Supply Chain Coordination with Contracts under Bilateral Asymmetric Information

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Abstract

This paper investigates coordination in a supply chain with contracts under bilateral asymmetric information. We study wholesale price, buy-back, revenue share, quantity discount and quantity flexibility contracts. Addressing how to allocate the total supply chain profit between retailer and supplier according to their advantages of information, we establish the bridge, transfer payment scheme, connecting the centralized solution and decentralized solution. In particular, all contracts are shown to coordinate the supply chain under bilateral asymmetric information except the wholesale price contract. Results are illustrated with numerical examples.

Keywords
supply chain coordination; asymmetric information; contract

1. Introduction

Supply chain performance might be suboptimal owing to the different and usually conflicting objectives that the supply chain members attempt to optimize. Classic double marginalization problem takes place. To improve the coordination, designing what kind of contracts such that each member's objective becomes aligned with the supply chain's objective is attractive. Clearly, the standard wholesale price contract cannot achieve the coordination.

This paper explores supply chain coordination with contracts under two-sided incomplete information. We first study the model under complete information as a benchmark. All parties know all parameters and distributions of market demand and production cost. Furthermore, they also know that others know this as well. Beyond questions, no coordination can be achieved with the contracts based on a single mechanism. Furthermore, the supplier and the retailer enter a Stackelberg game in which the supplier acts as the leader and the retailer acts as the follower. We assume the following order of events: the suppler offers the retailer a take-it-or-leave-it contract; if the retailer agrees with the terms of the contract, the retailer submits an order quantity q to the supplier; the supplier produces and delivers to the retailer; demand is realized; transfer payments are made and profits are collected. Otherwise, the game ends and each firm earn a default payoff.

Our article contributes to the existing literature in several ways. First, ours is the first to research on supply chain coordination with contracts under bilateral asymmetric information which is commonly observed in practice. Second, we have analytically obtained how to allocate the total supply chain profit between the retailer and the supplier according to their advantage of information. Third, we have established the bridge, transfer payment scheme, connecting the centralized solution and the decentralized solution. Finally, we examine whether coordination can be achieved with contracts based on a single policy under our setting. Consequently, wholesale price contracts lead to double marginalization while other five contracts can coordinate the supply chain under bilateral asymmetric information. Furthermore, we find out that the key to coordinate supply chain with contracts is trade-off between suppliers and retailers who possess their own information advantages.

2. Related literature

Our study stands on the interface among supply chain coordination, asymmetric information and supply contracts, all of which have attracted a vast amount of research.

As the supply contract literature whether can achieve the coordination is vast and our related contracts include six contracts based on a single mechanism, we ought to select the representative literature to review these six types of contracts briefly. Cachon (2003) explicitly illustrates the six types of contracts. For the basic wholesale price contract, it leads to double marginalization in several setting (e.g. Bernstein and Federgruen, 2003; Cachon and Lariviere (2005); Cachon and Kok, 2010). As for buy back contract, also called return policies, Pasternack (1985) shows that the supplier can offer an infinite variety of such contracts to coordinate the channel , as long as they satisfy a coordination condition. Taylor (2001) analyzes midlife returns and end-of-life returns for channel coordination under a price protection scheme in dynamic markets. Chambers and Snir (2007) consider a supplier selling to a retailer with private information. They demonstrate that the supplier can optimally offer a single buy back contracts to coordinate the channel with certain restriction on the demand distribution. As to revenue sharing contracts, beyond doubts, Cachon and Lariviere (2005) explicitly illustrate that the contract can coordinate a channel either with a single retailer, or with retailers competing in quantities. In addition, revenue sharing does not coordinate with demand that depends on costly retail effort. Tsay (1999) investigates the quantity flexibility (QF) contract in a complex model, which incorporates the issues of capacity planning, information updating, and channel coordination. The setting is which the buyer first estimates a purchase quantity for a given selling season, the supplier then commits to production, and finally the buyer makes the actual purchase based on the updated information. Tsay and Lovejoy (1999) consider the quantity commitment contract in a multi-echelon setting, allowing for non-stationary demand with information updating. Lian and Deshmukh (2008) demonstrate that the further in advance the commitment is made, the larger the discount. Taylor (2002) is the first to examine whether the linear rebate policy and the target sales rebates policy can coordinate the channel, respectively, both of which with sales effort effects. Krishnan et al. (2004) focus on the case in which the retailer chooses an order quantity, a signal of demand is observed and then effort is exerted. Saha (2012) proposes three different types of rebate induced contract for coordination and demonstrates that under certain conditions, both retailer and manufacturer can gain more profit by means of appropriate coordination contracts. At last, as for the quantity discount contract, Weng (1995) shows that either form of discount can be used to achieve channel coordination, and that the supplier would be indifferent between the two. Cachon and Lariviere (2001) use information asymmetric in a single period model of a capacity reservation contract. Tomlin (2003) models with both quantity discount and quantity premium contracts to achieve the coordination. Cachon and Kok (2010) compare the quantity discount with the wholesale price contract and the two-part tariff for the coordination in the setting in which competing manufacturers sell products to a retailer.

To sum, this paper will examine whether above six contracts could achieve the coordination individually in the setting in which both supplier and retailer possess their private information known only by themselves. The research more similar to our work can be found out in Guler and Keskin (2013). They examine whether five types of contracts aforementioned, excluding the target sales rebate contract, can achieve the coordination in a channel with random yield and random demand (SCRYRD). In particular, all contracts are shown to coordinate the channel except the wholesale price contract.

