Equal Distribution of Shortages in Supply Chain Problem of Food Corporation of India

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Abstract: The purpose of this research is to develop a twin objectives formulation considering cost and back order minimization of supply chain operational problem faced by Food Corporation of India (FCI). Cost minimization problem is a standard LP (Linear Programming) and we consider quadratic objective formula for the back order minimization problem which turns out to be NLP (Non Linear Programming). We attempt solving NLP by Langrangian relaxation. In this paper, few computational results have been given for the representative problems such as overall costs and the total back orders. Based on the empirical investigation, we discover many solutions which are obtained that make tradeoffs between overall cost and total back orders. In different instances of the problem, the proposed formulation with Lagrangian relaxation provides good bound in comparatively less computational time. These solutions are of practical value to supply chain managers. The paper will be helpful to any industry in making good tradeoffs between total back orders and operational costs. The research provides insight into the differences in the values of Lagrangian relaxation with total back orders and operational costs.

Keywords: Twin Objectives, Cost Minimization, Back Order Minimization, Food Corporation of India, Non-Linear Programming, Lagrangian Relaxation

I. Introduction

The food grain distribution in India is channeled in two stages where procured or imported/exported grains are stored in about 2000 warehouses for the onward issue to the demand points. The base of this channel is mainly dependent on rail and road transport which helps to move the stored quantities to the necessary points like public distribution for poverty alleviation/employment or household development controlled by the state government or army and paramilitary organizations controlled by central government. The public distribution system is established by the government of India for the welfare of the poor people by providing subsidized food and food processed items. As of now about 4.99 lakh Fair Price Shops (FPS) are distributed across India (Report on Public Distribution System, 2000), that is controlled by the ministry of consumer affairs, food, and public distribution and managed jointly with state government. But the biggest challenge faced in practice of the food grains supplied by the PDS is much less than the consumption needs of the consumer. Besides this, the delayed supply due to the lag of proper planning schedule is also a big issue. It has also come in the picture that the PDS has been biased to urban people ignoring the remote and poorer sections of the population in their needs. From procurement point Punjab, Haryana, Andhra Pradesh, MP, and UP & Chhattisgarh. Punjab, Haryana, and MP are the major wheat surplus states where Punjab, Andhra Pradesh, Chhattisgarh, and Haryana play the major role in rice surplus states in India. From state to more granular level, procurement is done either from the farmers directly or by the agents (Arhtias) in the mandis. At the starting of every season, the Government of India fixes the minimum support price (MSP) to procure the food grain to maintain uniformity across the country. In general, the greater the desired price stability, the greater the quantities that public agencies would need to hold in storage (Gulati et al., 1996). The implementation of the process is very old-fashioned. As of now, FCI is managing the entire exercise of movement by a manual process with the vast practical knowledge acquired by the planners over the years. So, in case of railways, the
main aim is to supply in full rake to reduce the operational cost and time. In a time of practice, the demands of nearby railheads are combined such that the full rake concept can be implemented where the single rail head demand is not enough to operate is a full rake. But in some emergency scenarios like army, flood, the FCI has been observed to operate in half rake movement to ease the situation. Freight terminal (FT) planning is broadly influenced by two criteria: cost of transportation of goods (variable cost), cost of location freight terminal (setup/one-time cost), cost of empty haulage of goods trains (goods trains have to travel empty to pick up the goods available for transportation) and the delays in transporting the goods. So locate freight terminals based on purely cost considerations (transportation and location cost) using the first problem and then obtain an estimate of delays in the system by simulation and estimate of empty haulage of the goods trains. It will be better if “multi-commodity multi rake” trains are developed. Due to limited section capacities and limited yard capacities, it may lead to congestion and we are unable to move the goods on time and it results in delays. The operations are considered to be efficient if they ensure a high return to producers at a low unit cost of distribution. The aim of the paper in the parts that follow is to provide a review of location planning models where minimizing the overall costs & backorders and to address issues these decisions have on resource utilization. We take many assumptions but relax some of them and try to solve by using Lagrangian relaxation and GAMS (General Algebraic Modeling System). GAMS is high-level modeling system for mathematical optimization, it can solve linear, nonlinear optimization and mixed integer type problem.

