

# Inventory-Transportation Model Considering Carbon Cap

**Thina Ardliana**

Shipbuilding Institute of Polytechnic Surabaya  
Doctoral Student at Industrial Engineering Department  
Sepuluh Nopember Institute of Technology  
Surabaya, Indonesia  
[ardlianathina@gmail.com](mailto:ardlianathina@gmail.com), [thina15@mhs.ie.its.ac.id](mailto:thina15@mhs.ie.its.ac.id)

**I Nyoman Pujawan and Nurhadi Siswanto**

Industrial Engineering Department  
Sepuluh Nopember Institute of Technology  
Surabaya, Indonesia  
[pujawan@ie.its.ac.id](mailto:pujawan@ie.its.ac.id), [siswanto@ie.its.ac.id](mailto:siswanto@ie.its.ac.id)

## Abstract

Carbon emissions is a subject of growing importance in recent years as companies and countries are under pressure to reduce such emissions to a level below what is called carbon cap. They are expected to find the best solution to reduce the amount of carbon emissions while minimizing the total cost. In this research we developed an integrated model of inventory and transportation from a single supplier to multiple customers considering carbon emission. Our objective is to find an optimum solution that considers both total cost and carbon emissions. Our numerical examples show that the carbon cap affect the total emission and total cost. In addition we also show relationships between the inventory level kept by the customer and the carbon cap.

## Keywords

Carbon cap, emission, mixed integer linear programming

## 1. Introduction

In the last decades, research related to the environmental issues has grown rapidly. The increasing attention of the academics on the environmental issue is due to, among others, the decreasing quality of the environment due to electronic waste and emissions from Green House Gases (GHG). The electronic waste is increasing with the trend of people using electronic products having short product life cycle (Gan *et al.*, 2015). The GHG is a greenhouse gas that has a major contribution to global warming and climate change. Based on the Intergovernmental Panel on Climate Change (IPCC) research report, the current global temperature reaches temperatures of less than 2 degrees Celsius, whereas the maximum carbon emission limit (called carbon budget) is 2 degrees. World Resource Institute (WRI) explained that carbon emissions of the earth have reached 52% of the total carbon budget in 2011 ([www.wri.org](http://www.wri.org)). Meanwhile, the target is to reduce global carbon emissions at least 50% until 2050 (Pan *et al.*, 2014a). From 1850 to 2011, the five countries that contribute the most to this pollutant are the United States (27%), European Union (25%), China (11%), Russia (8%) and Japan (4%) ([www.wri.org](http://www.wri.org)).

In the supply chain area, all activities related to inventory storage, production, and transportation have an impact on carbon emissions. Different decisions related to transportation, for example, would lead to different costs as well as different carbon emissions. Somehow, these two objectives could be a trade-offs, where lower carbon emission maybe achieved with higher cost, or the vice versa. It is therefore important for companies to not only consider traditional objectives such as minimizing costs, but also to take into account carbon emissions when making any supply chain decisions related to production, transportation and inventory (Hua *et al.*, 2011).

Several studies have proposed solution to transportation problems by considering the impact of carbon emissions (Pan et al., 2013; Shaw et al., 2014; Konur, 2014; Konur & Schaefer, 2014; Schaefer & Konur, 2015). The selection of transportation modes is one of the important things that has an effect on reducing carbon emissions in logistics system and the results are significant for both the economic and environmental sectors (Mohammed *et al.*, 2017). Some researchers investigated the effects of various carbon emission policies on modes of transportation. Hoen *et al.* (2010) initiated the study by examining the effects of two regulatory mechanisms (emission costs and constraint costs) on transport mode selection decisions and suggesting policy makers to constrain the emissions of transportation. On the other side, Pan *et al.* (2013) proposed an approach by pooling suppliers of retailers that affect the distribution using two modes of transportation (road and rail) which also affects the quantity of shipments. The objective function was to minimize the total cost of transportation emissions by applying the MILP method. Moreover, Shaw *et al.* (2013) used the goal programming to minimize total costs while maximizing carbon credits.

One way to develop an effective, competitive and eco-friendly supply chain is to integrate transportation and inventory decisions by considering carbon emissions. Konur (2014) developed an optimization model for transportation and inventory considering various types of trucks using heuristic methods. Other relevant studies were presented by Konur & Schaefer (2014) and (2015) where comparisons of two rules of transportation (less than truckload and truckload) were made. The two were only different in terms of the approach used to solve the problems, where one was using analytic and the other one was using Pareto Front Generation.

