A Goal Programming Model for Nurse Scheduling at Emergency Department

Nasr Al-Hinai
Department of Mechanical & Industrial Engineering
Sultan Qaboos University
Muscat, Oman
nhinai@squ.edu.om

Noor Al-Yazidy
Department of Mechanical & Industrial Engineering
Sultan Qaboos University
Muscat, Oman
u103313@student.squ.edu.om

Anfal Al-Hooti
Department of Mechanical & Industrial Engineering
Sultan Qaboos University
Muscat, Oman
u102973@student.squ.edu.om

Ekhlas Al-Shereiqi
Department of Mechanical & Industrial Engineering
Sultan Qaboos University
Muscat, Oman
u101176@student.squ.edu.om

Abstract

The nurse scheduling problem is among the most widely studied problem in literature. Nurse scheduling is concerned about allocating nurses to conduct specific tasks. The various objectives and constraints of this problem that changes from country to country and from hospital to hospital makes it very hard to solve. Furthermore, lack of resources makes this scheduling task very challenging. Many hospitals are still executing nurses scheduling manually, which is indeed a cumbersome and time consuming task. This, however, may lead to high chances and risks of not fulfilling all rules set by the hospital. Therefore, this article presents a goal programming framework model to scheduling nurses working at the emergency department with the objective to increase fairness in shifts distribution. The experimental results highlights the importance of developing heuristic or meta-heuristic approaches to solve such NP-hard problem.

Keywords

Nurse Scheduling Problem; Goal Programming; Multi-objective Optimization; Emergency Department; Scheduling Fairness

I. Introduction

Due to its complexity, the nurse scheduling problem has been a widely studied topic in the last few decades. Nurse scheduling is the procedure of developing rosters or timetables for nurses working at the hospital or at a specific ward in such a way that these rosters fulfil the staff requirements according to the number of patients, and respect the workplace regulations and policies. The schedules assign nurses to different shifts within a given time horizon such as a week or a month. Scheduling can greatly affect the performance and efficiency of the work and healthcare services. Nurse scheduling is usually done manually in many hospitals. However, it is cumbersome task to manually develop a schedule that covers the requirements, yet distributes shifts fairly amongst employees, and takes into account their preferences (Morizawa and Nagasaw, 2007).

The Emergency Department (ED) serves patients with medical emergencies such as accidents and injuries and other probably life-threatening conditions. The ER runs 24 hours, 7 days a week. Sufficient nurse staffing level is very important, because shortages in the number of nurses will greatly affect the quality and performance of the health services. Thus, this paper presents a general mathematical model for nurse scheduling working at an Emergency Department. The developed mathematical model aims to divide the load of work amongst the nurses equally and fairly, and improve their satisfaction.

The rest of the paper is structured as follows. Section II briefly explores the past research carried out in this area. Section III discusses general assumptions and constraints incorporated in this research. Section IV presents and discusses proposed mathematical model. Section V discusses the Numerical example carried out to test the efficiency of proposed mathematical model. Finally, section VI gives the concluding remarks and future research directions.

II. Literature Review

Nurse scheduling is indeed an important aspect in healthcare. Nurse scheduling problem becomes one of the most complicated problems, because the hospitals are dissimilar from other organizations. In general, it is related to staff scheduling aiming at satisfying certain objectives to improve the overall satisfaction or increase productivity. Scheduling problems and applications differ from one to another. These differences mandate the use of different solution methods and approaches.

Al-Najjar (2011) summarized the importance of staffing and scheduling Emergency Rooms. The author identified the scheduling as allocation of the tasks to the workers. Al-Najjar classified scheduling into three types: personnel, demand and operation scheduling. In this work, five possible points were recognized as a possible measure of the efficiency of the produced nurse schedule. These includes coverage, quality, stability, flexibility and cost.

According to Blochliger (2003), the modeling process for scheduling problems is usually divided into 4 major steps: data collection, constraints analysis and classification, development of the objective function, and finally the development of the model. Following these steps, Azaiez and Sharif (2005) developed an automated nurse scheduling system to replace the old manual scheduling system at Al-Kharj hospital in Saudi Arabia. The automated nurse scheduling system was developed based on 0-1 programming.

0-1 goal programming was also used to solve the nurse scheduling problem at Tafo Government hospital in Khumasi-Ghana by Agyei et al. (2015). The developed mathematical model aimed to improve the satisfaction of nurses by improving fairness and considering nurse preference. Jafari et al. (2016) presented a fuzzy mathematical programming model to maximize the nurses' preferences to work in their favorable shifts as well as to minimize the total surplus nurses to cover the demands of each day. Recently, Hakim et al. (2017) proposed three models to schedule nurses in Bogor State Hospital. They proposed two goal programming based model to assign nurses a certain inpatient unit and to manage nurses who are assigned to an inpatient room as well as at Polyclinic room as conjunct nurses, respectively.

