Reducing The Capacitated Lot Sizing Problem (CLSP) With Set Up, Production, Shortage And Inventory Cost To CLSP With Set Up Production and Inventory Cost

RRK Sharma
Department of Industrial Engineering
IIT Kanpur, 208016, India
rrks@iitk.ac.in

Vinayak A. Drave
Department of Industrial Engineering
IIT Kanpur, 208016, India
vinayak@iitk.ac.in

Abstract

Here a capacitated lot sizing problem is formulated differently (by using the formulation given by Vimal (2012 in a totally different context of warehouse location distribution problem); and it leads to new formulation of CLSP with set up, production and inventory costs; and hence valid inequalities given in Miller et. al. (2000) are applicable with few modifications. It opens up new area of research, and we give an additional valid in equality CLSP in this paper (see (9) below). It is to be noted that the model proposed here has additional inequality (apart from less number of variables as in Sharma et. al (2017)), and it would lead to further computational advantage.

Keywords
Reformulation of general CLSP; Promising valid inequalities for CLSP with no shortages allowed

1. Introduction

Capacitated single item lot sizing problem (CLSP) with setup, production, and inventory is a well-studied problem (see Miller et al. (2000) for a detailed literature review). They give several valid inequalities of CLSP which resulted in a reformulation that can be solved much more easily, as shortage variables are eliminated and we have a model that has less numbers of variables which leads to computational advantage (compared to effort required to solve the 0-1 mixed integer programming formulation of CLSP with set up, production, inventory and shortage costs). We pose capacitated single item lot sizing problem (CLSP) with setup, production, backorders and inventory as CLSP with set up, production and inventory costs by using a new formulation given by Vimal (2012). We can then use valid inequality given in Miller et al. (2000) with few alterations. Additional valid inequality is given in this paper (see (9).

2. Problem Formulation

Indices Used
t: Set of Time period from $1..T$

Constants:
f_t : fixed cost in time period 't';
p_t : per unit variable (production) cost in time period 't';
c_t : production capacity in time period 't';
D_t : demand in time period 't';
h_t : per unit inventory carrying cost in time period 't';

sh_t : per unit shortage cost in time period 't';

Definition of Variables:
x_t : amount produced in time period 't';
y_t : 1, if machine setup to produce in time period 't', 0, otherwise;
s_t : Ending shortage in time period 't';
I_t : Entering inventory in time period 't'

Model A1:

Minimize Z $= \sum_{t=1}^{T} f_t{*}y_t + \sum_{t=1}^{T} p_t{*}x_t + \sum_{t=1}^{T} h_t{*}I_t + \sum_{t=1}^{T} sh_t{*}s_t$ (1)

s.t.

$I_0 + \sum_{t=1}^{t_1} x_t + s_{t1} = \sum_{t=1}^{t_1} D_t + I_{t1}\ \forall\ t_1 = 1..T$ (2)

$x_t \leq c_t . y_t\ \forall\ t = 1..T$ (3)

$x_t, I_t, s_t >= 0$; and $y_t = (0,1)$ (4)

This formulation is based on the formulation given for the location-distributed problem with shortages and inventory by Vimal (2012). Traditionally the problem CPLP is formulated as given below. It is reproduced here for the sake of completeness.

Model A2:

Min (1)
$x_t + s_t - s_{t-1} = D_t + (I_t - I_{t-1})\ \forall\ t = 1..T$ (5)

and (3) & (4).

In the conventional formulation of the problem (Model A-2), shortage variables cannot be eliminated, and this can be done in Model A-1 above.

This is done in model A-3 given below

In model (A1), we substitute

$s_{t1} = \sum_{t=1}^{t1}(D_t) + I_{t1} - I0 - \sum_{t=1}^{t1}(x_t)\ \forall\ t_1 = 1..T$ (6)

(6) is substituted in (1) and we get:

Model A3:

$\sum_{t=1}^{T}(p_t{*}x_t){*}y_t + \sum_{t=1}^{T}(h_t){*}I_t + \sum_{t1=1}^{T}(sh_{t1}{*}(\sum_{t=1}^{t1}(D_t) + I_{t1} - I_0 - \sum_{t=1}^{t1}(x_t))$ (7)

And (3) and (4).