Generally speaking, any one of above six contracts cannot coordinate the decentralized channel, thus channel coordination with contracts literature can be classified into two streams. See Cachon (2003) for a review of this literature. The focus of one stream of research is on the composite contract. Taylor (2002) designs a contract by combing a target rebate contract and a returns contract to achieve the coordination and a win-win outcome. Bernstein and Federgruen (2005) demonstrate the coordination can be achieved by the 'linear price-discount sharing' (LPDS) scheme based on discounted wholesale price and returns policies. Xiong et al. (2010) introduce a composite contract based on buy back and quantity flexibility contracts which has advantages in terms of channel coordination and profit allocation. Chiu et al. (2011) show that a policy that combines the use of wholesale price, channel rebate, and returns, known as the price rebate and returns (PRR), can coordinate the channel. Another stream of research focuses on the setting consisting of either one manufacturer and multiple retailers (e.g. Bernstein and Federgruen, 2003) or competing manufacturer and one retailer. Cachon and Kok (2010) examine whether three types of contract can coordinate in the setting where multiple manufacturers sell through a single retailer. They demonstrate both quantity-discount contract and two-part tariff could achieve while the wholesale price contract fails. The second stream is more similar to our work which focuses on the setting under two-sided incomplete information.

Other than the aforementioned studies, the works most related to this paper involve the study of channel coordination with contracts under asymmetric information. Beyond questions, the channel literature that explicitly models asymmetric information can be sorted into two types. The first types is on asymmetric information in production cost (e.g. Corbett and De Groote (2000) and Ha (2001)). Kayy et.al (2013) talk about either delegation or control can yield substantially higher expected profit for the manufacturer because of information asymmetry about suppliers' production costs and the use of simple quantity discount or price-only contracts. On the contrary, the counterpart focuses on asymmetric information in market demand and forecast (e.g. Porteus and Whang (1999) and Cachon and Lariviere (2001)). Babich et al. (2011) design a hybrid contract that combines the buyback contract and the two-part tariff to achieve the coordination for a supplier who is working with a retailer who possesses private information about the demand distribution. Obviously, to coordinate, the composite contract is still indispensable under unilateral asymmetric information. Nevertheless, whether a single policy can achieve the coordination under two-sided incomplete information is such significant. As for two-sided incomplete information, literature pay attention to the information structure. Esmaeili and Zeephongsekul (2009) propose several seller-buyer channel models under two-sided incomplete information. In particular, a semi-cooperative model, where sharing marketing expenditure is used as an incentive strategy to reveal information, is proposed.

3. Model

3.1 Notation and Formulation

Consider a two-echelon supply chain that consists of a risk-neutral supplier whose private information is production cost and a risk-neutral retailer who possesses exclusive information with regard to market demand. Since both parties possess their own private information, it's possible to harm one's profit with arbitrary profit allocations. In other words, both parties don't require arbitrary profit allocations and this setting is different from general supply chain coordination. The product is produced by the supplier(she) at a unit cost c possessing a cumulative distribution function (CDF) H(·) and a probability density function (PDF) h(·) and sold by the retailer(he) to customers at an exogenous retailer price p. The CDF H(·) is defined over an interval $\left[\underline{c},\bar{c}\right]$. The retailer earns v<c per unit unsold at the end of season where v is net of any salvage expenses. The retailer faces stochastic demand D_ξ which is modeled as $D_\xi = D+\xi$ with D being a continuous random variable with (CDF) F(·) and (PDF) f(·) over its support $\left[-\varepsilon/2, \varepsilon/2\right]$. Let ξ denote this retailer's private information about demand forecast. The supplier resorts to a prior belief and considers ξ to be a zero-mean continuous random variable that takes values in $\left[\underline{\xi},\bar{\xi}\right]$ with (CDF) G(·) and (PDF) g(·). It is also assumed that $\xi-\varepsilon/2\geq 0$ for all ξ to ensure $D_\xi \geq 0$. Let $S(q)=\min(q,D_\xi)$ be expected sales and $I(q)=(q-D_\xi)^+ = q-S(q)$ be the expected left over inventory. Additionally, it is assumed that the shortage penalty is zero.

3.2 Centralized solution

The retailer and the supplier establish a supply chain through a menu of contracts $\{q(c,\xi),t(c,\xi,D_\xi)\}$. Given this menu, the supplier chooses a particular contract $\{q(c',\xi),t(c',\xi,D_\xi)\}$ that maximizes her profit. By doing so, she announces her production cost to be c', which could differ from her true production cost c. Similarly, the retailer selects the contract $\{q(c,\xi'),t(c,\xi',D_\xi)\}$ announcing his forecast information to be ξ'. By assigning the contract $\{q(c',\xi'),t(c',\xi',D_\xi)\}$, the expected profit of supplier and retailer could be defined as

$$\pi_s\left[q(c',\xi'),c\right] = -cq(c',\xi') + E_{\varepsilon\xi}t(c',\xi',D_\xi) \qquad (1)$$

$$\pi_r\left[q(c',\xi'),\xi\right] = p\min(q(c',\xi'),D+\xi) + vI[q(c',\xi')] - E_{\varepsilon c}t(c',\xi',D_\xi) \qquad (2)$$