This paper presents a Mixed Integer Linear Programming (MILP) method model to optimize the total costs related to transportation and storage considering the carbon cap limitation. The model is an attempt to make integrated decision involving inventory and transportation, a subject that has not much explored in the literature, but we believe is an important issue nowadays. In the following sections we will present the description of the problem, the model and the results from numerical examples.

## 2. Mathematical Model

### 2.1 Problem Description

We address a system consists of a single supplier,  $V(0)$ , and multiple customers,  $V(1, 2, \dots, 5)$ . The supplier manages inventory at his own facility as well as the delivery to the customers. Demand for each customer for a certain planning horizon is known. The customers also hold inventory at their own facilities. Initial inventories at each facility is assumed to be zero. We set the policy that no shortage is allowed both at the supplier as well as at the customer.

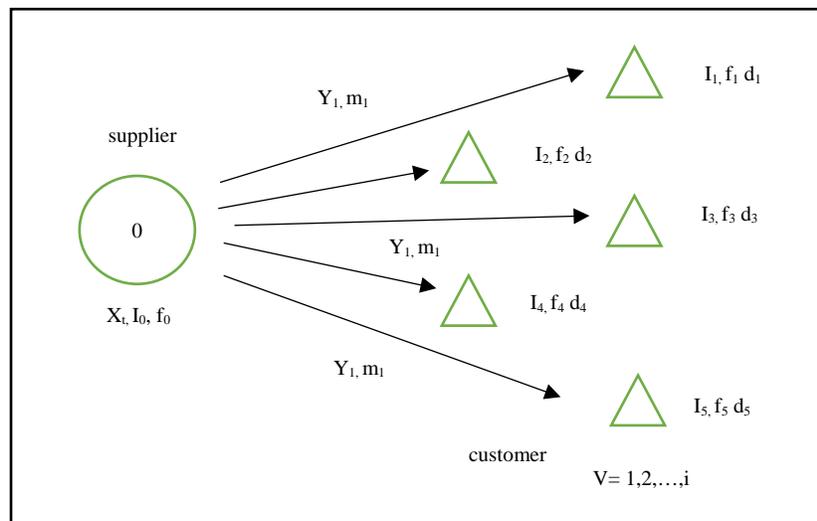


Figure 1. System Configuration

For delivery purposes, each vehicle is assumed to have a certain capacity. There is a cost associated with transporting goods to the customers, holding inventory at the supplier, as well as holding inventory at the customers. Carbon emissions is associated with keeping inventory at any facility and delivery activity from supplier to customers. Figure 1 illustrates the system being studied. The objective is to minimize total costs, but keep the level of carbon emission below the Carbon Cap. Carbon Cap is assumed to be known for the planning horizon.

## 2.2 Notations

This is the definition for index, variable, and parameter in this model:

### Index

- T: Set of time (*planning horizon*): 1, 2, 3, ..., t
- 0: Supplier
- i: customer: 1,2,3, ..., i
- V: Set of customers.

### Inventory variable

- $I_0^t$  : inventory at *supplier* in the end of period  $t$ .
- $I_i^t$  : inventory at *customer i* in the end of period  $t$ .
- $I_0^{t-1}$  : inventory at *supplier* in previous period  $t$ .

### Delivery variable

- $y_i^t$  : quantity of product delivered to *customer i* at period  $t$ .
- $x^t$  : equal to 1 if get the order at period  $t$

### Delivery Parameter

- $d_i^t$ : demand quantity of *customer i* at period  $t$ .
- $K_i$ : maximum capacity at *customer i*.
- $Q$  : vehicle capacity

### Carbon emission Parameter

- Cap : Carbon cap for the planning horizon
- $f_0$  : fixed carbon emission (in tons) associated with inventory at the supplier
- $f_i$  : fixed carbon emission (in tons) associated with inventory at the customer
- $m_i$  : fixed carbon emission (in tons) associated with distribution from supplier to customer  $i$
- $o^t$  : fixed carbon emission (in tons) associated with order

### Parameters for the objective functions

- $c_i$ : Shipping cost from supplier to customer  $i$ .
- $h_0$  : holding cost at *supplier*.
- $h_i$  : holding cost at *customer i*.
- $P^t$  : fixed cost