It can be easily seen that coefficient of I_t is positive; and coefficient of y_t can be positive, negative or zero. It can be easily seen that Model A3 is capacitated lot sizing problem as given in Miller et. al. (2000). And hence valid inequalities developed in Miller et. al. (2000) can be applied with little adjustments. Below we give a valid inequality which is not frequently considered in literature:

$S_t >= 0$ (8)

$$\sum_{t=1}^{T} y_t * c_t \geq \sum_{t=1}^{T} D_t \quad (9)$$

Thus we give model A4 which is Model A3 + (9). We note that equation (9) is not used frequently in literature. Sharma et.al (2017) have shown that Model A3 will offer computational advantages. It is to be noted that equations (9) is given not in Miller et. al. (2000); Nadjib et al. (2006); Karimi et al. (2003) and Wolsey (2002). It is hoped that we may get better results by solving A4 (due to additional constraint (9)) and that these are not considered in Miller (2000)). This ensures that 'feasibility' of vector of y_j (in Model A4) that is considered in solution method and that it reduces search space.

3. Conclusion

We note that Sharma et. al (2017) posed capacitated lot sizing problem (CLSP) with set up, production, inventory and shortage can be reduced to CLSP with production, set up, and inventory costs using the ideas given in Vimal (2012) (as this eliminated shortage variables and offered computational advantages). We are in a process of empirically validating usefulness of inequality (9) (over the results shown in Sharma et. al (2017)) in Model A4 (not considered in Miller (2000), which was found to be effective in the context of warehouse location problems (Sharma, 1991)and (Sharma and Berry, 2017).

4. References

Karimi, F, and Wilson, JM; The capacitated lot sizing problem: A review of models and algorithms, *Omega*, vol. 3, pp. 365-378, 2003.

Miller, AJ, Nemhauser, GL and Savelsberg, M.W.P; On capacitated lot sizing and continuous 0-1 knapsack polyhedral, *European Journal of Operational Research*, vol. 125, pp. 298-315, 2000.

Nadjib, B., Stephene, DP and Najib MN and Atle, N., Single item lot sizing problem, *European Journal of operational Research*, vol. 168, pp. 1-16, 2006.

Nilanjan, DK, A new formulation for single item capacitated lot sizing problem with shortage and inventory; *Unpublished M Tech thesis*, Dept of Industrial And Management Engineering, IIT Kanpur 208106, India; Thesis Supervisor: Prof. RRK Sharma.

Vimal Kumar, Equal distribution of shortages in supply chain of food corporation of India: Using Lagrangian Relaxation Methodology, *Unpublished M TECH thesis*, Department of Industrial and Management Engineering, Indian Institute of Technology, Kanpur, 208016, 2012.

Sharma, RRK, Modeling a fertilizer distribution system, European Journal of Operational Research; vol. 51, pp. 24-34, 1991.

Sharma, R.R.K. and Berry, V., Developing New Formulations and Relaxations of Single Stage Capacitated Warehouse Location Problem (SSCWLP): Empirical Investigation for Assessing Relative Strengths and Computational Effort, *European Journal of Operational Research*, vol. 177, pp. 803-812, 2007.

Sharma, RRK, Ankita, M, Vimal Kr, Vinay Singh and Pritee Agarwal, Application of Modified Benders Decomposition to Single-Stage Multi-Commodity Multi-Period Warehouse Location Problem: An Empirical Investigation. *American Journal of Operational Research*, vol. 6, pp. 245-259, 2016.

Wolsey, LA, Solving Multi-Item Lot-Sizing Problems with an MIP Solver Using Classification and Reformulation, Vol. 48 (12), pp. 1587 – 1602, 2002.

Sharma, RRK, Vimal Kumar and Nilanjan DK, "Developing a New Reformulation of Single Level Capacitated Lot Sizing Problem (SLCLSP) with Set Up, Shortage and Inventory Costs", American Journal of Operational Research, V 7(5), 2017, pp. 282-284.

Biographies

Prof. R.R.K. Sharma is a professor in the Department of Industrial and Management Engineering, Indian Institute of Technology, Kanpur, India. He has published more than 170 articles of International repute. (Email id- rrks@iitk.ac.in)

Vinayak A. Drave is pursuing his Ph.D. in the Department of Industrial and Management Engineering, IIT, Kanpur, India. (Email id – vinayak@iitk.ac.in)