Bullwhip Effect Measures in a Non-Stationary Seasonal Supply Chain

Cherian Samuel
Department of Mechanical Engineering
Indian Institute of Technology (BHU)
Varanasi, India
csamuel.mec@itbhu.ac.in

Narayan Kumar
Faculty of Mechanical Engineering
Sri Ramswroop Memorial University
Lucknow, India
narayan.kumar.mec13@itbhu.ac.in

Abstract

The objective of this paper is to study the impact of seasonal demand parameters and the lead time of the retailer on the variables like order quantity, bullwhip effect and on-hand inventory to elaborate the supply chain dynamics. The objectives are achieved through analysis of a simple two echelon supply chain with retailer at the lower level and the supplier at the upper level. The time series of the end customer demand data exhibit a non-stationary SARIMA process. Through expression of bullwhip effect we show an increase in their value due to the presence of seasonal effect and non-stationary in the end customer demand. Our results show an increase in the variance of the expressions of both order quantity and on-hand inventory due to the presence of seasonality in the demand. Further with an increase in the retailer's lead time the bullwhip effect increases with a constant rate till the lead time crosses one season. As the lead time crosses one season, the slope suddenly increases. Further when the value of seasonal moving average parameter changes from positive to negative value, the bullwhip effect decreases.

Keywords

Bullwhip effect, seasonal demand, IMA process, MSME

1. Introduction

1.1 Overview

Supply chain of a product consists of manufacturer, retailer and customers called as end customers. These end customers place demands of various products to the retailer. The retailer in turn satisfies their demand which varies based upon the type of product and the customers in the target market. There are supply chains of some products in which the end customer's demand is seasonal. According to Baron [1] and Hartman [2], the seasonality in demand of the product is due to two main factors: natural and institutional. Butler [3] mentions three causes of seasonality as social pressure or fashion, sporting season, and inertia or tradition. Due to the seasonal nature of demand, several products get deteriorate due to the fact that in some period the sale of these products is low in comparison to other time periods. For the case of apparel products, Al-Zubaidi and Tyler [4] describes the following two basic reasons for deterioration of the above stated demand. (1) The decreasing shelf life of apparel: the supply system has been built around basic goods, i.e. merchandise sold the year round, and (2) the long lead-time: clothing lead-times are traditionally long (Mattila et al. [5]).

The phenomenon of demand variability amplification from end customer to manufacturer in a supply chain is known as bullwhip effect. Evidence of the bullwhip effect is first pointed out by Forrester [6], who discussed its causes and possible remediation in the context of industrial dynamics. After that, several researchers such as Blinder [7], Blanchard [8], Burbidge [9], Caplin [10] and Khan [11] also recognized the existence of the bullwhip effect in supply chains. Sterman [12] explored and illustrated the bullwhip effect through an experiment on the well-known "beer game". Several authors have quantified the bullwhip effect with stationary AR (1) end customer demand. Though the assumption of stationary demands is an adequate approximation, in practice, demands often exhibit strong seasonality. Moreover a large portion of make to stock producers faces seasonal and stochastic demand, (Metters [13]). A partial listing of such industries includes weather related industries (e. g., pharmaceutical products, lawnmowers, canned foods), back-to-school industries (e.g., pencils, clothing), and holiday related industries (e. g., toys, wrapping paper). Grave [14] studied the amplification in demand considering end customer demand process as non-stationary. The author studied the effect of demand parameters on various performance measure like on hand inventory variance and safety stock.

In order to analyse such a problem, a two stage serial supply chain with retailer at the lower level and the wholesaler at the upper level (Figure 1) is considered. The end customer demand of the product is not constant and repeats over a fix interval of time. More, specifically, the time series of the end customer demand data is non-stationary and exhibit seasonal or periodic fluctuations and is represented by an IMA (0, 1, 0)×$(0,0,1)_s$ process. The choice of a suitable forecasting model helps in reducing the bullwhip effect phenomena. Therefore, in order to quantify bullwhip measure we use the Minimum Mean Square Error (MMSE) forecasting model for the demand process under consideration.