## 2.3 Model Formulation

The mathematical model of this research:

### Minimize :

$$\sum_{t \in T} h_0 I_0^t + \sum_{i \in V} \sum_{t \in T} h_i I_i^t + \sum_{t \in T} P^t x^t + \sum_{i \in V} \sum_{t \in T} c_i y_i^t \quad (2.1)$$

**Subject To :**

$$I_0^t = I_0^{t-1} + \sum_{i \in V'} y_i^t, \quad \forall t \in T \quad (2.2)$$

$$I_0^t \geq 0, \quad \forall t \in T \quad (2.3)$$

$$I_i^t = I_i^{t-1} + y_i^t - d_i^t, \quad \forall i \in V, \forall t \in T \quad (2.4)$$

$$I_i^t \geq 0, \forall i \in V, \quad \forall t \in T' \quad (2.5)$$

$$\sum_{t \in T} f_0 I_0^t + \sum_{i \in V} \sum_{t \in T} f_i I_i^t + \sum_{t \in T} o^t x^t + \sum_{i \in V} \sum_{t \in T} m_i y_i^t \leq Cap \quad (2.6)$$

$$I_i^t \leq C_i, \quad \forall i \in V, \forall t \in T \quad (2.7)$$

$$y_i^t \leq C_i - I_i^t, \quad \forall i \in V, \forall t \in T \quad (2.8)$$

$$\sum_{j \in V} y_j^t \leq Q, \quad \forall t \in T \quad (2.9)$$

$$y_i^t \geq 0, \quad \forall i \in V, \forall t \in T \quad (2.10)$$

$$x^t \in \{0,1\}, \quad \forall t \in T \quad (2.11)$$

Equation (2.1) is an objective function that minimizes the total cost which is consisting of inventory costs in both the suppliers and the customers, the fixed cost and the shipping cost. Equation (2.2) is the inventory balance equation at the supplier. Equation (2.3) is to ensure that there is no shortage at the supplier. Equation (2.4) is the inventory balance equation at the customer. Equation (2.5) is to ensure that there is no stock out at the customer. Equation (2.6) is the limit of carbon emission. Equation (2.7) limits the maximum inventory level at the customer. Equations (2.8) is to ensure minimum delivery volume is satisfied. Equation (2.9) is a limitation that ensures the volume delivered does not exceed the vehicle capacity. Equations 2.10 and 2.11 are integrality and non-negativity constraints. The model presented here is based on Benjaafar *et al.* (2010) but with extensions to accommodate the integration of inventory and transportation decisions.

### 3. Result

This research involved carbon emission calculation and carbon cap policy as constrains in the model. Carbon Cap (C) is carbon capacity allowed on the supply chain activities. In previous research, Benjaafar *et al.* (2010), Hoen *et al.* (2010), Palak *et al.* (2014) and Mohammed *et al.* (2017) had involved Carbon Cap as one of the parameters to determine its effect on cost optimization and carbon emissions. However, the previous research had not mentioned its impact on customer inventory. Therefore, to fill this gap, we added an analysis of the effect of carbon cap on the amount of customer inventory. Unlike Benjaafar *et al.* (2010) that presented a model with single supplier and single customer, we consider multiple suppliers in our model.

In the model test scenario, we used the following carbon cap values: 220, 250, 270, 300, 330, 350, 400, and 450 (in tons). We ran this model in two periods using one supplier and five customers. Planning horizon was set to be two weeks. Warehouse capacity for each customer was 50, 40, 40, 40, 40 and 30. Capacity of supplier vehicle was 200. Inventory storage cost was \$ 5. Travelling cost was \$ 10. Demand of each customer in the first period was 40, 30, 50, 30, 30 and in the next period was 40, 40, 30, 40, 20. Then, the value of each emission of inventory, order and vehicle emissions were \$3, \$2 and \$10. This numerical data then became input for the mixed integer linear programming model and solved by using Lingo 11.0.