II. Literature Review

The model developed in this paper is a distribution model where the transportation cost is driving cost. Most of the practical problems can be modeled as the distribution problem which is primarily a complex network problem. In the network the points or vertices can be locations like plants, central depots, warehouses, retailers, and customers depending on the problematic aspects and the edges can be the connecting links between the point like airways, ships, railways and roadways (Jawahar et al., 2012). The solution approach of the problem can also be simplified like the fixed charge transportation problem (FCTP). In this approach two kinds of cost are considered, (i) variable cost: a continuous cost that generally increases linearly with the transported quantity, and (ii) fixed charge: which is a fixed amount incurred to set up a non-zero quantity transport between any source and a destination introduced in (Diaby 1991). Though the concept of facility location problem was introduced by Balinski (1965), where the cost of locating a new facility was considered as a minimization problem of associated costs.

In a broad way, the facility location problem can also be viewed as a subset of organizational problem where we are only considering the optimized transport of materials or products. In the work of Xiao & Zheng (2010), it can be found that the entire product is not manufactured or assembled in a single workstation. There are always several workstations that work simultaneously for effective production. But to get going the production, continuous movement of parts occurs in between the stations and costs associated with it. Besides the production, every manufacturing firm needs to supply high-quality products to its customers within the time at a minimum possible cost to stay in the competitive market. This aims to an efficient distribution system that delivers the products in the prescribed scenario. Besides supply of finished products, raw material supply also comes into the distributional planning. This defines that the prime aim of any organization is to search for different policies that reduce the total cost of operation (both variable and fixed cost) throughout their supply chain (Demirli & Yimer, 2008) to retain maximum profit. It becomes more difficult especially when the distance between the production facilities or suppliers increases from the principal customers as the variable cost increases with distance (Akbalika & Penzb, 2011). This concept has also reflected in the work of (Silva and Figuera, 2007) where transportation costs(variable) and location-specific (fixed costs) has played a major component in determining the price (cost) of goods.

Here, we are trying to solve the distribution problem as facility location problem in the context of food grain distribution. Facility location problems can be capacititated, where the supplying or storing capacity of any warehouse is predetermined which is also very unlikely in the practical scenario. This leads to the Uncapacitated Facility Location (UFL), where the capacity of any warehouse is not fixed. The idea
behind is that the delivery centers (depots, warehouses, etc.) can be established keeping the demand into consideration. This also leads to the concept of dynamic demand. It simply signifies that the future demand can’t be forecasted exactly. To resolve this kind of scenarios two primary models have been suggested by the researchers. One suggests that the increased capacity can be installed at the planning period considering the recent trend of demand (Erlenkotter 1981, Shulman 1991, Drezner 1995, Canel & Khumawala 1996, 1997, 2001). The other is the concept of chasing demand (Roodman & Schwarz 1975), considers that the facilities can be modified (increased or decreased) if there is a high change in demand. The Uncapacitated Facility Location (UFL), comes under the NP-hard category which is hard to solve in polynomial time (Francis et al. 1983, Krarup & Pruzan 1983, Mirchandani, 1990, Drezner 1995). Adding the dynamic demand concept and the newly introduced product mix concept (Vecihi et al., 2006) further increased the complexity of the problem.

In the context of changing customer demand conditions, facility location plays a significant role in minimizing the cost and increasing efficiency. In the fast-changing economic scenario, the price change happens to very frequently changing the demand location simultaneously. So, the warehouse in proper location can lead to less transportation cost as well as availability in lesser time. The interrelation of logistics costs, service, and customer satisfaction with intelligent facility location can be seen from (Slabinac, 2013). The criterion considered in facility location are (1) identify those customers presently located in the supply point or the existing delivery route, (2) specify the potential facility location, (3) the space where the facilities to be delivered to the customers, (4) the distance matrix between customer and facility and the expected time corresponding to it. These concepts are discussed in detail in (ReVelle & Eiselt, 2005).