1.2 Objective

The objective of this work is to quantify the bullwhip effect for its use in a seasonal and non-stationary demand process and study the impact of seasonal end customer demand parameter and lead time on the bullwhip effect. The objectives are achieved through analytical and numerical analysis of two stage serial supply chain with single retailer at the lower level and single wholesaler at the upper level. The time series of the end customer demand data exhibit IMA process which is a non-stationary model. This work is different from other works in following aspects:

1. Rather than considering non-seasonal demand, we employ seasonal demand process.
2. The end customer demand is non-stationary demand process.
3. Rather than relying on simple forecasting techniques such as moving average and exponential smoothing minimum mean square error technique to forecast the lead time demand is used.
4. The exact measure of the bullwhip effect is derived unlike deriving only the lower bound on bullwhip effect.

2. Literature Review

A large number of literatures are available on the study of the bullwhip effect phenomena. The most of the demand process has been modelled as either a stationary or a non-stationary process. The stochastic process $\{x_t : t = 1, 2, \ldots\}$ is stationary if for every collection of time indices $1 \le t_1 < t_2 \cdots < t_m$, the joint probability distribution of $(x_{t1}, x_{t2}, \cdots, x_{tm})$ is the same as the joint probability distribution of $(x_{t1+h}, x_{t2+h}, \cdots, x_{tm+h})$ for all integer $h \ge 1$. Most of the papers including Jhonson and Thompson [15], Lee et al. [16], Aviv [17], Alwan et al. [18] and So and Zheng [19] have assumed the demand as a stationary process and have modelled it as an Autoregressive Moving (ARMA) type process of the first order. The work of Lee et al. [16] deals with positively correlated demand in which the variance of the replenishment order at the retailer is larger than that of retail sales, and shows that the degree of variance amplification increases with an increase in the replenishment lead time. Aviv [17] considered the demand model in which a vector autoregressive time series has been described as a linear state space form, and uses an adaptive replenishment policy. Alwan et al. [18] studied the effect of different forecasting models on a serial supply chain with AR(1) demand. Comparison of a two-level supply chain with deterministic as well as varying lead-time for the AR(1) demand process has been studied by So and Zheng [19] and the authors proved that in the case of varying lead-time, the variance in the order quantity is more in comparison to that of the deterministic lead-time case.

For the case of non-stationary demand process, one of the most referred paper is Grave [14] wherein he considered the demand process as autoregressive integrated moving average [ARIMA (0, 1, 1)], and showed that the net inventory in case of the non-stationary demand is more than for the case of independent and identically distributed demand.

For the IMA type demand process, the following three forecasting models have been studied, (i) Moving Average (MA); (ii) Exponentially Weighted Moving Average (EWMA) and (iii) Minimum Mean Square Error (MMSE). Grave [16] used a EWMA forecasting method for the ARIMA (0, 1, 1) demand process. Chen et al. [20] used MA and EWMA model both of which are not optimal for AR(1) demands. Moreover, they showed that when an MA and EWMA method is used, the next upstream node faces a complex demand process in terms of the higher moving average terms. The bullwhip effect may be mitigated by eliminating its main causes (Lee et al. [16]). Among various causes of the bullwhip effect, forecasting methods are considered as one of the most important causes because the inventory system of a supply chain is directly affected by the forecasting method.

Jhonson and Thompson [15] showed that the optimal policy for a periodic ordering system is myopic for both stationary and non-stationary demand processes when the order-up-to policy is used along with the minimum mean square forecasting method. Most of the author used periodic review order up to inventory replenishment policy.
Bullwhip effect is defined as an amplification effect as the phenomena where orders to the wholesalers tend to have large variance than the sales to the buyer (Lee et al. [16]). According to the authors, this distortion propagates upstream in an amplification form. A quantitative measure of the bullwhip effect was proposed by the authors as the ratio of variance of order quantity at the echelon under consideration to the variance of demand of the end customer. They identified five root causes for the amplification effect in supply chains as: (i) demand forecast updating; (ii) lead time; (iii) batch ordering; (iv) supply shortages and (v) price variations. Chen et al. [20] study the effect of two factors namely demand forecasting and order lead times on the bullwhip effect for the case of two stage serial supply chain.