Also, $E_{\varepsilon\xi}t(c',\xi',D_\xi) = E_{\varepsilon c}t(c',\xi',D_\xi)$. The expected centralized supply chain's profit can be expressed as

$$\pi\left[q(c',\xi')\right] = \pi_s\left[q(c',\xi')\right] + \pi_r\left[q(c',\xi')\right] = (p-v)\min(q(c',\xi'),D+\xi) - (c-v)q(c',\xi') \quad (7)$$

$\pi_s[q(c',\xi'),c]$ denotes the supplier's profit if she chooses the contract $\{q(c',\xi'),t(c',\xi',D_\xi)\}$ when her true production cost is c. Likewise, $\pi_r[q(c',\xi'),\xi]$ denotes the profit of the retailer with his true demand forecast information being ξ if he selects the contract $\{q(c',\xi'),t(c',\xi',D_\xi)\}$.

Therefore, the following programming can depict above bilateral adverse selection about this centralized supply chain.

$$\max_{q(c,\xi)} E_{c\xi}(\pi[q(c,\xi)])$$

s.t. (IC_s) $\pi_s[q(c,\xi),c] \geq \pi_s[q(c',\xi),c],\ \forall c' \in \{\underline{c},\bar{c}\}$ (3)

(IC_r) $\pi_r[q(c,\xi),\xi] \geq \pi_r[q(c,\xi'),\xi],\ \forall \xi' \in \{\underline{\xi},\bar{\xi}\}$ (4)

(IR_s) $\pi_s[q(c,\xi)] \geq 0$ (5)

(IR_r) $\pi_r[q(c,\xi)] \geq 0$ (6)

Among these, incentive compatible constraints (IC_s) and (IC_r) can motivate both parties to tell their true private information. Individual rational constraints (IR_s) and (IR_r) can ensure non-negative revenue toward the lowest type of suppliers and retailers.

Lemma 1. (IC_r) and (IC_s) can be simplified as two differential equations and two monotonic conditions,

(1) $\dfrac{\partial \pi_s(c,\xi)}{\partial c} = -q(c,\xi), \dfrac{\partial \pi_r(c,\xi)}{\partial \xi} = (p-v)F(q(c,\xi)-\xi)$

(2) $\dfrac{\partial q(c,\xi)}{\partial c} \leq 0, \dfrac{\partial F(q(c,\xi)-\xi)}{\partial \xi} \geq 0$

The proofs of all lemmas and propositions can be found in the Appendix A. Part 1 implies that the supplier's profit will be decreasing in c while the retailer's profit will be increasing in ξ. Part 2 shows that order quantity will be decreasing in c but increasing in ξ. According to lemma 1 and individual rational constraints (IR_s) and (IR_r) ,we could get the information rent of both parties. The supplier's information rent is

$$R_s = \pi_s(\bar{c},\xi) + \int_c^{\bar{c}} q(x,\xi)dx = \int_c^{\bar{c}} q(x,\xi)dx \text{ since } \pi_s(\bar{c},\xi) = 0.$$

On the other hand, the retailer's information rent is

$$R_r = \pi_r(c,\underline{\xi}) + \int_{\underline{\xi}}^{\xi}(p-v)F(q(c,y)-y)dy = (p-v)\int_{\underline{\xi}}^{\xi} F(q(c,y)-y)dy \text{ since } \pi_r(c,\underline{\xi}) = 0.$$

Combining R_S with R_r, we can obtain total information rent $R(q(c,\xi))$

$$R(q(c,\xi)) = R_s + R_r = \int_c^{\bar{c}} q(x,\xi)dx + (p-v)\int_{\underline{\xi}}^{\xi} F(q(c,y)-y)dy \quad (7)$$

Meantime, the profit of whole supply chain will be

$$\pi[q(c,\xi)] = (p-v)\min(q(c,\xi), D+\xi) - (c-v)q(c,\xi) \quad (8)$$

By $R(q(c,\xi))$, we can know the total information rent is related to (CDF) of supplier's production cost, retailer's demand forecast information and the market demand. Total information rent is also determined by $q(c,\xi)$, hence, we continue to solve it.

3.1.1 Optimal quantity solution

Based on above analysis, if the supplier reports her production cost to be c' while the retailer announces his demand forecast information to be ξ' , the centralized supply chain's profit will be

$$\pi[q(c',\xi')] = (p-v)\min(q(c',\xi'), D+\xi') - (c'-v)q(c',\xi') \quad (9)$$

Thus, optimal order quantity will be $q^*(c',\xi')=F^{-1}(1-\frac{c'-v}{p-v})+\xi'$ and the proof is similar to proposition 1. If the supplier would like to maximize her profit when both her incentive compatibility constraint and individual rationality constraint are satisfied, her reported cost c' should be equal to be her true production cost c. Likewise, the retailer's best choice is also to announce his true demand forecast information $\xi'=\xi$. Then the optimal quantity will be

$$q^*(c',\xi')=F^{-1}(1-\frac{c-v}{p-v})+\xi \text{ if } \xi'=\xi,\ c'=c, \text{ then } q^*(c,\xi)=q^\circ(c,\xi).$$

Thus the first condition for coordination has been satisfied while it is also the only thing we can optimize. Adding up another dimension of asymmetric information implies adding up another incentive compatible constraint to be satisfied. In other words, one degree of freedom in the contract is consumed to satisfy the additional incentive compatible constraint. It implies that there is not enough degree of freedom to arbitrarily allocate profit to coordinate. However, both parties don't require arbitrary profit allocations under bilateral asymmetric information since arbitrary profit allocations might hurt one party's profit. We can also regard this under bilateral asymmetric information as the difference from general supply chain coordination. Thus it's still possible to coordinate under bilateral asymmetric information and we need to set one rule to allocate profit.