Sharma (1996) considered the food grains distribution problems as faced by FCI, a public-sector corporation with the food grain distribution in India. They presented a mix integer linear programming formulation of the problem as faced by the FCI which turns out to be a single stage ware location problem (SSWLP), and suggest a branch and bound based procedure for a solution. At each node of the branch and bound procedure results a minimum cost flow problem which can be solved O (n2 ln(n)) time by using best known procedures, but they presented Lagrangian Relaxation method which obtains a good bound for the associated minimum cost flow problem in O (n2) time, thus offering substantial savings in computations which become particularly significant while solving large sized real-life problem as faced by the FCI. In this research, they gave a few computational results for the representative problems.

Sharma (1996) took the set of potential trans-shipment locations are known and each point has an associated fixed cost with it. The problem is to choose a sufficient number of trans-shipment points such that the sum total of fixed location cost and transportation cost of shipping food grains to markets in (from plant to warehouses and from warehouses to market) minimized while satisfying the demand at each market point. It is assumed that sufficient warehouse capacity is available at the trans-shipment points and that only a single commodity is considered for distribution. The assumption of a single commodity is justified in this case because various commodities made available to fair price shops are in accordance with a predetermined ratio. The solution method applied in the work is the Lagrangian relaxation. In this method instead of solving the entire solution, we solve a relaxed problem and update the Lagrange multipliers in each iteration to get the close to optimal solution.

At the industrial level, the management of logistics is not an independent quantity. To maintain lower operation cost we need to store products at lower inventory costs which helps the firm to provide the product at lower cost at varying demand scenario when we have excess produced quantity. To ease transportation and storage cost good facility location is to be identified as well as proper vehicle routing algorithm is required. When the precision making is done for one period it is known as single period transportation model. But in practice, the decisions are extended more than one period (days/weeks/months) which is known as multi-period transportation model. Considering the multiperiod model, the demand is dynamic in most of the cases. This cause the shortage of supply over the period which is terminated as shortage or backorders. The shortage of any period is usually transmitted as backorder of the next period. All this decision-making for multiple periods is coming under management science.
III. Research framework

In the state-sponsored distribution system, FCI procures the food grains from locations called “Mandis” where farmers deliver their product. The procurement prices are fixed by the government in advance, and hence the quantities available at the “Mandis” for procurement can be assumed to be estimated with reasonable accuracy. The food grains are then moved to the large-sized warehouse owned by the FCI, and it may be assumed for the practical purpose that warehouse sizes are unlimited. This is so because the food grains get the topmost priority for storage space available and then the remaining space is allotted to other commodities. From these largely sized warehouses, the food grains are moved to smaller sized warehouses which supply the food grains to the government-owned Fair Price shops that are attached to it. Thus the quantity to be supplied to the smaller sized warehouses can be known by the summing quantities of food grains to be supplied to Fair Price shops that are attached to them. The operational problem of the FCI is to choose a sufficient number of large-sized and small-sized warehouses and managing quantities available there so as to minimize the sum of location and transportation costs. In this paper we focus on operational problem whereby we seek to minimize transportation cost plus cost of inventory plus cost of back order plus cost of space that is the first objective and back order minimization will be the second objective which represents the comprehensive research work review in the area of facility location problems; it covers different techniques and relative strength of Lagrangian relaxation.

A. Distribution model of food grain from Punjab to U.P. for single/composite commodity:

In India, FCI distributes mainly Wheat and Rice. As we know Punjab is a surplus state of food grain and has 50% contribution of procurement of food grain by FCI in PDS (public distribution system), U.P. produces sufficient quantity of food grain, but huge amount of food grains are consumed by the people so has little bit contribution of food grain in PDS by FCI. Here we took Punjab as a overproduce state of food grain and food grain distribution from Punjab to U.P. with different mode (Rail/Road). There are 75 districts in U.P. and out of the 45 districts are connected with Broad Gauge and other 30 districts are connected with Meter Gauge. Here we are using aggregate planning for the single/multi-commodity. Aggregate planning is used to reduce a multi-commodity problem to a single composite commodity. Its approach is predicated on an existence of an aggregate unit of production. When the types of items produced are similar, an aggregate production unit can correspond to an “average” item, but if many different types of items are produced, it would be more appropriate to consider aggregate units in terms of weight (tons of steel), volume (gallons of gasoline), amount of work required (worker-year of programming time), or dollar value (value of inventory in dollars). It depends on the context of the particular planning problem and the level of aggregation required (Zhibin, 2009).