3. Model Development

Seasonal behaviour can be modelled using Seasonal Autoregressive Integrated Moving Average (SARIMA) model. Seasonal ARIMA models allow us to model fluctuations that relate to events not just in the immediate past, but also in the previous cycles. In this work, the seasonal behaviour of the end customer demand is represented by a multiplicative integrated moving average IMA $(0, 1, 0)\times(0,0,1)_s$ model. Both retailer and wholesaler follow an order-up-to-level periodic review policy for the review of their inventories and replenish its inventory from the upstream party every period. Both retailer and wholesaler employ MMSE forecasting method to calculate the lead time demand. The order placed by the retailer at the end of any time period t arrives at the end of $t+L+1^{\text{th}}$ period, where L is the deterministic lead time, which is the time, elapsed between the placement of an order and the arrival of that order.

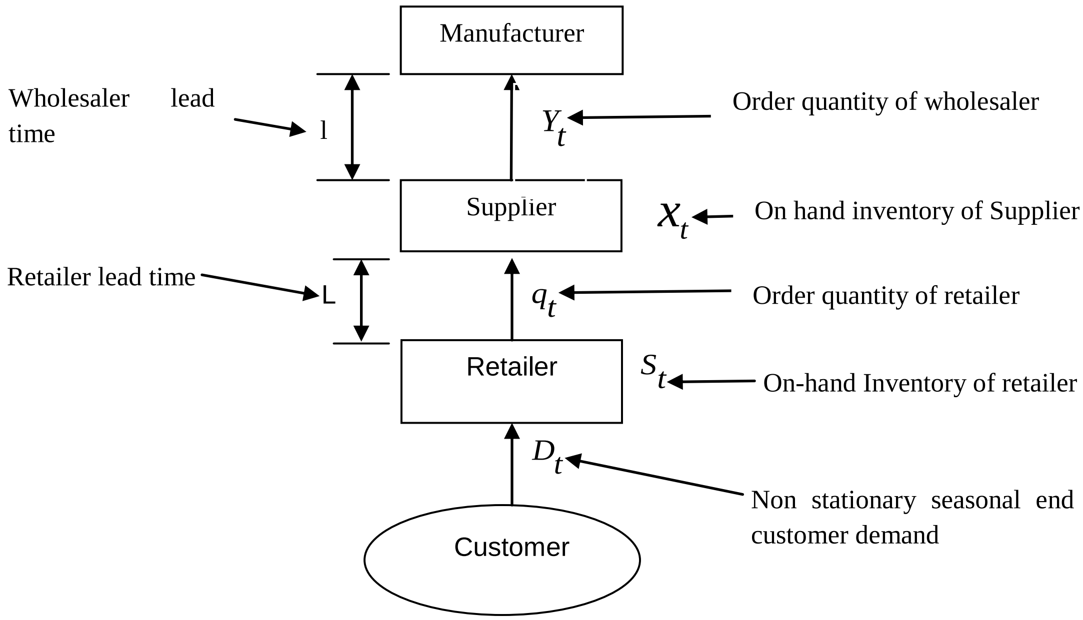

Figure 1: Two level serial supply chain model.

3.1 Assumptions

1. A two echelon supply chain that consists of one wholesaler and one retailer is considered.
2. The retailer's lead time L, is a fixed amount of time between the arrival of the replenishment from the wholesaler and the time at which the retailer's order was placed to the wholesaler.
3. The wholesaler's lead time l, is a fixed amount of time between the arrival of the replenishment from the manufacturer and the time at which the wholesaler's order was placed to the manufacturer.
4. The length of lead time is assumed to be a multiple of the inventory review interval.
5. Both retailer and wholesaler follow an order-up-to-level periodic review policy for the review of their inventories and replenish its inventory from upstream party every period.
6. Both retailer and supplier have an unlimited capacity.
7. Shortages are fully backlogged.
8. The retailer faces external demand for a single product from the end customers, where the underlying process is IMA $(0,1,0)\times(0,0,1)_s$.
9. To forecast the customer demand both retailer and wholesaler employ MMSE technique.
10. The lead times are independent of the order size and the ordering period.
11. If the manufacturer does not have enough stock to fill the order, then we assume that the manufacturer will meet the shortfall by obtaining some units from an alternative source.