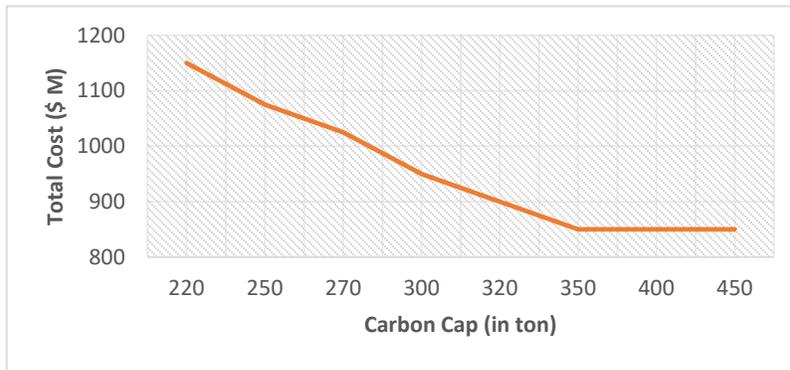


Figure 2. Total Cost vs Carbon Cap

Figure 2 shows the effect of carbon cap constrain on the total system cost. The graph shows the relationship inversely. If the carbon emission capacity increases, the lower is the total cost. This result is in line with some previous studies (Benjaafar *et al.*, 2010; Palak *et al.*, 2014; and Muhammed *et al.*, 2017). When the value of Carbon Cap is above 350 tons, the cost of the system reached an optimal value of \$ 850. However, for carbon cap below 350 tons, the level of emission was proportional to the carbon cap (see Figure 2). The results of this test are consistent with some previous researches (Benjaafar *et al.*, 2010; Hoen *et al.*, 2010; and Muhammed *et al.*, 2017). From the two tests, we can identify a trade-off between total cost and carbon emissions, where the lower emission level is achieved with higher cost.

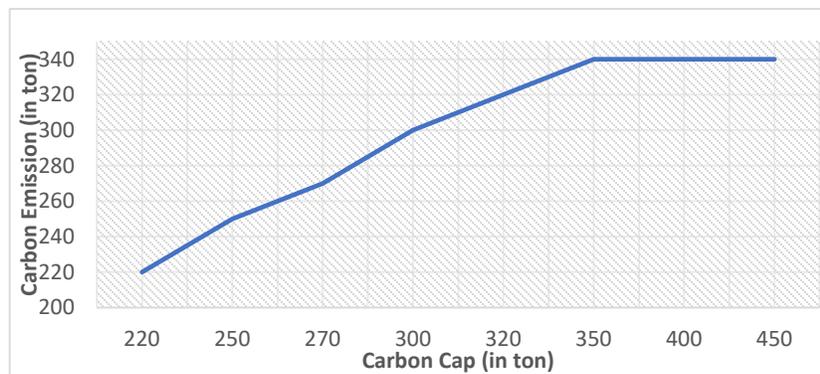


Figure 3. Total Carbon Emission vs Carbon Cap

This research also examined the impact of Carbon Cap restrictions on the amount of inventory held by the customer (shown in Figure 3). The experiments were using four parameter of Carbon Cap, namely C270, C300, C320, and C350 for the entire planning horizon. The results show that the level of inventory held by the customers generally increase when we relax the restriction on the Carbon Cap. The average inventory levels for the above four Carbon Cap values are 135, 150, 160, and 170 respectively. However, when the value of Carbon Cap is kept increased, there will be a limit where the average inventory level will remains the same. This is logical since there are other constraints that play a role, so relaxing the Carbon Cap constraint at some point would not affect the results. This is also consistent with the results presented in Figure 3 above.

Table 1. Inventory held at customers under different Carbon Cap values

Carbon Cap	Average Inventory held by Customers
270	135
300	150
320	160
350	170

## 4. Conclusion

In this research we attempt to model the inventory and delivery decisions by considering carbon cap. We also explore the impact of varying carbon cap values on the total costs and the inventory level. We show that there is a trade-off between emission and total cost, that is, lower emission is achieved at higher cost. The results is in line with earlier works such as Benjaafar *et al.*, 2010; Hoen *et al.*, 2010; Palak *et al.*, 2014; and Muhammed *et al.*, 2017. An implication to this is that there is a potential that companies need to absorb more costs when the objective related to the environment is given a higher priority. In such a case an important future works is to develop a framework for making decisions under different situations, for example, under a different level of willingness to absorb additional costs in the supply chain. In this research, we also attempt to explore the relationships between inventory level kept by the customer and the carbon emission. The results show that customer tends to keep higher inventory when the carbon cap is relaxed. However, to find a firm conclusion on this, further works involving more extensive set of experiments would be needed.