3.1.2 Information rent

To motivate them tell their truth, it is indispensable to pay sufficient information rent to both parties. Information rent refers to monetary income, which is utilized to prevent the information prevailing party telling lies about his private information.

Definition. Based on above total expected rent and respective rent, the proportion of profit division, H_r and H_s, are given below, where $H_s=R_s/R(q^*(c,\xi))$; $H_r=R_r/R(q^*(c,\xi))=1-H_s$

According to the definition of proportion, we can know the proportion is affected by both c and ξ. It makes sense that supplier's information rent will be decreasing in c, but it is an interesting question how it varies with retailer's asymmetric information ξ. So we introduce the interaction of information rent to explain the relationship between two parties' information rent.

Interaction of information rent. Since $R_s=\int_c^{\bar{c}} q^*(x,\xi)dx$, $R_r=(p-v)\int_{\underline{\xi}}^{\xi} F(q^*(c,y)-y)dy$, we could know the bridge to interaction is optimal quantity. Especially, the optimal quantity acts as one part of integrand. Supplier's own asymmetric information influences the integral bounds while retailer's asymmetric information affects the integrand via the bridge. Further, one party's asymmetric information works on not only his own rent directly but also other side's rent by the interaction indirectly. Surely, the proportion will vary with the interaction. These conclusions are all illustrated in Theorem 1.

Theorem 1. (1) $R(q^*(c,\xi))$ is increasing in $q^*(c,\xi)$;

(2) Both $R(q^*(c,\xi))$ and $\pi[q^*(c,\xi)]$ are decreasing in c while increasing in ξ;

(3) Efficient information promotes other side's rent each other;

(4) H_s is decreasing in both c and ξ while H_r is opposite.

Clearly, part 1 could be derived from $\partial R(q^*(c,\xi))/\partial q^*(c,\xi)\geq 0$. Part 1 implies that total information rent is increasing in the optimal quantity. Combining Lemma 1 and part 1, we can know $R(q^*(c,\xi))$ is decreasing in c while increasing in ξ. An agent with higher ξ and lower c is more efficient so that information rent is increasing in the agent's efficiency. Likewise, the performance of whole supply chain is also influenced by the agent's efficiency. Part 2 shows that a more efficient agent will improve the system's performance.

Efficient information implies higher ξ for retailer and lower c for supplier, respectively. Efficient information will not only enlarge his own rent's integral range, but also promotes optimal quantity as the integrand. So part 3 shows that efficient information promotes other side's rent each other. In contrast, inefficient information will decrease both his own rent and opposing rent, further, total information rent will be harmful.

Both retailer's information rent and supplier's are decreasing in c, so the proportion is determined by the comparison between two parties' decline. Especially, the retailer's profit is affected by c indirectly due to decreasing quantity. Part 4 shows that supplier's proportion will be decreasing in c. Since the proportion is based on their advantage of information, it makes sense that proportion will be increasing in their own information rent. Higher c leads to inefficient, so that the supplier with higher c will lose the competition power. On the contrary, the retailer with higher ξ possesses larger competition power, the proportion of profit for supplier will be decreasing in ξ.

Then the dilemma comes to retailer. From the perspective of interaction of information rent, he prefers the supplier with lower cost while on the other hand, to improve his proportion of profit, inefficient supplier is better for him. In other words, he has to make a choice between larger proportion within a smaller whole profit and smaller proportion within a larger whole profit. To solve this dilemma, the second part of the menu of contract, transfer payment, might work since the division of profit has been determined.

3.1.3 Transfer payment

By the proportion of profit division, the supplier's and retailer's profit is respectively

$$\pi_s\left[q^*(c,\xi)\right]=H_s\pi\left[q^*(c,\xi)\right]=H_s\left\{(p-v)S\left[q^*(c,\xi)\right]-(c-v)q^*(c,\xi)\right\} \quad (10)$$

$$\pi_r\left[q^*(c,\xi)\right]=H_r\pi\left[q^*(c,\xi)\right]=H_r\left\{(p-v)S\left[q^*(c,\xi)\right]-(c-v)q^*(c,\xi)\right\} \quad (11)$$

According to both the supplier's and retailer's profit under bilateral asymmetric information, we can obtain

Lemma 2. $t(q^*(c,\xi))=H_s(p-v)S\left[q^*(c,\xi)\right]+(H_s v+H_r c)q^*(c,\xi)$ (12)

Proof. The supplier's and retailer's profit is respectively

$$\pi_s\left[q^*(c,\xi)\right]=-cq^*(c,\xi)+t(q^*(c,\xi)) \quad (13)$$

$$\pi_r\left[q^*(c,\xi)\right]=(p-v)S\left[q^*(c,\xi)\right]-vq^*(c,\xi)-t(q^*(c,\xi)) \quad (14)$$

Combining (10) and (13), we can get (12). Hence we can get Theorem 2.