3.2 Demand model

The retailer faces an external demand for a single product from end customers, where the underlying demand process is a IMA $(0, 1, 0)\times(0,0,1)_s$ demand process. Let D_t, $t = 1, 2, \cdots$, be the demand process at time period t

$$D_t = D_{t-1} + e_t - \Theta e_{t-s} \tag{3.1}$$

Where D_t is the observe demand for the period t, Θ is constant coefficient expressing the degree of correlation between the demand at the present period and the demand at the previous seasonal period s, $s = 1, 2, \cdots$. Also, e_t is the error term, which is independent and identically normally distributed with mean 0 and variance σ^2.

3.3 Forecasting model

The minimum mean square error (MMSE) forecasting model based on an ARIMA is analysed so that the bullwhip effect can be reduced by suitable design of a forecasting model. The end customer demand process is considered as IMA (0, 1, 0)×(0,0,1)$_s$ process for which Exponential Weighted Moving Average (EWMA) is the MMSE forecasting model. The seasonal demand process is a non-stationary process for which a first order exponential weighted moving average provides the minimum mean square forecast (Agrawal [22]).

Proposition 1: The MMSE-based lead time demand forecast depends on the seasonal cycle, s, and can be expressed as

$$F_t^L = \begin{cases} L * D_{t-1} - \Theta * L\{e_{t-s} + e_{t-s+1} + e_{t-s+2} + \cdots + e_{t-1}\} & L \geq s \\ L * D_{t-1} - \Theta \sum_{i=0}^{L-1} \left((l-i) e_{t-s+i}\right) & L < s \end{cases} \tag{3.2}$$

Proof: It is given in appendix A.

Proposition 2: The variance of the lead time demand forecast error based on MMSE does not depend on the period, t, but does depend on the seasonal cycle, s, and can be given as

$$\left(\hat{\sigma}_t^L\right)^2 = \begin{cases} \dfrac{(L+1)(L+2)}{2}\sigma_e^2 & L < s \\ \sigma_e^2 \left(\dfrac{(L+1)(L+2)}{2} + \Theta^2 \dfrac{(L-s)(L+1-s)}{2} - 2\Theta\left\{ \sum_{i=0}^{L-s-1} \left((L+1-i)(L-s-i)\right) \right\} \right) & L \geq s \end{cases} \tag{3.3}$$

Proof: It is given in appendix B.

3.4 Replenishment policy

Within each period, the retailer's ordering process occurs in the following sequences. Before the end of time period $t, t = 1, 2, \cdots$, the retailer first observes the arriving demands, then decides an order size to bring his inventory position to order-up level S_t and places an order of size q_t. This order placed by the retailer is received at the beginning of time period $t + L + 1$. Any orders for the retailer that are not fulfilled immediately due to excess demand are backordered with penalty cost.

Next, the wholesaler operates his ordering process as follows. At the end of time period $t, t = 1, 2, \cdots$, the wholesaler receives and delivers the required order size q_t to the retailer. After the wholesaler observes q_t, the wholesaler places an order with his upstream party at the end of period t to bring his inventory position to order-up level x_t. This order will arrive at the beginning of $t + l + 1$.

4. Bullwhip Effect at Retailer

The retailer employs a periodic review order-up-to inventory replenishment policy. In this periodic review order up to inventory replenishment policy, an order is placed at the beginning of each period so as to increase the inventory level up to a predetermined level. To accommodate the variances and correlation effects of the demand process, it should therefore be determined for each period separately after observing the recant demand.

Bases on the periodic review order up to inventory replenishment policy discussed above, the retailer places an order of quantity q_t to the wholesaler at the beginning of period t. the order quantity q_t can be given as

$$q_t = S_t - S_{t-1} + D_{t-1}. \tag{4.1}$$

Where S_t is the base-stock level in period t, i.e., the inventory position at the beginning of period t (after the order is placed).