Constantino et al. (2014) presented multi-assignment problem-based algorithm (MAPA) deterministic heuristic to schedule with the objective to maximize the satisfaction of nurses' preferences and minimize the violation of soft constraints. Chen et al. (2016) developed a two-stage algorithm based on goal programming. The objective was to determine the minimum required medical staff using worst-case scenario in the first stage of the heuristic. In the second stage of their approach, they applied the analytic hierarchy process (AHP) to determine the penalty of soft constraints.

III. Constraints And Assumptions

As stated before, nurse scheduling problem is very complicated due to the fact that hospitals are governed by the healthcare rules and regulations which differs from one country to the other and from hospital to the other.

Therefore, in this research the authors have adopted the healthcare regulations imposed by the Ministry of Health at the Sultanate of Oman and followed the policies and rules followed by the Emergency Department at Sultan Qaboos University Hospital. Thus, the following rules and regulations are used to develop the mathematical:

1. Night shift should not be followed by a day shift.
2. Night shift should not be followed by an evening shift.
3. An evening shift should not be followed by a day shift.

4. The number of working hours is 140 hours for each worker per one month.
5. There is a minimum and a maximum number of nurses for each shift.
6. The minimum number of Days off per four week is 7 days and maximum is 8 days.
7. There are three level of nurses, junior, intermediate, and senior nurses.
8. Each shift must have a minimum number of senior nurses.
9. Not more than 6 Night shifts should be assigned to one nurse per 4 weeks.
10. No payments are given for overtime. However, a nurse is compensated with a leave equivalent to overtime hours.

IV. PROPOSED MODEL

The following mathematical model describes the nurse scheduling problem. The model consists of general constraints of scheduling problem.

$$\text{MIN } z = P_1 \sum_{k=1}^{d} (d1^-_k + d3^-_k + d2^-_k) + P_2 \sum_{k=1}^{d} [\ \sum_{j=1}^{n} (f1^+_{j,k} + f1^-_{j,k}) + \sum_{i=1}^{m} (f2^+_{i,k} + f2^-_{i,k}) + \sum_{s=1}^{p} (f3^+_{s,k} + f3^+_{s,k})\ ] \quad (1)$$

Subject to

$$JD_{j,k} + JE_{j,k} + JN_{j,k} + JO_{j,k} = 1 \quad \forall\ j = 1, 2, \ldots, n\ ;\ k = 1, 2, \ldots, d \quad (2)$$
$$ID_{i,k} + IE_{i,k} + IN_{i,k} + IO_{i,k} = 1 \quad \forall\ i = 1, 2, \ldots, m\ ;\ k = 1, 2, \ldots, d \quad (3)$$
$$SD_{s,k} + SE_{s,k} + SN_{s,k} + SO_{s,k} = 1 \quad \forall\ s = 1, 2, \ldots, p\ ;\ k = 1, 2, \ldots, d \quad (4)$$
$$\sum_{c=0}^{md} JO_{j,k+c} \geq 1 \quad \forall\ k = 1, 2, \ldots, d-md\ ;\ j = 1, 2, \ldots, n \quad (5)$$
$$\sum_{c=0}^{md} IO_{i,k+c} \geq 1 \quad \forall\ k = 1, 2, \ldots, d-md\ ;\ i = 1, 2, \ldots, m \quad (6)$$
$$\sum_{c=0}^{md} SO_{i,k+c} \geq 1 \quad \forall\ k = 1, 2, \ldots, d-md\ ;\ s = 1, 2, \ldots, p \quad (7)$$
$$\sum_{K=1}^{d} JO_{j,k} \geq mo \quad \forall\ j = 1, 2, \ldots, n \quad (8)$$
$$\sum_{K=1}^{d} IO_{i,k} \geq mo \quad \forall\ i = 1, 2, \ldots, m \quad (9)$$
$$\sum_{K=1}^{d} SO_{s,k} \geq mo \quad \forall\ s = 1, 2, \ldots, p \quad (10)$$
$$\sum_{j=1}^{n} JD_{j,k} + \sum_{i=1}^{m} ID_{i,k} + \sum_{s=1}^{p} SD_{s,k} + d1^-_k - d1^+_k = D_k \quad \forall\ k = 1, 2, \ldots, d \quad (11)$$
$$\sum_{j=1}^{n} JE_{j,k} + \sum_{i=1}^{m} IE_{i,k} + \sum_{s=1}^{p} SE_{s,k} + d2^-_k - d2^+_k = E_k \quad \forall\ k = 1, 2, \ldots, d \quad (12)$$
$$\sum_{j=1}^{n} JN_{j,k} + \sum_{i=1}^{m} IN_{i,k} + \sum_{s=1}^{p} SN_{s,k} + d3^-_k - d3^+_k = N_k \quad \forall\ k = 1, 2, \ldots, d \quad (13)$$
$$\sum_{K=1}^{d} [(dh)JD_{j,k} + (eh)JE_{j,k} + (nh)JN_{j,k} + f1^-_{j,k} - f1^+_{j,k}] = h \quad \forall\ j = 1, 2, \ldots, n \quad (14)$$
$$\sum_{K=1}^{d} [(dh)ID_{i,k} + (eh)IE_{i,k} + (nh)IN_{i,k} + f2^-_{i,k} - f2^+_{i,k}] = h \quad \forall\ i = 1, 2, \ldots, m \quad (15)$$
$$\sum_{K=1}^{d} [(dh)SD_{i,k} + (eh)SE_{i,k} + (nh)SN_{i,k} + f3^-_{s,k} - f3^+_{s,k}] = h \quad \forall\ s = 1, 2, \ldots, p \quad (16)$$
$$d1^-_k,\ d1^+_k,\ d12^-_k,\ d2^+_k,\ d3^-_k,\ d3^+_k \geq 0 \quad (17)$$
$$f1^-_{j,k},\ f1^+_{j,k},\ f2^-_{i,k},\ f2^+_{i,k},\ f3^-_{s,k},\ f3^+_{s,k} \geq 0 \quad (18)$$
$$JD_{j,k},\ JE_{j,k},\ JN_{j,k},\ JO_{j,k},\ ID_{i,k},\ IE_{i,k},\ IN_{i,k},\ IO_{i,k},\ SD_{s,k},\ SE_{s,k},\ SN_{s,k},\ SD_{s,k} = (0,1)$$
$$\forall\ k = 1, 2, \ldots, d;\ j = 1, 2, \ldots, n\ ;\ i = 1, 2, \ldots, m\ ;\ s = 1, 2, \ldots, p \quad (19)$$