Backorder is an order for a good or service that cannot be filled at the current time due to a lack of available supply. The higher the number of items backorder, it means the demand is higher for the item. It is an important factor in inventory management analysis. If a company consistently sees items in backorder then this could be taken as a signal that it is running too lean - and that it is losing out on business by not providing the products demanded by its customers. XPWS1_{ij} is quantity moved from Punjab to warehouse stage 1 which is directly connected with Broad gauge connection and XWS1WS2_{ijk} is Quantity moved from warehouse stage 1 to warehouse stage 2 which is connected with Meter gauge connection. We have developed a Mathematical Model for food grain distribution.

We have taken some assumptions which are following:
1. Quantities available at the “Mandis” for procurement can be assumed to be estimated with reasonable accuracy.
2. Warehouse sizes are unlimited.
3. Aggregate planning is used to reduce a multi-commodity problem to a single composite commodity.

We are considering the supply points, warehouse points 1 and warehouse points 2. In my formulation I have taken “i” is the mode of transportation, the supply point Punjab is represented by “p”, warehouse stage 1 point by “j” and warehouse stage 2 point by “k”.
B. Formulation for Distribution
We have taken \( t \) for four time periods \( t_1, t_2, t_3, t_4 \) and there are twenty warehouses at both stages in the first case and forty warehouses at both stages in the second case, so finally we have

- **i=2**: Mode of transportation (Rail/Road)
- **t**: time period \((t=1,...,4)\) as \( t_1,t_2,t_3,t_4 \)
- **j**: number 20 warehouses at stage 1 (in the first case of sample problems)
- **k**: number 20 warehouses at stage 2 (in the first case of sample problems)
- **j**: number 40 warehouses at stage 1 (in the second case of sample problems)
- **k**: number 40 warehouses at stage 2 (in the second case of sample problems)

C. Parameters

- **CPWS1_{ij}**: Cost of transportation from Punjab to ws1 with mode ‘i’ in time period ‘t’ at location ‘j’.
- **CWS1WS2_{ijk}**: Cost of transportation from ws1 to ws2 with mode ‘i’ in time period ‘t’ from location ‘j’ to location ‘k’.
- **CInv_WS1_{tj}**: unit cost of inventory at ws1 in time period ‘t’ at location ‘j’.
- **CInv_WS2_{tk}**: unit cost of inventory at ws2 in time period ‘t’ at location ‘k’.
- **CWS1_BO_{tj}**: the cost of backorder at ws1 in time period ‘t’ at location ‘j’.
- **CWS2_BO_{tk}**: the cost of backorder at ws2 in time period ‘t’ at location ‘k’.
- **C_{Space WS1_{j}}**: the cost of warehouse space in warehouse stage 1 at location ‘j’.
- **C_{Space WS2_{k}}**: the cost of warehouse space in warehouse stage 2 location ‘k’.
- **D_{WS1_{tj}}**: demand at Warehouse stage1 in time period ‘t’ at location ‘j’.
- **D_{WS2_{tk}}**: demand at Warehouse stage2 in time period ‘t’ at location ‘k’.
- **Supply_{PWS1_{ij}}**: upper limit on supply from plant to warehouse stage 1 by mode ‘i’ time period ‘t’ at location ‘j’.

D. Decision Variables

- **XPWS1_{ij}**: Quantity moved from Punjab to warehouse stage 1 with mode ‘i’ in time period ‘t’ at location ‘j’.
- **XWS1WS2_{ijk}**: Quantity moved from warehouse stage 1 to warehouse stage 2 with mode ‘i’ in time period ‘t’ from location ‘j’ to location ‘k’.
- **XWS1_BO_{tj}**: Backorder quantity at ws1 in time period ‘t’ at location ‘j’.
- **XWS2_BO_{tk}**: Backorder quantity at ws2 in time period ‘t’ at location ‘k’.
- **Inv_{WS1_{tj}}**: Inventory at warehouse stage 1 at time period ‘t’ at location ‘j’.
- **Inv_{WS2_{tk}}**: Inventory at warehouse stage 2 in time period ‘t’ at location ‘k’.
\( \text{Inv}_\text{WS1}^{(t-1)j} \): is the number of units of inventory minus the number of units of back orders at the beginning of the month at warehouse stage 1 at location 'j'.