Note that we allow q_t being negative, in which case we assume, that this excess inventory is returned without cost. The optimal order-up level S_t can be determined by the lead time demand as

$$S_t^* = F_t^L + z\hat{\sigma}_t^L \tag{4.2}$$

Where z is the normal z -score chosen to meet a desired service level. The optimal order-up level S_t^* can be determined from the inventory holding and shortage costs. However, since these costs cannot be accurately estimated in practice, the service level approach is often employed when the order-up level is to be determined (Duc et al. [23]).

Note that given a service level, the base-stock level S_t is determined based on lead time demand forecast F_t^L and the standard deviation of the lead time demand forecast error $\hat{\sigma}_t^L$.

Proposition 3: The order quantity q_t depends on the seasonal cycle s and can be given as

$$q_t = \begin{cases} D_{t-1} + L(1-\Theta)e_{t-1} & L \geq s \\ D_{t-1} + Le_{t-1} - \Theta\left(e_{t-s} + e_{t-s+1} + \cdots + e_{t-s+L-1}\right) & L < s \end{cases} \tag{4.3}$$

Proof: It is given in appendix C.

In general, the seasonal cycle has a significant impact on inventory management at each stage in a supply chain. Therefore, from proposition 3, the retailer faced with seasonal demand should replenish inventory considering seasonal phenomena. From Eq. 4.3, the following proposition can be derived.

Proposition 4: The variance of the order quantity q_t depends on the seasonal cycle s and can be expressed as following.

$$Var(q_t) = \begin{cases} (L+1-\Theta L)^2 + (1-\Theta)^2(t-2s-1) + (s-2) & L \geq s \\ (L+1)^2 + (s-L-1) + (1-\Theta)^2(t+L-2s-1) & L < s \end{cases} \tag{4.4}$$

Proof: It is given in appendix D.

From Eq. 4.4, the bullwhip effect measure denoted as B' can be expressed as shown below.

$$B' = \begin{cases} \dfrac{(L+1-\Theta L)^2 + (1-\Theta)^2(t-2s) + (s-2)}{s-1+(1-\Theta)^2(t-2s)} & L \geq s \\ \dfrac{L^2 + (s-L-1) + (1-\Theta)^2(t+L-2s)}{s-1+(1-\Theta)^2(t-2s)} & L < s \end{cases} \tag{4.5}$$

From the above equation it can be seen that when lead time is greater than seasonal cycle

1. Bullwhip effect at retailer increases with increase in lead time.
2. For a given lead time bullwhip effect increases first with increase in Θ, than reaches maximum for $\Theta_{\max}$ and then decreases.
3. Several authors stated that when lead time is greater than seasonal cycle bullwhip effect always increases for stationary demand. Form the above figure it is clear that it is not always true. Lee et al. [21] also stated the same phenomena.
4. The behaviour of bullwhip effect when lead time is less than seasonal cycle is different when lead time is greater than seasonal cycle.
5. For a given lead time bullwhip effect increases with increase in Θ.
6. Bullwhip effect increases with increase in lead time.

5. Conclusion

In the present work a two level supply chain with one retailer and one wholesaler is considered where the objective is to quantify the bullwhip effect for use in a seasonal and non-stationary demand process and investigate the effect of lead time and the seasonal moving average coefficient on bullwhip effect measure. Both the echelons consider an order-up to inventory replenishment policy and experiences deterministic lead time. The retailer faces end customer demand, represented by time series process IMA $(0,1,0)\times(0,0,1)_s$. Analytical results show that with an increase in

the retailer's lead time the bullwhip effect increases with a constant rate till the lead time crosses one season. As the lead time crosses one season, the slope suddenly increases. Further when the value of seasonal moving average parameter changes from positive to negative value, the bullwhip effect decreases. The possible reason for this reduction may be due to the fact, that as seasonal moving average parameter changes from positive to negative, retailer order quantity variance increases does not increases at a faster rate as the end customer demand variance increases.
The results obtained are applicable to multistage supply chain because the order process at each successive level remains seasonal ARIMA process.