The objective function (1) of this problem is to minimize the shortages of nurses in different shifts and minimize variations in workload distribution between them.

The constraints of the model are divided into two types: soft constraint, and hard constraints.

1. Hard constraints:

Equations (2-4) force nurses to be assigned to only one shift per day or given a day off for junior, intermediate, and senior nurses, respectively. Equations (5-7) indicate that nurses cannot work more than (md) consecutive days for junior, intermediate and senior nurses, respectively. Equations (8-10) force the assignment of at least (mo) days off for all junior, intermediate, and senior nurses.

2. Soft constraints:

Equations (11-13) are to avoid any shortage or surplus of nurses in all day, evening, and night shifts, respectively. The deviational variables ($d1^-_k$, $d2^-_k$, $d3^-_k$) represent slack or shortage of nurses. On the other hand, ($d1^+_k$, $d2^+_k$, $d3^+_k$) represent surplus or excess nurses assigned to a shift. Equations (14-16) are to distribute the workload amongst the nurses equally. Deviational variables ($f1^-_{j,k}$, $f1^+_{j,k}$, $f2^-_{i,k}$, $f2^+_{i,k}$, $f3^-_{s,k}$, $f3^+_{s,k}$) are added to represent overtime or undertime. ($f1^+_{j,k}$, $f2^+_{i,k}$, $f3^+_{s,k}$) represent that nurse j/i/s works overtime on day k. on the other hand, ($f1^-_{j,k}$, $f2^-_{i,k}$, $f3^-_{s,k}$) represent that nurse i/j/s works undertime on day k.

Lastly, Equations (17-18) are the non-negativity constraint for the deviational variables ($d1^-_k$, $d1^+_k$, $d12^-_k$, $d2^+_k$, $d3^-_k$, $d3^+_k$) and the non-negativity constraint for the deviational variables ($f1^-_{j,k}$, $f1^+_{j,k}$, $f2^-_{i,k}$, $f2^+_{i,k}$, $f3^-_{s,k}$ – $f3^+_{s,k}$), respectively. Equation (19) defines the variables as binary variables.