Theorem 2. (1) $t(q^*(c,\xi))$ is increasing in $q^*(c,\xi)$;

(2) $t(q^*(c,\xi))$ is decreasing in c while increasing in ξ;

(3) $\pi_s\left[q^*(c,\xi)\right]$ is decreasing in c and $\pi_r\left[q^*(c,\xi)\right]$ is also increasing in ξ while not only how $\pi_r\left[q^*(c,\xi)\right]$ changes in c but also how $\pi_s\left[q^*(c,\xi)\right]$ changes in ξ both depend on c and ξ.

Part 1 shows that transfer payment will also be increasing in quantity under bilateral asymmetric information. Part 2 implies transfer payment will increase in the efficiency of both parties. In accordance with the form of transfer payment, we learn about that even though retailers share parts of profits from suppliers due to asymmetric information, retailers also spend parts of cost of suppliers owing to opposite private information.

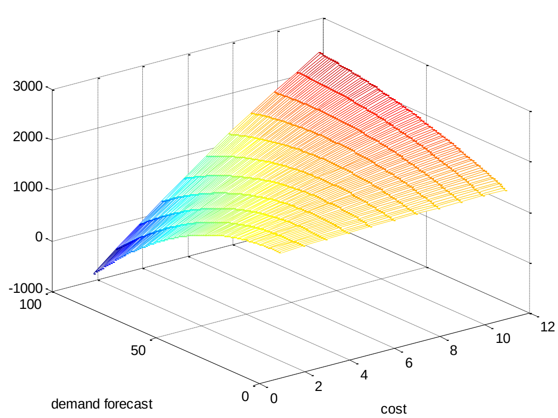

Fig. 1 $B(c,\xi)$

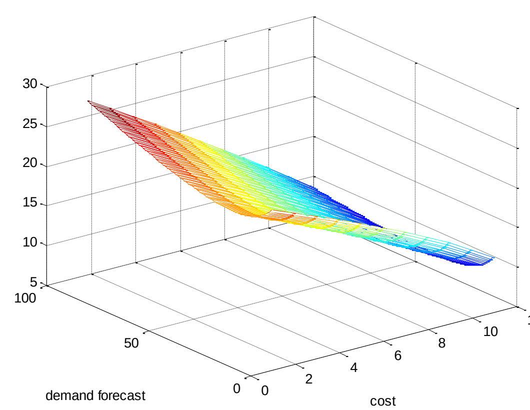

Fig. 2 Supplier's profit

Part 3 implies how the retailer's profit and supplier's profit changes with asymmetric information according to above rules, i.e., $\pi_s\left[q^*(c,\xi)\right]=H_s\pi\left[q^*(c,\xi)\right]$ consisting of two parts differs from $\pi_s\left[q^*(c,\xi)\right]=\int_c^{\bar{c}} q^*(x,\xi)dx$.

Equally, $\pi_s\left[q^*(c,\xi)\right]$ is still decreasing in c. Since both supply chain's profit $\pi\left[q^*(c,\xi)\right]$ and the proportion H_r are increasing in ξ, the retailer's profit will increase with ξ beyond questions. However, supply chain's profit will be decreasing in c while the retailer's proportion will be increasing. Differing from general case, i.e., retailer's profit will decrease with c, how it changes with c depends on both c and ξ under bilateral asymmetric information. In other words, retailer's profit might increase with c if his own private information ξ takes some values. To illustrate this conclusion in a better way, we provide the following example, which indicates that it's possible that $\pi_r\left[q^*(c,\xi)\right]$ will be increasing in c.

Example. Suppose that the demand is uniformly distributed over [0,A] , then we could calculate that $B(c,\xi) := (-\xi R_S + q^*(c,\xi)R_r)\pi\left[q^*(c,\xi)\right] - (\xi + \frac{A(p-2v)}{(p-v)^2}(p-c))R_r R(q^*(c,\xi))$ could determine the sign of $\partial\pi_r / \partial c$. To be more specific, $\xi = 10$, if $c = \bar{c}$, $\partial\pi_r / \partial c > 0$; if $c = \underline{c}$, $\partial\pi_r / \partial c < 0$.

Fig. 1 shows $B(c,\xi)$ with respect to demand forecast and cost. The parameter set is p=12, v=2, A=10, the range of cost is [3,4.5]. By Fig. 2, we can continue to depict how the retailer's profit and supplier's profit changes with asymmetric information. Beyond questions, supplier's profit is decreasing in c while there exists a minimum for any given cost with respect to demand forecast.

Optimal profit is achievable if both parties coordinate by contracting on a set of transfer payments such that individual party's objective becomes aligned with the centralized supply chain's objective. Thus according to advantage of information, allocating the centralized supply chain's profit can coordinate the supply chain. Nevertheless, whether single contract based on established transfer payment scheme could coordinate the supply chain is another problem. Next, we investigate how bilateral asymmetric information changes the contract structure and the parameters in the decentralized setting.

3.2 Decentralized solution

Based on above established transfer payment scheme, the firms can adjust their terms of trade via a contract in order to maximize their profits. In the decentralized setting, the supplier and the retailer maximize their own profit so that they might deviate from the optimal solution of the centralized supply chain. The contracts modify the players' profit using above transfer payment scheme to make both parties choose the optimal solution of the centralized supply chain. First, we focus on the most simple and fundamental wholesales price contract. Based on this, we concentrate on other five contracts based on a single mechanism.