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APPENDIX A

The end customer demand is given as

$$D_t = D_{t-1} + e_t - \Theta e_{t-s} \tag{A.1}$$

It is noted that the lead time demand at the retailer can be expressed as

$$D_t^L = D_t + D_{t+1} + \cdots + D_{t+L} = \sum_{i=0}^{L} D_{t+i} \tag{A.2}$$

In addition, with the MMSE technique, the lead time demand forecast F_t^L that the retailer will face over L future periods can be given as

$$F_t^L = F_t + F_{t+1} + \cdots + F_{t+L} = \sum_{i=0}^{L} F_{t+i} \tag{A.3}$$

According to (Box 1976), for the SARIMA (0, 1, 0)×(0,0,1)$_s$ process F_t^L can be determined by

$$F_{t+i} = E\left[D_{t+i} \middle| D_{t-1}, D_{t-2} \cdots\right] \tag{A.4}$$

From which an exact expression of F_{t+i} can be derived. The lead time demand at the retailer can be described using a recursive relation for a constant correlation coefficient. By applying the following equations,

$$D_{t+i} = D_{t-1} + \{e_t + e_{t+1} + \cdots + e_{t+i}\} - \Theta(e_{t-s} + e_{t-s+1} + \cdots + e_{t-s+i}) \tag{A.5}$$

$$D_t^L = L * D_{t-1} + \sum_{i=0}^{L-1} \big((L-i)e_{t+i}\big) - \Theta \sum_{i=0}^{L-1} ((L-i)e_{t-s+i}) \tag{A.6}$$

Taking the conditional expectation from Eq. A.6

$$E\left[e_{t+i} \middle| D_{t-1}, D_{t-2} \cdots\right] = \begin{cases} 0 & i \geq 0 \\ e_{t+i} & i < 0 \end{cases} \tag{A.7}$$

Therefore,

$$F_{t+i} = \begin{cases} D_{t-1} - \Theta(e_{t-s} + e_{t-s+1} + \cdots + e_{t-1}) & i \geq s \\ D_{t-1} - \Theta(e_{t-s} + e_{t-s+1} + \cdots + e_{t-s+i}) & i < s \end{cases} \tag{A.8}$$

From Eq. A.7 and Eq. A.8, we can derive the following preposition.

$$F_t^L = \begin{cases} L * D_{t-1} - \Theta * L\{e_{t-s} + e_{t-s+1} + e_{t-s+2} + \cdots + e_{t-1}\} & L \geq s \\ L * D_{t-1} - \Theta \sum_{i=0}^{L-1} \big((l-i)e_{t-s+i}\big) & L < s \end{cases} \tag{A.9}$$

APPENDIX B

From Eq. A.2 and Eq. A.8

$$\left(D_t^L - F_t^L\right) = \begin{cases} \{(L+1)e_t + (L)e_{t+1} + (L-1)e_{t+2} + \cdots + e_{t+L}\} & L < s \\ \begin{Bmatrix} \{(L+1)e_t + Le_{t+1} + \cdots + e_{t+L}\} \\ -\Theta\{(L+1-s)e_t + (L-s)e_{t+1} + \cdots + e_{t+L-s}\} \end{Bmatrix} & L \geq s \end{cases} \tag{B.1}$$

Taking the variance of Eq. B.1

$$Var\left(D_t^L - F_t^L\right) = \begin{cases} \dfrac{(L+1)(L+2)}{2}\sigma_e^2 & L < s \\ \left\{\sigma_e^2 \begin{pmatrix} \dfrac{(L+1)(L+2)}{2} + \Theta^2 \dfrac{(L-s)(L+1-s)}{2} \\ -2\Theta\left\{\sum_{i=0}^{L-s-1}((L+1-i)(L-s-i))\right\} \end{pmatrix}\right\} & L \geq s \end{cases} \tag{B.2}$$