V. Computational Results

The nurse scheduling problem was proven to be NP-hard problem. Nevertheless, to test the efficiency of the proposed model a small scale test case problem was randomly generated. Details of the test case problem is as follows:

- Number of senior nurses = 12
- Number of intermediate nurses = 12
- Number of junior nurses = 12
- Number of scheduling horizon days = 14
- Minimum days off per nurse per schedule = 5
- Maximum night shifts per nurse per schedule = 6
- Maximum consecutive working days = 5
- Average number of shifts per nurse per schedule = 10
- Required number of nurses per shift = 8
- Minimum number of senior nurses per shift = 1
- Minimum number of intermediate nurses per shift = 3
- Minimum number of senior nurses per shift = 2

The test case problem was solved by implementing the mathematical model using LINGO version 13.0. However, LINGO was unable to handle this size. The infeasibility was due to the constraint of having integer and binary deviational variables. When the integer deviational variables constraint was relaxed, a feasible solution was found, however, it makes no sense to have number of nurses in real numbers.
This finding emphasis on the importance of developing heuristic or meta-heuristic approaches to solve such NP-hard problem.

VI. Conclusion And Futurework

This paper developed a mathematical model to solve the nurse scheduling problem. The mathematical model considered nurses working at an emergency department where there is three level of nurses' skills. The objective of the model is to minimize the shortages of nurses in a given shift while minimizing the variation in workload distribution between them during the scheduling horizon. The constraints of the model were designed based on the regulations that are considered at the Emergency Department at Sultan Qaboos University Hospital.

This problem is a well-known NP-hard problem. The developed mathematical model can be used to solve only small size problem. It is necessary to develop some intelligent method such as heuristic or meta-heurist to deal with medium to large size real problem. Therefore, in the near future, some intelligent methods will be developed and extensive numerical analysis will be conducted to compare them with mathematical model developed in this research.

Acknowledgment

Authors would like to extend their gratitude and appreciations to Sultan Qaboos University for providing the necessary funding and assistance to conduct this work.

References

Al-Najjar, S. M., and Ali, S. H., Staffing and Scheduling Emergency Rooms in Two Public Hospitals: A case study, *International Journal of Business Administration*, vol. 2, No. 2, pp. 137-148, 2011.

Azaiez, M.N, and Sharif, S.S., A 0-1 goal programming for nurse scheduling, *Computers and Operations Research* vol. 32, issue 3, pp. 491-507, 2005.

Agyei, W., Obeng-Denteh, W., and Andaam, E. A., Modeling Nurse Scheduling Problem Using 0-1 Goal Programming : A Case Study of Tafo Government Hospital, Kumasi-Ghana, *International Journal of Scientific and Technology Research*, vol. 4, issue 3, pp. 5-10, 2015.

Blöchliger, I., Modeling staff scheduling problems. A tutorial, *European Journal of Operational Research*, vol. 158, issue 3, pp. 533-542, 2004.

Chen, P.-S., Lin, Y.-J., and Peng, N.-C., A two-stage method to determine the allocation and scheduling of medical staff in uncertain environments, *Computers and Industrial Engineering*, vol. 99, pp. 174-188.

Constantino, A.A., Landa-Silva, D., de Melo, E.L., de Mendonça, C.F.X., Rizzato, D.B., and Romão, W., A heuristic algorithm based on multi-assignment procedures for nurse scheduling, *Annals of Operations Research*, vol. 218, issue 1, pp. 165-183, 2014.

Jafari, H., Bateni, S., Daneshvar, P., Bateni, S., and Mahdioun, H., Fuzzy Mathematical Modeling Approach for the Nurse Scheduling Problem: A Case Study, *International Journal of Fuzzy Systems*, vol 18, issue 2, pp. 320-332.

Hakim, L., Bakhtiar, T., and Jaharuddin, The nurse scheduling problem: A goal programming and nonlinear optimization approaches, *IOP Conference Series: Materials Science and Engineering*, vol. 166, issue 8, 8 February 2017, Article number 012024.

Morizawa, K., and Nagasawa, H., A Heuristic Method in Static Nurse Scheduling, *proceedings of 19th International Conference on Production Research*, pp. 599-8531, 2007.

BIOGRAPHY

Nasr Al-Hinai is an Assistant Professor, in Mechanical and Industrial Engineering in Sultan Qaboos University (SQU). He earned his B.Eng. in Mechanical Engineering from SQU, Masters of Sceince in Advanced Manufacturing Technology and Systmes Management from UMIST-UK, and PhD in Production Planning from University of Manitoba-Canada. His research interests include production planning and control, optimization, meta-heuristics, product development, manufacturing, simulation, scheduling and Six Sigma.

Noor Al-Yazidy is a research assistant at the Department of Mechanical and Industrial Engineering, Sultan Qaboos University, Sultanate of Oman. She received her B.Eng. in Industrial Engineering from SQU. Her research interests include production planning and control, operations and supply chain management.

Anfal Al-Hooti received her B.Eng. in Industrial Engineering from SQU. Her research interests include production planning and control, operations and supply chain management.

Ekhlas Al-Shereiqi received her B.Eng. in Industrial Engineering from SQU. Her research interests include production planning and control, operations and supply chain management.