2}{n^2}\right)\right) +$$
$$\frac{(h_v + w_e)}{T}\left[\frac{(P-D)T_{v1}^2}{2}\left(1 - \frac{\theta T_{v1}}{3}\right) + \frac{DT_{v2}^2}{2}\left(1 + \frac{\theta T_{v2}}{3}\right) - \left(\frac{DT^2}{2n}\left(1 + \frac{1}{3}\frac{\theta T}{n}\right)\right)\right] + \tag{26}$$
$$\frac{(d_{cv} + d_{ev})}{T}\left(PT_{v1} - DT - \frac{1}{2}\frac{D\theta T^2}{n}\right) + \frac{o}{T} + r_c\frac{n}{T} + (h_b + w_e)\frac{DT}{2n}\left(1 + \frac{1}{3}\frac{\theta T}{n}\right) + (d_{cb} + d_{eb})\frac{D\theta T}{2n}$$

3.3. Solution Procedure

The objective of this model is to minimize the expected total cost function with carbon emission consideration TC_E. For this purpose we need to determine the optimal number of deliveries (n^*). Because n is a discrete variable, we can derive n as follows:

Step 1. Substitute Eq. (18) and (19) to (26).
Step 2. Input all the system parameters.
Step 3. Determine the partial derivative of TC_E with respect to T_{v2} and set it to zero.
Step 4. For a range of n-value, denote the resulting minimum value of T_{v2}. For each n and T_{v2} find the corresponding value of T_{v1} and T from Eq. (18) and (19).
Step 5. Derive the optimal number of deliveries (n^*) such that satisfy $TC_E(n^*-1) \geq TC_E(n^*) \leq TC_E(n^*+1)$.
Step 6. Find the optimal delivery quantity, Q, from equation (5).
Step 7. Find the optimal vendor's production quantity, R.

4. Numerical Example

To illustrate the performance of the developed model, we adopt the numerical example from Yang & Wee (2000) and Lee & Kim (2014) with some modification. The parameters are as follows:

P	= 2,000,000 units/year,	D	= 500,000 units/year,
C	= \$2,000/order,	S	= \$100,000 /setup,
θ	= 0.1,	r_c	= \$500/delivery,
h_b	= \$60/unit/year,	h_v	= \$40/unit/year,
d_{cb}	= \$600/unit,	d_{cv}	= \$400/unit.

To incorporate the carbon emission, we use several data and assumption as follow:

V = 2.6 $kgCO_2$/litre diesel fuel (US. EPA)
E = 500 $grCO_2$/kWh (CCC – UK Parliament)
c_1 = 30 litre/100 km (Volvo truck report for Long haul)
c_2 = 0.45 litre/100 km/ton payload (Volvo truck report for Long haul)
tx = \$61.8/tonCO_2 (Taiwan government's plan; Chan, 2009)
w = 100 kWh/unit/year
b = 4 kg/unit
v_{de} = 4 $kgCO_2$ /unit
b_{de} = 5 $kgCO_2$ /unit

Therefore,

e_1 = \$ 0.048/km, e_2 = \$ 2.89 x 10^{-6} /unit/km, w_e = \$ 3.09/unit/year,
d_{eb} = \$ 0.31/unit, d_{ev} = \$ 0.25/unit.

Further, to solve our developed model we need several additional parameters as follow:

d = 100 km, t_f = \$ 500/trip, t_v = \$ 0.75/litre,

By implementing the proposed solution procedure, we found the optimal value of n that minimize the integrated total cost as n^* = 8 with the TC_E^* = \$3,246,283. The corresponding time parameter and total cost are given in Table 1. From Eq. (5), the order delivery size Q^* is 5,372 units. Further, the optimal vendor's production quantity (R) is 43,052 units. From Eq. (6) and (14), the buyer's and vendor's inventories are 2,685 units and 13,444 units respectively. Fig. 2 shows the graphical representation of TC_E concavity when n^* = 8.

As T = 0.08590, the total demand during the order cycle time (DT) is 42,950 units. Considering the loss due to deterioration at the vendor and buyer inventory, the vendor can fulfill the demand by producing 43,052 units per order cycle time. In one unit of time (e.g. one year), the total vendor's production quantity is 501,206 units. Comparing this result to the demand (D), we can see a significant amount of loss due to item deterioration.