\( \text{Inv}_\text{WS2}^{(t-1)k} \): is the number of units of inventory minus the number of units of back orders at the beginning of the month at warehouse stage 2 at location 'k'.

\( \text{XWS1\_Qty\_moved}^{tj} \): quantity moved at warehouse stage one in time period 't' at location 'j'.

\( \text{XWS2\_Qty\_moved}^{tk} \): quantity moved at warehouse stage two in time period 't' at location 'k'.

\( \text{XWS1\_Qty\_Supplied}^{tj} \): quantity supplied at warehouse stage one in time period 't' at location 'j'.

\( \text{XWS2\_Qty\_Supplied}^{tk} \): quantity supplied at warehouse stage two in time period 't' at location 'k'.

E. Mathematical formulation

Objective function 1

Min of (Cost of transportation + Inv. Carrying cost + B.O. cost + Cost of warehouse space)

\[
\sum_{i, t, j} (\text{CPWS1}_i) \cdot (\text{XPWS1}_itj) + \sum_{i, t, j, k} (\text{CWS1\_WS2}_{ijk}) \cdot (\text{XWS1\_WS2}_{itjk}) + \\
\sum_{i, t, j} (\text{CInv\_WS1}_i) \cdot (\text{Inv}_\text{WS1}^{tj}) + \sum_{i, t, j, k} (\text{CInv\_WS2}_i) \cdot (\text{Inv}_\text{WS2}^{tk}) + \\
\sum_{i, t, j} (\text{CWS1\_BO}_i) \cdot (\text{XWS1\_BO}^{tj}) + \sum_{i, t, j, k} (\text{CWS2\_BO}_i) \cdot (\text{XWS2\_BO}^{tk}) + \\
\sum_{j} (\text{CSpace\_WS1}_j) \cdot \max_t (\text{Inv}_\text{WS1}^{tj}) + \sum_{k} (\text{CSpace\_WS2}_k) \cdot \max_t (\text{Inv}_\text{WS2}^{tk})
\]

Objective function 2

We have taken a uniform distribution of back orders at all points. If rail capacity is low then it will produce a plan that has a uniform distribution of backorder across warehouse points.

Min. \( \sum_{i, j} (\text{XWS1\_BO}_i)^2 + \sum_{i, k} (\text{XWS2\_BO}_i)^2 \)

Subjected to

\( (\text{Inv}_\text{WS1}^{(t-1)j}) - (\text{XWS1\_Qty\_Supplied}_{ij}) + \sum_j \text{XPWS1}_{ij} - \sum_{i, k} \text{XWS1\_WS2}_{ijk} = \text{Inv}_\text{WS1}_{ij} \) for all \( t \) and \( j \) …… (1)

\( (\text{Inv}_\text{WS2}^{(t-1)k}) - (\text{XWS2\_Qty\_Supplied}_{ik}) + \sum_{i, k} \text{XWS1\_WS2}_{ijk} = \text{Inv}_\text{WS2}_{ik} \) for all \( t \) and \( k \) ……………….. (2)

\( \sum_{t=1}^{t_1} \text{XWS1\_Qty\_Supplied}_{ij} + \text{XWS1\_BO}_{ij} = \sum_{t=1}^{t_1} \text{DWS1}_i + \text{Inv}_\text{WS1}_{ij} \quad \forall \ j \text{ and } t_1 = 1 \ldots T \) ……………….. (3)

\( \sum_{t=1}^{t_1} \text{XWS2\_Qty\_Supplied}_{ik} + \text{XWS2\_BO}_{ik} = \sum_{t=1}^{t_1} \text{DWS2}_i + \text{Inv}_\text{WS2}_{ik} \quad \forall \ k \text{ and } t_1 = 1 \ldots T \) ……………….. (4)