3.2.1 Wholesale price contract

With a wholesale price contract the supplier charges the retailer w per unit purchased: $T_w(w,q) = wq$. Then the retailer's profit function is

$$\pi_r(q,w) = (p-v)S(q) - (w-v)q \qquad (15)$$

To show wholesale price contract could coordinate supply chain, we need to show $q = q^*(c,\xi)$. Substituting $q^*(c,\xi)$ into (15), we can get $\pi_r(q^*(c,\xi),w) = (p-v)S(q^*(c,\xi)) - (w-v)q^*(c,\xi)$. Compared it with $\pi_r\left[q^*(c,\xi)\right] = H_r(p-v)S\left[q^*(c,\xi)\right] - H_r(c-v)q^*(c,\xi)$, we can know $H_r = 1$, $w = c$. Both $H_r = 1$ and $w = c$ imply that the supplier's profit is zero.

Apparently, wholesale price contract could never coordinate the supply chain owing to the double marginalization even though under the bilateral asymmetric information. If wholesale price is larger than supplier's cost, order quantity will be smaller than $q^*(c,\xi)$. To coordinate, a single wholesale price contract without any mechanism, e.g. return policies, channel rebate policies, is obviously inefficient.

3.2.2 Revenue sharing contract

With a revenue sharing contract the supplier charges the retailer w_r per unit purchased plus the retailer gives the supplier a percentage of his revenue. Assume that all revenue is shared, i.e., salvage revenue is also shared between the firms. (It is also possible to design coordinating revenue sharing contracts in which only regular revenue is shared). Let Φ be the fraction of supply chain revenue the retailer keeps, so (1-Φ) is the fraction the supplier earns.

Based on classical revenue sharing contract, obviously, $\Phi = H_r$ in our setting, since both parties don't require arbitrary profit allocations under bilateral asymmetric information.

The realized transfer payment with revenue sharing is

$$T_r(w_r, q, \Phi, D_\xi) = (w_r + (1-\Phi)v)q + (1-\Phi)(p-v)\min(q, D_\xi) \quad (16)$$

A revenue sharing contract can coordinate the chain if $q = q^*(c,\xi)$, so substituting both $\Phi = H_r$ and $q^*(c,\xi)$ into (16), we can obtain the retailer's profit function

$$\pi_r(w_r, q^*(c,\xi), H_r) = H_r(p-v)\min(q^*(c,\xi), D_\xi) - (w_r - H_r v)q^*(c,\xi) \quad (17)$$

Comparing (17) with (11), we can get $w_r(c,\xi) = H_r c$ .The expression of wholesale price is similar to other literature while the difference is that proportion of profit could be seen as a endogenous variable in our setting since it is affected by both parties' asymmetric information. If $H_s = 0$, the revenue sharing contract will reduce to wholesale price contract since $w_r(c,\xi) = c$.

To illustrate the revenue sharing contract in a better way, we can suppose v=0 and substitute $w_r(c,\xi) = H_r c$ into (17), thus $T_r(w_r, q^*(c,\xi), \Phi, D_\xi) = H_r c q^*(c,\xi) + H_s p \min(q^*(c,\xi), D_\xi)$. No doubt that $p\min(q^*(c,\xi), D_\xi) > cq^*(c,\xi)$ if v=0. The retailer minimizes the transfer payment while the supplier's goal is to maximize it. Between $p\min(q^*(c,\xi), D_\xi)$ and $cq^*(c,\xi)$, the retailer prefers the latter while the counterpart is opposite. As for $w_r(c,\xi) = H_r c$, the retailer's better choice is to raise it up, and with the increase of ξ, increasing H_r leads to increasing w_r. Since the effect of increasing ξ is equivalent to the effect of increasing c for retailer, w_r will also be increasing in c.

Compared with other wholesale price in general literature, some interesting conclusions may be obtained since the fraction the supplier earns could be seen as a endogenous variable. From the perspective of supplier, her "one stone"--decreasing her cost--kill three "birds". Decreased c results in decreased w_r directly, meantime, it might also leads to decreasing H_r so that w_r will be decreasing indirectly. Apart from w_r, order quantity will also rise up owing to decreasing c. To sum up, the supplier who possesses more advantage of information could get more transfer payment due to three effects.

3.2.3 Buy back contract

With a buy-back contract, i.e. buyback agreement, return policies, the supplier charges the retailer w_b per unit purchased, but pays the retailer b per unit remaining at the end of the season:

$$T_b(w_b, q, b, D_\xi) = w_b q - bI(q) = b\min(q, D_\xi) + (w_b - b)q \quad (18)$$

Substituting $T_b(w_b, q, b, D_\xi)$ into (21), we can obtain the retailer's profit function

$$\pi_r(q, w_b, b) = (p - v - b)\min(q, D_\xi) - (w_b - b - v)q \quad (19)$$

A buyback contract can coordinate the chain if $q = q^*(c,\xi)$, so substituting $q^*(c,\xi)$ into (19), we can get $\pi_r(q^*(c,\xi), w_b, b) = (p - v - b)\min(q^*(c,\xi), D_\xi) - (w_b - b - v)q^*(c,\xi)$. Compared it with (11), Theorem 3 could be obtained.

Theorem 3. (1) $w_b(c,\xi) = H_s p + H_r c$, $b(c,\xi) = H_s(p-v)$;

(2) $w_b(c,\xi)$ will decrease in ξ while how it changes in c depends on c and ξ.