From Table 1, we found that when the decision is made solely from the buyer's perspective, the optimal value of n is $n^\#$ = 24 and the minimum TC_b is \$269.238. In this situation, the total cost TC_E both for vendor and buyer is \$3,357,490 which is 3.31% higher than the integrated total cost with joint decision. This result shows the cost reduction due to the supply chain joint decision. From Eq. (5), the order delivery size $Q^\#$ is 1978 units. Further, the optimal vendor's production quantity is 47,584 units. From Eq. (6) and (14), the buyer's and the vendor's inventories are 989 units and 16,836 units respectively.

The total carbon emission per year calculation for n^* are as follow:

- Fuel consumption = 1 year/0.08590 year/order cycle x 8 delivery/order cycle x [(2 x 100 km/delivery x 30 litre/100 km) + (100 km/delivery x 0.45 litre/100 km/ton x 4 kg/unit x 1.10^{-3} ton/kg x 5,372 unit)] = 6488.56 litre
- Fuel emission = 6488.56 litre x 2.6 $kgCO_2$/litre 1.10^{-3} ton/kg = 16.87 ton CO_2
- Vendor's warehouse emission = 156,506.64 unit x 100 kwh/unit x 500 $grCO_2$/kwh x 1.10^{-6} ton/gr = 7,825.33 ton CO_2
- Vendor's deteriorated item emission = 937.31 unit x 4 $kgCO_2$/unit = 3.75 ton CO_2
- Buyer's warehouse emission = 250,089.48 unit x 100 kwh/unit x 500 $grCO_2$/kwh x 1.10^{-6} ton/gr = 12,504.47 ton CO_2
- Buyer's deteriorated item emission = 268.68 unit x 5 $kgCO_2$/unit = 1.34 ton CO_2

Therefore, the total carbon emission (TE) from the vendor and the buyer is 20,351.77 ton of CO_2 per year. A similar carbon emission calculation was performed for $n^\#$ = 24 and the result is 21,417.75 ton of CO_2/year. It is 4.98% higher than the total carbon emission for n^*.

Table 1. Result of optimal solution n

n	Tv_2 (10^{-5})	Tv_1 (10^{-5})	T (10^{-5})	TC_b (10^3)	TC_v (10^3)	TC_E (10^3)
1	4935	1649	6585	2067.001	1737.468	3804.469
2	5587	1867	7454	1188.198	2279.829	3468.027
3	5884	1967	7851	850.482	2503.280	3353.762
4	6064	2027	8092	672.301	2627.992	3300.293
5	6191	2070	8260	563.141	2709.006	3272.147
6	6288	2103	8391	490.145	2766.806	3256.951
7	6368	2129	8498	438.466	2810.789	3249.256
8*	**6437**	**2153**	**8590**	**400.404**	**2845.879**	**3246.283***
9	6498	2173	8671	371.565	2874.905	3246.470
10	6553	2192	8745	349.259	2899.609	3248.868
20	6973	2332	9305	272.180	3048.286	3320.466
21	7009	2344	9353	270.747	3058.794	3329.541
22	7044	2356	9400	269.823	3068.928	3338.751
23	7078	2368	9446	269.339	3078.734	3348.074
24#	**7112**	**2379**	**9491**	**269.238#**	**3088.252**	**3357.490**
25	7146	2390	9536	269.470	3097.513	3366.983

When all the carbon emission costs are set to 0, we found the optimal value of n that minimizes the integrated total cost TC'' is n''= 9. The total cost ($3,194,209) is lower because the emission cost is excluded. The corresponding time parameter and total cost are given in Table 2. By substituting the result into Eq. (26), one has TC_E = $3,246,970. It is 0.021% higher than the total carbon emission from n^*. From Eq. (5), the order delivery size Q'' is 4916 units. Further, the optimal vendor's production quantity is 44329 units. From Eq. (6) and (14), the buyer's and vendor's inventories are 2457 units and 14149 units respectively. A similar carbon emission calculation was performed for n'' = 9 and the result is 20,526.97 ton CO_2/year. It is 0.854% higher than the total carbon emission for n^*. Table 3 provides the summary of cost and emission savings from the integrated inventory model considering carbon emissions.