Solving Eq. B.2 we get,

$$\left(\hat{\sigma}_t^L\right)^2 = \begin{cases} \dfrac{(L+1)(L+2)}{2}\sigma_e^2 & L < s \\ \left\{\sigma_e^2 \begin{pmatrix} \dfrac{(L+1)(L+2)}{2} + \Theta^2 \dfrac{(L-s)(L+1-s)}{2} \\ -2\Theta\left\{\sum_{i=0}^{L-s-1}((L+1-i)(L-s-i))\right\} \end{pmatrix}\right\} & L \geq s \end{cases} \tag{B.3}$$

APPENDIX C

The optimal order quantity q_t is given by

$$q_t = S_t - S_{t-1} + D_{t-1} \tag{C.1}$$

The optimal on hand inventory level at retailer is calculated by

$$S_t^* = F_t^L + z\hat{\sigma}_t^L \tag{C.2}$$

From Eq. B.3 it is clear that $\hat{\sigma}_t^L$ is independent of time. Substituting Eq. C.2 in Eq. C.1 we get

$$q_t = S_t^* - S_t^* + D_{t-1} = \left(F_t^L - F_{t-1}^L\right) + z\left(\hat{\sigma}_t^L - \hat{\sigma}_{t-1}^L\right) + D_{t-1} = \left(F_t^L - F_{t-1}^L\right) + D_{t-1} \tag{C.3}$$

Next, by substituting Eq. A.7 in Eq. C.3, we obtain Eq. C.4.

$$q_t = \begin{cases} D_{t-1} + L(1-\Theta)e_{t-1} & L \geq s \\ D_{t-1} + L\,e_{t-1} - \Theta\left(e_{t-s} + e_{t-s+1} + \cdots + e_{t-s+L-1}\right) & L < s \end{cases} \tag{C.4}$$

APPENDIX D

By the recursive relationship Eq. A.1 can be written as

$$D_t = \left(e_t + e_{t-1} + \cdots + e_{t-s+1}\right) + (1-\Theta)\left(e_{t-s} + e_{t-s-1} + \cdots + e_s\right) \tag{D.1}$$

From Eq. C.4 and Eq. C.5

$$q_t = \begin{cases} \left(e_{t-1} + e_{t-2} + \cdots + e_{t-s}\right) + (1-\Theta)\left(e_{t-s-1} + e_{t-s-2} + \cdots + e_s\right) + L(1-\Theta)e_{t-1} & L \geq s \\ \begin{Bmatrix} \left(e_{t-1} + e_{t-2} + \cdots + e_{t-s}\right) + (1-\Theta)\left(e_{t-s-1} + e_{t-s-2} + \cdots + e_s\right) \\ + Le_{t-1} - \Theta\left(e_{t-s} + e_{t-s+1} + \cdots + e_{t-s+L-1}\right) \end{Bmatrix} & L < s \end{cases} \tag{D.2}$$

After rearranging the error terms in sequence and taking the variance we get

$$Var(q_t) = \begin{cases} (L+1-\Theta L)^2 + (1-\Theta)^2(t-2s-1) + (s-2) & L \geq s \\ (L+1)^2 + (s-L-1) + (1-\Theta)^2(t+L-2s-1) & L < s \end{cases} \tag{D.3}$$

Biographies

Cherian Samuel is a faculty member in the department of Mechanical Engineering, Indian Institute of Technology, Banaras Hindu University, Varanasi, India. He obtained his Bachelor degree in Mechanical Engineering and Masters in Production Engineering from Calicut University and doctoral degree from IIT Kharagpur. He has several publications in peer reviewed journals. His areas of interest include production and operations management, supply chain management etc.

Narayan Kumar is an associate Professor in Faculty of Mechanical Engineering, Sri Ramswroop Memorial University, Lucknow, India. Mr. Kumar holds a Bachelor of Technology degree in Mechanical Engineering from IIITDM, Jabalpur and a Master of Technology degree in Industrial Management from IIT (BHU), Varanasi. He has taught courses in quality management, production & operations management and concurrent engineering. His research interests include forecasting, supply chain management, optimization and applied numerical analysis.