(3) $b(c,\xi)$ is decreasing in both c and ξ;

Similar to revenue sharing contract, buyback contract in our setting is also based on traditional one. The expression of part 1 is similar to other literature, while the difference is that proportion of profit could be regarded as a endogenous variable in our setting, since it is affected by both parties' asymmetric information. The player who possesses larger proportion of profit owns greater competition power. The case is so suitable in our setting. If $H_s = 0$

, the buyback contract will reduce to a wholesale price contract since $w_b(c,\xi)=c$, $b(c,\xi)=0$; likewise, if $H_s=1$, the retailer's profit will be zero since $w_b(c,\xi)=p$. Owing to $0<H_s<1$, $w_b(c,\xi)=H_s p+H_r c$ can be viewed as a competition between the supplier and the retailer. The retailer minimizes the wholesale price while the supplier's goal is opposite. Thus, the greater competition power the retailer owns, the smaller the wholesale price will be. Comparing p with c, the retailer's best choice is to let minor c be endowed with greater weights. Combined with part 2, we can discuss further.

To comprehend in a better way, we can rearrange the price's expression to be $w_b(c,\xi)=p-H_r(p-c)$. Since the retailer's profit proportion H_r increases with ξ, $w_b(c,\xi)$ will be decreasing in advantage of retailer's private information. Generally, wholesale price will increase with c if the profit allocation proportion is exogenous while the case is opposite in our setting. Owing to endogenous proportion similarly, H_r is increasing in c. By the condition, we can also identify the revenue sharing differs from the buyback though they are generally equivalent. First, unlike revenue sharing, $(p-\bar{c})^2>2\Delta c(p-v)$ as one of sufficient conditions for $\partial w_b(c,\xi)/\partial c<0$ implies that coordinating buyback parameters depend on the retail price. Second, players focus on the wholesales price for buyback contract while they will pay attention to transfer payment for revenue sharing.

Similar to the wholesale price, part 3 shows that efficient supplier will also own a larger buyback rate. It makes sense that supplier's goal is to maximize both wholesale price and buyback rate while the retailer is opposite.

3.2.4 Quantity flexibility contract

With a quantity flexibility (QF) contract the supplier charges w_q per unit purchased but then compensates the retailer for his losses on unsold units. To be specific, the retailer receives a credit from the supplier at the end of the season equal to $(w_q-v)\min(I,\delta q)$ where $\delta\in[0,1]$ is a contract parameter. With the quantity flexibility contract the transfer payment is

$$T_q(w_q,q,\delta,D_\xi)=w_q q-(w_q-v)\min((q-D-\xi)^+,\delta q) \qquad (20)$$

where the last term is the retailer's compensation for unsold units, up to the limit of δq units. The retailer's and supplier's expected profit function under this contract are given, respectively.

$$\pi_r(q,w_q,\delta)=(p-v)S(q)-(w_q-v)q+(w_q-v)\int_{(1-\delta)q-\xi}^{q-\xi}F(x)dx$$

$$\pi_s(q,w_q,\delta)=(w_q-c)q-(w_q-v)\int_{(1-\delta)q-\xi}^{q-\xi}F(x)dx$$

Proposition 1. Expected profit function of retailer in the QF contract is concave in q under bilateral asymmetric information while supplier's concavity is ambiguous.

To coordinate, $\partial\pi_r(q,w_q,\delta)/\partial q=\partial\pi/\partial q$, then

$$w_q(c,\xi,\delta)=\frac{c-v}{(c-v)/(p-v)+(1-\delta)F((1-\delta)q^*(c,\xi)-\xi)}+v \qquad (21)$$

The parameter δ exists implicitly in w_q, thus the price w_q is a function of δ. If $\delta=0$, this contract will reduce to a wholesale price contract since $w_q(c,\xi,0)=c$; likewise, if $\delta=1$, the retailer's profit will be zero even negative since $w_q(c,\xi,1)=p$. Besides, $\partial w_q(c,\xi,\delta)/\partial\delta>0$ implies that the supplier clearly requires larger wholesale price when she gives more credits.

The parameter δ might resemble H_s in the buyback contract so that the two contracts could be comparative. First, unsold parts addressed by the supplier result in the participated expected left over inventory in both contracts. Thus the quantity flexibility contract fully protects the retailer on a portion of the retailer's order whereas the buyback contract gives partial protection on the retailer's entire order. (The retailer continues to salvage left over inventory, which is why the salvage value is not include in each unit's credit). Second, for the parameter δ and H_s, effects on wholesale price is different though they play the similar role. Suppose that the demand is uniformly distributed over

[0,A], then $(1-\delta)F((1-\delta)q^*(c,0))=(1-\delta)^2F(q^*(c,0))$ if $\xi=0$. Thus the price will increase with δ in a square number so that the price gap between a small δ and a large one is so great even though they are still continuous.