Sensitivity analyses were carried out for several parameters by 20% increase or decrease. Table 4 - 8 shows the result of the sensitivity analysis. The main conclusion drawn from the sensitivity analyses are as follows:

1. The range of percentage of cost reduction (*PoCR*) and percentage of emission reduction (*PoER*) is from 0.006% to 0.043% and from 0% to 0.856% respectively.
2. The values of n^*, *PoCR* and *PoER* are most sensitive to the parameter θ.
3. The higher the value of the parameter tx, c_1 & c_2, w and d, the smaller the values of n^*.
4. The higher the value of the parameter θ, the higher the value of n^*.

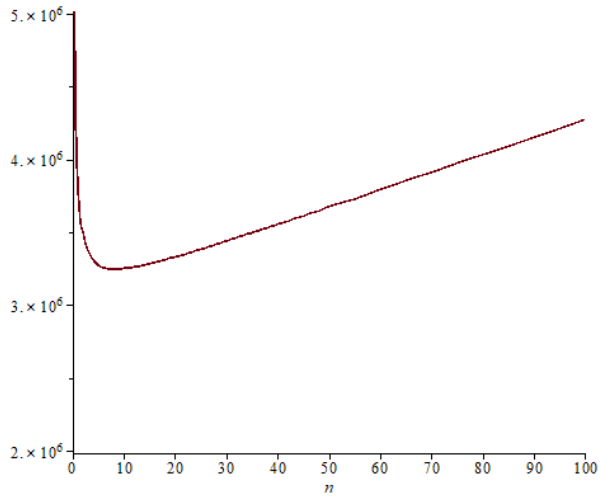

Fig. 2 Graphical representation of TC_E concavity when n^* = 8

Table 2. Optimal solution of n when emission costs are 0

n	Tv_2 (10^{-5})	Tv_1 (10^{-5})	T (10^{-5})	TC_b (10^3)	TC_v (10^3)	TC_E (10^3)
6	6409	2143	8553	486.197	2720.463	3206.660
7	6493	2171	8664	434.876	2763.350	3198.226
8	6564	2195	8759	397.039	2797.567	3194.606
9"	**6627**	**2216**	**8844**	**368.341**	**2825.868**	**3194.209"**
10	6685	2236	8920	346.120	2849.949	3196.069

Table 3. Summary of cost and emission saving

	Buyer's individual decision (a)	*Integrated decision considering emission (b)*	*Saving ((a-b)/a)*	*Integrated decision (c)*	*Saving ((c-b)/c)*
n	24	8		9	
TC_E ($/year)	3,357,490	3,246,283	3.31%	3,246,970	0.021%
TE (tonCO_2/year)	21,417.75	20,351.77	4.98 %	20,526.97	0.854%

Table 4. Sensitivity analysis when *tx* is changed by 20%

tx ($/ton$CO_2$)	37.08	49.44	{61.8 }	74.16	86.52
*n**	9	8	8	8	8
TC_E(n*)	3225684	3236027	3246283	3256504	3266687
TE(n*)	20526.89	20351.80	20351.77	20351.74	20351.71
n"	9	9	9	9	9
$TC_E(n")$	3225866	3236420	3246970	3257524	3268077
$TE(n")$	20526.97	20526.97	20526.97	20526.97	20526.97
PoCR	0.006%	0.012%	0.021%	0.031%	0.043%
PoER	0.0004%	0.853%	0.854%	0.854%	0.854%

* The optimal integrated decision of *n* that minimizes TC_E by considering carbon emission
" The optimal integrated decision of *n* without considering carbon emission
PoCR, percentage of cost reduction = $[TC_E(n")-TC_E(n^*)]/ TC_E(n")$
PoER, percentage of emission reduction = $[TE(n")- TE(n^*)]/ TE(n")$
{}, base column

Table 5. Sensitivity analysis when c_1 & c_2 are changed by 20%

c_1 (litre/km) c_2 (litre/km/ton)	18/100 0.27/100	24/100 0.36/100	{30/100} {0.45/100}	36/100 0.54/100	42/100 0.63/100
*n**	9	8	8	8	8
TC_E(n*)	2974016	3110168	3246283	3382405	3518525
TE(n*)	20519.42	20348.39	20351.77	20355.14	20358.52
n"	9	9	9	9	9
$TC_E(n")$	2974524	3110750	3246970	3383198	3519423
$TE(n")$	20519.67	20523.32	20526.97	20530.61	20534.25
PoCR	0.018%	0.019%	0.021%	0.023%	0.026%
PoER	0.0012%	0.852%	0.854%	0.855%	0.856%