## References

- Benjaafar, S., Li, Y., Daskin, M., (2010), "Carbon Footprint and the Management of Supply Chains: Insights from Simple Models". *IEEE Trans. Autom. Sci. Eng.*, 10 (1), 99–116.
- Gan, S. S., Pujawan, I N., Widodo, B., and Suparno (2015), "Pricing decision model for new and remanufactured short-life cycle products with time-dependent demand". *Operations Research Perspectives*, 2, 1-12.
- Hua, G., Cheng, T. C. E., & Wang, S., (2011), "Managing Carbon Footprints in Inventory Management", *International Journal of Production Economics*, 132(2), 178–185.
- Hoen, K.M.R., Tan, T., Fransoo, J.C., Houtum, G.J., (2010), "Effect of carbon emission regulations on transport mode selection in supply chains", [http://cms.ieis.tue.nl/Beta/Files/WorkingPapers/Beta\\_wp308.pdf](http://cms.ieis.tue.nl/Beta/Files/WorkingPapers/Beta_wp308.pdf), accessed on 20/05/2010.
- Konur, D., (2014), "Carbon constrained integrated inventory control and truckload transportation with heterogenous freight trucks", *Int J Prod Econ*, 153: 268-279.
- Konur, D., & Schaefer, B., (2014), "Integrated Inventory Control and Transportation Decisions under Carbon Emissions Regulations: LTL vs. TL Carriers", *Transportation Research Part E: Logistics and Transportation Review*, 68, 14–38.
- Mohammed, F., Selim, S. Z., Hassan, A., & Syed, M. N., (2017), "Multi-Period Planning of Closed-Loop Supply chain with Carbon Policies under Uncertainty", *Transportation Research Part D: Transport and Environment*, 51, 146–172
- Palak, G., Eksioglu, S.D., Geunes, J., (2014), "Analyzing the impacts of carbon regulatory mechanisms on supplier and mode selection decisions: an application to a biofuel supply chain", *Int. J. Prod. Econ.*, 154, 198–216
- Pan, S., Ballot, E., & Fontane, F., (2013), "The Reduction of Greenhouse Gas Emissions from Freight Transport by Pooling Supply chains", *International Journal of Production Economics*, 143(1), 86–94
- Shaw, K., Shankar, R., Yadav, S. S., & Thakur, L. S., (2013), "Production Planning & Control : The Management of Operations Modeling a Low-Carbon Garment Supply chain", *Production Planning & Control*, 24(8–9), 851–865
- WRI, (2013). "World's Carbon Budget to be Spent in Three Decades". World Resource Institute, Washington, DC. [online]. Available: <http://www.wri.org/blog/2013/09/world%E2%80%99s-carbon-budget-be-spent-three-decades>. Accessed on 11 April 2017

## Biographies

**Thina Ardliana** is a Lecturer at Shipbuilding Institute of Polytechnic Surabaya (PPNS). She received her bachelor diploma in Mathematics from Sepuluh Nopember Institute of Technology (ITS), Surabaya, Indonesia. She earned her Master's degree in Industrial Engineering Department from ITS. She is currently a PhD Candidate at Industrial Engineering Department, Sepuluh Nopember Institute of Technology (ITS), Surabaya, Indonesia. Her research area is optimization in supply chain management.

**I Nyoman Pujawan** is Professor of Supply Chain Engineering at the Department of Industrial Engineering, Sepuluh Nopember Institute of Technology (ITS), Surabaya, Indonesia. He received a bachelor degree in Industrial Engineering from ITS, M.Eng in Industrial Engineering from Asian Institute of Technology (AIT), Thailand, and PhD

in Management Science from Lancaster University, UK. He has published articles in various international journals including the *European Journal of Operational Research*, *International Journal of Production Research*, *International Journal of Production Economics*, *Production Planning and Control*, *Business Process Management Journal*, *International Journal of Industrial and Systems Engineering*, among others.

**Nurhadi Siswanto** is currently the Head of Industrial Engineering Department Sepuluh Nopember Institute of Technology (ITS), Indonesia. He holds a Bachelor of Science degree in Industrial Engineering from Sepuluh Nopember Institute of Technology (ITS), Surabaya, Indonesia, Master of Science in Industrial Engineering (MSIE) from Purdue University, West Lafayette, Indiana, USA and PhD in Computer Science with specialization in Operation Research from University of New South Wales (UNSW), Canberra, Australia.