Since $\min(I,\delta q)$ is determined by δ, comparing δ with $K(c,\xi)=I(q^*(c,\xi))/q^*(c,\xi)$, we can investigate QF contract further. If δ takes a large value δ_L, transfer payment will be

$$T_q(w_q,q,\delta_L,D_\xi)=w_qq-(w_q-v)I(q)$$

Substituting $T_q(w_q,q,\delta_L,D_\xi)$ into the supplier's expect utility function, we can get

$$\pi_s(q,w_q,\delta_L)=(w_q-v)\min(q,D_\xi)-(c-v)q$$

To coordinate, $\partial\pi_s(q,w_q,\delta_L)/\partial q=\partial\pi/\partial q$, then $w_q(\delta_L)=p$ leads to retailer's zero profit. Clearly, if $\delta_L=1$, retailer's profit will be negative. On the other hand, if δ takes a small value δ_S, transfer payment will be

$$T_q(w_q,q,\delta_S,D_\xi)=(w_q-(w_q-v)\delta_S)q$$

Substituting $T_q(w_q,q,\delta_S,D_\xi)$ into the retailer's expect utility function, we can get

$$\pi_r(q,w_q,\delta_S)=(p-v)\min(q,D_\xi)+vq-(w_q-(w_q-v)\delta_S)q$$

To coordinate, $\partial\pi_r(q,w_q,\delta_S)/\partial q=\partial\pi/\partial q$, then $w_q(\delta_S)=(c-v)/(1-\delta_S)+v$. Substituting $w_q(\delta_S)$ into transfer payment, we can conclude that supplier's profit will be zero. To sum up, a large δ_L leads to retailer's zero profit while a small δ_S results in supplier's zero profit. Coordinating the supply chain with QF contract, an intermediate δ_I ought to be considerable by both parties. Fig. 3 shows $K(c,\xi)$ at different values of production cost and demand forecast. The parameters are the same with the setting given in Section 4.1.4. It could be observed that there is a certain range for $K(c,\xi)$. In other words, δ can't work in the global range. Furthermore, both a large δ_L and a small δ_S make $\delta's$ degree of freedom be inefficient.

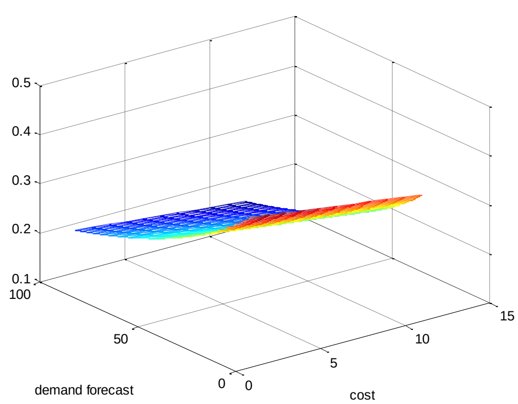

Fig. 3 $K(c,\xi)$

5. Managerial insights and conclusions

To improve the coordination between the supplier and the retailer, there are increasing interests in examining various types of supply chain contracts to achieve channel coordination as well as win–win situations for each partner of the chain. Since contracts based on a single policy, e.g. returns, quantity discounts, revenue sharing, rebates, fail to coordinate the chain, scholars attempt several approach. Beyond question, it is a better method to consider the composite contracts. Varying the setting is another good choice. On the one hand, modifying the supplier-retailer models could be considered, e.g. multiple suppliers single retailer, single supplier multiple retailers, even multiple suppliers multiple retailers. Actually, this is effective. On the other hand, information structure could also be altered, e.g. the retailer's asymmetric information about market demand, the supplier's private information about production cost. As a result, to coordinate, the composite contract is still indispensable under unilateral asymmetric information.

Clearly, this paper varies the setting. We preliminarily attempt to discuss the case under bilateral asymmetric information. Since one dimension of asymmetric information adds up, no additional degree of freedom left to satisfy increased incentive compatible constraint. This implies there is not enough degree of freedom to arbitrarily allocate profit. However, both parties don't require arbitrary profit allocations under bilateral asymmetric information since arbitrary profit allocations might hurt one party's profit. We can also regard this under bilateral asymmetric information as the difference from general supply chain coordination, hence some progress about the coordinated contracts in this paper based on traditional definition has been achieved. Then we have analytically obtained how to allocate the total supply chain profit between the retailer and the supplier according to their advantage of information. The dilemma comes to retailer (also supplier). From the perspective of interaction of information rent, he prefers the supplier with lower cost, while on the other hand, to improve his proportion of profit, inefficient supplier is better for him. In other words, retailer (or supplier) has to select between larger proportion within a smaller whole profit and smaller proportion within a larger whole profit.

Wholesale price contract is prone to double marginalization and still cannot coordinate the chain though under bilateral asymmetric information. Buyback contract could be regarded as the focus in this paper since all other contracts are compared with it. Wholesale price will increase with c if the profit allocation proportion is exogenous while the case is opposite in our setting.

From the perspective of supplier, her “one stone”--decreasing her cost--kills three “birds” with revenue sharing contract. Decreased c results in decreased w_r directly, meantime, it might also leads to decreasing H_r so that w_r will be decreasing indirectly. Apart from w_r, order quantity will also rise up owing to decreasing c. To sum up, the supplier who possesses more advantage of information could get more transfer payment due to three effects. Compared with buyback contract, players focus on the wholesale price for buyback contract while they will pay attention to transfer payment for revenue sharing. Moreover, revenue sharing contract doesn't depend on the retail price. As for quantity discount contract, there exists a price gap between quantity discount and buyback contract.

At last, compared with buyback contract, the quantity flexibility contract fully protects the retailer on a portion of the retailer's order whereas the buyback contract gives partial protection on the retailer's entire order though unsold parts addressed in both contracts. Furthermore, the price will increase with δ in a square number. Coordinating the supply chain with QF contract, an intermediate δ_I ought to be considerable by both parties. In other words, δ can't work in the global range.

(All the proof could be found in https://www.dropbox.com/s/xoor6lq5gi2lxxi/bilateral%20proof.pdf?dl=0)

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