Table 6. Sensitivity analysis when *w* is changed by 20%

w (kWH)	60	80	{100}	120	140
*n**	9	8	8	8	8
TC_E(n*)	3226266	3236296	3246283	3256231	3266141
TE(n*)	12325.56	16285.82	20351.77	24417.69	28483.58
n"	9	9	9	9	9
$TC_E(n")$	3226445	3236708	3246970	3257233	3267496
$TE(n")$	12325.55	16426.26	20526.97	24627.67	28728.38
PoCR	0.006%	0.013%	0.021%	0.031%	0.041%
PoER	0.0000%	0.855%	0.854%	0.853%	0.852%

Table 7. Sensitivity analysis when θ is changed by 20%

θ (kWH)	0.06	0.08	{0.1}	0.12	0.14
*n**	8	8	8	9	9
TC_E(n*)	3021552	3136514	3246283	3351501	3452519
TE(n*)	20342.83	20347.47	20351.77	20530.89	20534.70
n"	8	9	9	9	9
$TC_E(n")$	3022204	3137479	3246970	3351944	3452914
$TE(n")$	20342.88	20522.58	20526.97	20531.05	20534.89
PoCR	0.022%	0.031%	0.021%	0.013%	0.011%
PoER	0.0002%	0.853%	0.854%	0.0008%	0.00094%

Table 8. Sensitivity analysis when *d* is changed by 20%

d (km)	60	80	{100}	120	140
*n**	9	8	8	8	8
TC_E(n*)	2974014	3110165	3246283	3382401	3518519
TE(n*)	20519.42	20348.39	20351.77	20355.14	20358.52
n''	9	9	9	9	9
$TC_E(n'')$	2974522	3110746	3246970	3383194	3519417
$TE(n'')$	20519.67	20523.32	20526.97	20530.61	20534.25
PoCR	0.017%	0.019%	0.021%	0.023%	0.026%
PoER	0.0012%	0.852%	0.854%	0.855%	0.856%

5. Conclusion

In this study, we extend the single vendor-buyer integrated inventory model to consider deterioration and carbon emission cost. The model extends previous studies by considering carbon emission from warehousing as well as transport activities and from the disposal of deteriorated items**.** The numerical example shows that in an integrated supply chain decision, the delivery frequency (*n*) is smaller and the delivery quantity (*Q*) is bigger when compared with the decision made solely from the buyer's perspective. The integrated inventory model has significantly reduced the total cost and the total carbon emissions. It is also shown that by incorporating emission cost into the inventory model decreases the delivery frequency and increases the quantity. Incorporating carbon emission into the integrated inventory model will reduce the total carbon emissions and slightly reduce the total supply chain cost. Further research can be considered for different backordering situation and different emission control policies.

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References

Battini, D., Persona, A., and Sgarbossa, F., A sustainable EOQ model: theoretical formulation and applications, *International Journal of Production Economics*, vol. 149, no. 1, pp. 145-153, 2014.

Bazan, E., Jaber, M.Y., and Zanoni, S., Carbon emissions and energy effects on a two-level manufacturer-retailer closed-loop supply chain model with remanufacturing subject to different coordination mechanisms, *International Journal of Production Economics*, vol. 183, no. 1, pp. 394-408, 2017.

Benjaafar, S., Li, Y., and Daskin, M., Carbon footprint and the management of supply chains: insights from simple models, *IEEE Transactions on Automation Science and Engineering*, vol. 10, no. 1, pp. 99-116, 2013.

Bonney, M., and Jaber, M. Y., Environmentally responsible inventory models: non-classical models for a non-classical era, *International Journal of Production Economics*, vol. 133, no. 1, pp. 43-53, 2011.

Bouchery, Y., Ghaffari, A., Jemai, Z., and Dallery, Y., Including sustainability criteria into inventory models, *European Journal of Operational Research*, vol. 222, no. 2, pp. 229-240, 2012.

Bouchery, Y., Ghaffari, A., Jemai, Z., and Tan, T., Impact of coordination on cost and carbon emissions for a two-echelon serial economic order quantity problem, *European Journal of Operational Research*, vol. 260, no. 2, pp. 520-533, 2017.

Carter, C.R., and Rodgers, D.S., A framework of sustainable supply chain management: moving toward new theory, *International Journal of Physical Distribution & Logistics Management*, vol. 38, no. 5, pp. 360-387, 2008.