\( \text{XWS1\_Qty\_moved}_{ij} = \sum_j \text{XPWS1}_{ij} \quad \forall \ t, j \) …………………………………….. (5)

\( \text{XWS2\_Qty\_moved}_{ik} = \sum_{j} \sum_j \text{XWS1\_WS2}_{ijk} \quad \forall \ t, k \) …………………………………….. (6)

\( \text{XPWS1}_{ij} \leq \text{Supply\_PWS1}_i \quad \forall \ i, t, j \) …………………………………….. (7)

\( \text{XWS1\_WS2}_{ijk} \leq \text{Supply\_WS1\_WS2}_{ijk} \quad \forall \ i, t, j, k \) …………………………………….. (8)

\( \text{XPWS1}_{ij} \geq 0 \) for all \( i \) and \( j \) …………………………………….. (9)

\( \text{XWS1\_WS2}_{ijk} \geq 0 \) for all \( i, j \) and \( k \) …………………………………….. (10)
In our solution approach $\text{Inv}_{WS1}(t-1,j) = 0$ at the initial stage in the first month and in next month inventory will have some value, which will come due to the difference between supply and demand and it will be adjusted with back orders. It will be same for $(\text{Inv}_{WS2}(t-1,k))$. Equation (1) indicates that LHS is equal to RHS, here quantity distributed from location ‘j’ is equal to quantity supplied to location ‘k’ and quantities supplied to different PDS connected to ‘j’. Equation (2) indicates that that total inventory at the warehouse at stage 2 is equal to the difference between the sum of previous inventory and quantity moved at k and the supply of food grain. Equation (3) and (4) indicate that the sum of quantity supplied and backorder is equal to the sum of demand and inventory from p to j and j to k. Equation (5) and (6) indicate that the quantity moved is always equal to $XPWS1$ from p to j and $XWS1WS2$ from j to k. Equation (7) and (8) indicate that the quantity supplied is always greater than quantity moved from p to j and j to k.

**F. Procedure for Lagrangian relaxation:**

LR is techniques well suited for problems where the constraints can be difficult when difficult constraints are relaxed, and then reduced problem is easily solvable. Here equation (2) is bad constraints so Lagrangian relaxation is used in solution procedure using GAMS code only at k. The main idea is to relax the problem by removing the difficult constraints and putting them into the objective function, assigned with weights (the Lagrangian multiplier). We assume Lagrangian multiplier $U_k$. Each weight represents a penalty which is added to a solution that does not satisfy the particular constraint.

Step 1. Obtain an upper bound from the combined formulation above non linear equation. Objective function 2 is $(x_{t,j},x_{ws1_bo}(t,j) + x_{ws1_bo}(t,j)) + \sum((t,k),x_{ws2_bo}(t,k) \cdot x_{ws2_bo}(t,k))$ and $Axkminusb_k.l(t,k) = \text{inv}_{ws2}(l(t,1-k)) - x_{ws2_qty_supplied}(l(t,k)) + x_{ws2_qty_moved}(l(t,k)) - \text{inv}_{ws2}(l(t,k))$ is bad constraint.

Step 2. Obtain the value of Lagrangian coefficient as shown below.

\[
LR.. \text{bound}_{bo} = \sum((t,j),x_{ws1_bo}(t,j) * x_{ws1_bo}(t,j)) + \sum((t,k), x_{ws2_bo}(t,k) \cdot x_{ws2_bo}(t,k)) - \sum((t,k),U_k(t,k) \cdot (\text{inv}_{ws2}(t-1,k) - x_{ws2_qty_supplied}(t,k) + x_{ws2_qty_moved}(t,k) - \text{inv}_{ws2}(t,k))).
\]

Step 3. Update the values of elements as shown below.

We assume that $U_k(t,k) = 0.0005$ and upper bound = 3

Loop (iter_count$continue)

norm2 = sum((t,k),sqr (Axkminusb_k.l(t,k)))

Norm = norm2 + 5

Results (iter_count,'temp_counter') = temp_counter

Results (iter_count,'theta') = theta

Step size = theta*(upper bound-bound_{bo}.l)/norm

Results (iter_count,'norm') = norm

Results (iter_count,'step') = step size

$U_k(t,k) = U_k(t,k)+\text{stepsize}*Axkminusb_k.l(t,k)$

Step 4. Repeat the above steps for 20 iterations and obtain the bound as max.