Chan, Y., Taiwan plans taxes for energy and CO2 emissions by 2011, Available: https://www.businessgreen.com/bg/news/1800579/taiwan-plans-taxes-energy-co2-emissions-2011, October 20, 2009.

Chen, X., Benjaafar, S., and Elomri, A., The carbon-constrained EOQ, *Operations Research Letters*, vol. 41, no. 2, pp. 172-179, 2013.

Goyal, S.K., An integrated inventory model for a single supplier–single customer problem, *International Journal of Production Research*, vol. 15, no. 1, pp. 107-111, 1977.

Hovelaque, V., and Bironneau, L., The carbon-constrained EOQ model with carbon emission dependent demand, *International Journal of Production Economics*, vol. 164, no. 1, pp. 285-291, 2015.

Hua, G. W., Cheng, T. C. E., Zhang, Y., Zhang, J. L., and Wang, S. Y., Carbon-constrained perishable inventory management with freshness-dependent demand, *International Journal of Simulation Modelling*, vol. 15, no. 3, pp. 542-552, 2016.

Jaber, M.Y., Glock, C.H., and El-Saadany, A.M.A., Supply chain coordination with emissions reduction incentives, *International Journal of Production Research*, vol. 51, no. 1, pp. 69-82, 2013.

Jauhari, W.A., Pamuji, A.S., and Rosyidi, C.N., Cooperative inventory model for vendor-buyer system with unequal-sized shipment, defective items and carbon emission costs, *International Journal of Logistics Systems and Management*, vol. 19, no. 2, pp. 163-186, 2014.

Misra, R.B., Optimum production lot size model for a system with deteriorating inventory, *International Journal of Production Research*, vol. 13, no. 5, pp. 495–505, 1975.

Montoya-Torres, J.R., Gutierrez-Franco, E., and Blanco, E.E., Conceptual framework for measuring carbon footprint in supply chains, *Production Planning & Control*, vol. 26, no. 4, pp. 265-279, 2015.

Sundarakani B., Goh, M., de Souza, R., and Shun, C., Measuring carbon footprints across the supply chain, *Proceeding of the 13th International Symposium on Logistics (ISL2008): Integrating the Global Supply Chain*, Bangkok, Thailand, July 6 – 8, 2008.

Sundarakani, B., de Souza, R., Goh, M., Wagner, S.M., and Manikandan, S., Modeling carbon footprints across the supply chain, *International Journal of Production Economics*, vol. 128, no. 1, pp. 43-50, 2010.

The Parliamentary office of Science and Technology, Carbon footprint of electricity generation, Available: https://www.parliament.uk/documents/post/postpn_383-carbon-footprint-electricity-generation.pdf, June 2011.

United States Environmental Protection Agency, Emission facts: Average carbon dioxide emissions resulting from gasoline and diesel fuel, Available: https://nepis.epa.gov/, February 2005.

Volvo Truck, Emissions from Volvo's truck, Available: http://www.volvotrucks.com/content/dam/volvo/volvo-trucks/markets/global/pdf/our-trucks/Emis_eng_10110_14001.pdf, November 24, 2014.

Wahab, M.I.M., Mamun, S.M.H., and Ongkunarak, P., EOQ models for a coordinated two-level international supply chain considering imperfect items and environmental impact, *International Journal of Production Economics*, vol. 134, no. 1, pp. 151-158, 2011.

Wangsa, I.D., Greenhouse gas penalty and incentive policies for a joint economic lot size model with industrial and transport emissions, *International Journal of Industrial Engineering Computation*, vol. 8, no. 4, pp. 453-480, 2017.

Yang, P.C. and Wee, H.M., Economic ordering policy of deteriorated item for vendor and buyer: An integrated approach, *Production Planning & Control*, vol. 11, no. 5, pp. 474-480, 2000.

Zanoni, Z., Mazzoldi, L., and Jaber, M.Y., Vendor-managed inventory with consignment stock agreement for single vendor-single buyer under the emission-trading scheme, *International Journal of Production Research*, vol. 52, no. 1, pp. 20-31, 2014.

Zanoni, Z., Mazzoldi, L., Zavanella, L.E., and Jaber, M.Y., A joint economic lot size model with price and environmentally sensitive demand, *Production & Manufacturing Research*, vol. 2, no. 1, pp. 341-354, 2014.

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