**IV. Computational results**

In this section, we are going to present the results based on empirical investigations in order to verify the closeness and accuracy of the formulation in terms of time and lower bound obtained. The first part of the empirical investigation is on 2x4x20x20 matrix and the second empirical investigation is on 2x4x40x40 matrix. The problem size of 2x4x20x20 means that there is 2 modes of transportation, the time duration is 4 months where cycle time is fixed at the start of the month, a number of warehouses at stage one is 20 and number of warehouses at stage two is 20. The problem size of 2x4x40x40 means that there is 2 modes of transportation, the time duration is 4 months where cycle time is fixed at the start of the month, a number of warehouses at stage one is 40 and number of warehouses at stage two is 40.
Sample problems were created randomly for the different values. These problems were then written and solved using optimization software GAMS (Generalize Algebraic Modeling Structure). The values of input parameters along with the solution of both sets of problems are given below:

### Computational result for size $2 \times 4 \times 20 \times 20$: [$i = 2, t = 4, j = 20, k = 20$]

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<th>$Z_1$</th>
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**Table 1: Result table for size $2 \times 4 \times 20 \times 20$**

### Computational result for size $2 \times 4 \times 40 \times 40$: [$i = 2, t = 4, j = 40, k = 40$]

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**Table 2: Result table for size $2 \times 4 \times 40 \times 40$**
2x4x20x20 for Costs:

![Figure 2: Overall cost for problem size 2X4X20X20](image)

2x4x20x20 for BO:

![Figure 3: Total backorders for problem size 2X4X20X20](image)

2x4x40x40 for Cost:

![Figure 4: Overall cost for problem size 2X4X40X40](image)

2x4x40x40 for BO:

![Figure 5: Total backorders for problem size 2X4X40X40](image)
From the results, we can see that quantity supplied to various big warehouses can be calculated. These values have been calculated considering the given constraints and they minimize the backorder cost. As it can be inferred from the result and subsequent example that the resulting quantities supplied to any big warehouse come in the form of demand functions for the various time period. So with the demand estimates for various big warehouses for different time periods, total backorder cost can be minimized. When the Lagrangian values of cost (Z1) and BO (Z2) solving in the GAMS problem there are two parameters to solve the new values of Z1 and Z2:
1. When taking the Lagrangian value of Z1 is add as a constraint and solve for the value of Z2.
2. When taking the Lagrangian value of Z2 is add as a constraint and solve for the value of Z1.

The values of input parameters of Lagrangian values of Z1 and Z2 along with the new solution values if Z1 and Z2 both sets of problems and solutions are given below:
LR is successful when the reduced problem is in P (polynomial solvable). The way in which LR is attempted, the reduced problem is again solved as NLP (quadratic objective function s.t. linear constraints), hence LR is NOT successful. Moreover, LR gives a feasible solution, and GAMS gives the optimal solution. Hence, GAMS type of approach is preferable. Maybe later GAMS performance is to be compared to LDR type of approach developed by Singh (2012) which just requires a matrix inversion for a solution. It is expected that LR will be promising if used in conjunction with methodology due to Singh (2012) as then we have to invert a matrix of smaller size.

**Statistical Analysis for Problem 2*4*20*20 and 2*4*40*40**
Hypothesis tests are conducted as follows:
1) To check whether the Lagrangian results are significantly related to overall cost and total back orders for both type problems. We propose that different constraints of the supply chain problem are relaxed at Z1 and Z2 to generate different solutions. Therefore, we check the Lagrangian result is relaxed at Z1 is higher than the Lagrangian result is relaxed at Z2 for a given set of problems where their mean values are μ₁ and μ₂ respectively.
So, Null hypothesis, H₀: μ₁ < μ₂
Alternate hypothesis, Ha: μ₁ > μ₂
Paired sample t-test is used to test the hypotheses for the Lagrangian values are given below.

**Result:** From problem size 2*4*20*20, the statistical t-tables we have the critical value for t-stats at α = 0.05 as 2.086 for dof = 19 and for this t-test, t = 2.749 thus we can easily reject the null hypothesis. μ₁ = 49270052.55 and μ₂ = 1470170.20, therefore it supports the alternate hypothesis. From problem size 2*4*40*40, the statistical t-tables we have the critical value for t-stats at α = 0.05 as 2.086 for dof = 19 and for this t-test, t = 2.996 thus we can easily reject the null hypothesis. μ₁ = 4348281.50 and μ₂ = 464236.55, therefore, it supports the alternate hypothesis.

2) To check whether execution timing is significantly related to problem size 2*4*20*20 and 2*4*40*40 are considered as problem 1 and problem 2. Definitely, the execution timing of big complex problem is high. Therefore, we check the execution timing of both the problems is equal for the given set of problems where μ₁: Mean execution timing of problem 1 and μ₂: Mean of execution timing of problem 2.
So, Null hypothesis, H₀: μ₂ ≤ μ₁
Alternate hypothesis, Ha: μ₂ > μ₁
Paired sample t-test is used to test the hypothesis for problem 1 and problem 2, the values are given below.

**Result:** From the statistical t-tables we have the critical value for t-stats at α = 0.05 as 2.086 for dof = 19 and for this t-test, t = -30.30 thus we can easily reject the null hypothesis. μ₁ = 15.559 and μ₂ = 32.268, therefore, it supports the alternate hypothesis.
V. Conclusion and future research

We have proposed a solution set that optimizes both the values when the overall costs decreased then back orders are increased and vice-versa. This optimal solution is getting by using Lagrangian relaxation method. We decomposed the combined formulation into two different subproblems and solved it for different instances. Here the efficiency of decomposition approach for warehouse stages is shown in terms of the time and lower bound for backorder is obtained. In all the different instances of the problem the proposed formulation with Lagrangian relaxation provides good bound in comparatively less computational time. This method is compared to LDR rules (Holt et al, 1960) that it minimizes the quadratic problem statement into a linear function. From the results, we can see that quantity supplied to various big warehouses can be calculated. These values have been calculated considering the given constraints and they minimize the total backorder.

For future work, different obstacles are still left, as we have mentioned in the research problem, which can be included in the model. Few of them are: (1) no proper number of warehouses to procure food grain; (2) no circulation of food grains; (3) delay of rail; (4) high lead time; (5) no manpower; (6) gap between planned and actual movement and; (7) poor infrastructure which are faced by FCI as well as Indian Railway. For these, we can suggest FCI and Indian Railway develop the new warehouses at certain places to fulfill the demand of PDS without delay and these changes will give a different feasible solution in order to minimize the wastage of food grain. We propose for future work that different constraints of the supply chain problem be relaxed to generate different quality solutions. These may be compared with Linear Decision Rule type of methodology applied to NLP by Singh (2012). Also, we can change different parameters of LR such as step size; starting values of multipliers, halving of a parameter of LR etc that are making good tradeoffs between back orders and operational cost.

References


**BIOGRAPHY**

**Vimal Kumar** is an Assistant Professor at MANIT, Bhopal. He has done his Doctoral degree from the Department of Industrial & Management Engineering, IIT Kanpur, India. He completed his Masters in Supply Chain Management from the Department of Industrial & Management Engineering, IIT Kanpur in the year 2012. He completed his graduation (B.Tech) in Manufacturing Technology in the year 2010 from JSS Academy of Technical Education, Noida. Currently, he is pursuing research in the domain of TQM and Manufacturing Strategy. He has published fourteen articles in reputable international journals and presented nine papers at international conferences. He was invited to serve as session chair for Quality Control & Management at the International Conference on Industrial Engineering & Operations Management (IEOM-2016) at Kuala Lumpur, Malaysia. He is a contributing author in journals including *IJPPM, IJQRM, IJPMB, IJPQM, IJBIS, The TQM Journal, and Benchmarking: An International Journal*, etc. and also a guest reviewer of the reputable journals like *TQM & Business Excellence, Benchmarking: An International Journal, and JSIT*. 