Equal Distribution of Shortages in Supply Chain Problem of Food Corporation of India

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Abstract: *The purpose of this research is to develop a twin objectives formulation considering cost and back order minimization of supply chain operational problem faced by Food Corporation of India (FCI). Cost minimization problem is a standard LP (Linear Programming) and we consider quadratic objective formula for the back order minimization problem which turns out to be NLP (Non Linear Programming). We attempt solving NLP by Langrangian relaxation. In this paper, few computational results have been given for the representative problems such as overall costs and the total back orders. Based on the empirical investigation, we discover many solutions which are obtained that make tradeoffs between overall cost and total back orders. In different instances of the problem, the proposed formulation with Lagrangian relaxation provides good bound in comparatively less computational time. These solutions are of practical value to supply chain managers. The paper will be helpful to any industry in making good tradeoffs between total back orders and operational costs. There is a variety of different approaches that different constraints of the supply chain problem be relaxed to generate different quality solutions. The research provides insight into the differences in the values of Lagrangian relaxation with total back orders and operational costs.*

Keywords: *Twin Objectives, Cost Minimization, Back Order Minimization, Food Corporation of India, Non-Linear Programming, Lagrangian Relaxation*

I. Introduction

The food grain distribution in India is channeled in two stages where procured or imported/exported grains are stored in about 2000 warehouses for the onward issue to the demand points. The base of this channel is mainly dependent on rail and road transport which helps to move the stored quantities to the necessary points like public distribution for poverty alleviation/employment or household development controlled by the state government or army and paramilitary organizations controlled by central government. The public distribution system is established by the government of India for the welfare of the poor people by providing subsidized food and food processed items. As of now about 4.99 lakh Fair Price Shops (FPS) are distributed across India (Report on Public Distribution System, 2000), that is controlled by the ministry of consumer affairs, food, and public distribution and managed jointly with state government. But the biggest challenge faced in practice of the food grains supplied by the PDS is much less than the consumption needs of the consumer. Besides this, the delayed supply due to the lag of proper planning schedule is also a big issue. It has also come in the picture that the PDS has been biased to urban people ignoring the remote and poorer sections of the population in their needs. From procurement point Punjab, Haryana, Andhra Pradesh, MP, and UP & Chhattisgarh. Punjab, Haryana, and MP are the major wheat surplus states where Punjab, Andhra Pradesh, Chhattisgarh, and Haryana play the major role in rice surplus states in India. From state to more granular level, procurement is done either from the farmers directly or by the agents (Arhtias) in the mandis. At the starting of every season, the Government of India fixes the minimum support price (MSP) to procure the food grain to maintain uniformity across the country. In general, the greater the desired price stability, the greater the quantities that public agencies would need to hold in storage (Gulati et al., 1996). The implementation of the process is very old-fashioned. As of now, FCI is managing the entire exercise of movement by a manual process with the vast practical knowledge acquired by the planners over the years. So, in case of railways, the

main aim is to supply in full rake to reduce the operational cost and time. In a time of practice, the demands of nearby railheads are combined such that the full rake concept can be implemented where the single rail head demand is not enough to operate is a full rake. But in some emergency scenarios like army, flood, the FCI has been observed to operate in half rake movement to ease the situation. Freight terminal (FT) planning is broadly influenced by two criteria: cost of transportation of goods (variable cost), cost of location freight terminal (setup/one-time cost), cost of empty haulage of goods trains (goods trains have to travel empty to pick up the goods available for transportation) and the delays in transporting the goods. So locate freight terminals based on purely cost considerations (transportation and location cost) using the first problem and then obtain an estimate of delays in the system by simulation and estimate of empty haulage of the goods trains. It will be better if "multi-commodity multi rake" trains are developed. Due to limited section capacities and limited yard capacities, it may lead to congestion and we are unable to move the goods on time and it results in delays. The operations are considered to be efficient if they ensure a high return to producers at a low unit cost of distribution. The aim of the paper in the parts that follow is to provide a review of location planning models where minimizing the overall costs & backorders and to address issues these decisions have on resource utilization. We take many assumptions but relax some of them and try to solve by using Lagrangian relaxation and GAMS (General Algebraic Modeling System). GAMS is high-level modeling system for mathematical optimization, it can solve linear, nonlinear optimization and mixed integer type problem.

II. Literature Review

The model developed in this paper is a distribution model where the transportation cost is driving cost. Most of the practical problems can be modeled as the distribution problem which is primarily a complex network problem. In the network the points or vertices can be locations like plants, central depots, warehouses, retailers, and customers depending on the problematic aspects and the edges can be the connecting links between the point like airways, ships, railways and roadways (Jawahar et al., 2012). The solution approach of the problem can also be simplified like the fixed charge transportation problem (FCTP). In this approach two kinds of cost are considered, (i) variable cost: a continuous cost that generally increases linearly with the transported quantity, and (ii) fixed charge: which is a fixed amount incurred to set up a non-zero quantity transport between any source and a destination introduced in (Diaby 1991). Though the concept of facility location problem was introduced by Balinski (1965), where the cost of locating a new facility was considered as a minimization problem of associated costs.

In a broad way, the facility location problem can also be viewed as a subset of organizational problem where we are only considering the optimized transport of materials or products. In the work of Xiao & Zheng (2010), it can be found that the entire product is not manufactured or assembled in a single workstation. There are always several workstations that work simultaneously for effective production. But to get going the production, continuous movement of parts occurs in between the stations and costs associated with it. Besides the production, every manufacturing firm needs to supply high-quality products to its customers within the time at a minimum possible cost to stay in the competitive market. This aims to an efficient distribution system that delivers the products in the prescribed scenario. Besides supply of finished products, raw material supply also comes into the distributional planning. This defines that the prime aim of any organization is to search for different policies that reduce the total cost of operation (both variable and fixed cost) throughout their supply chain (Demirli & Yimer, 2008) to retain maximum profit. It becomes more difficult especially when the distance between the production facilities or suppliers increases from the principal customers as the variable cost increases with distance (Akbalika & Penzb, 2011). This concept has also reflected in the work of (Silva and Figuera, 2007) where transportation costs(variable) and location-specific (fixed costs) has played a major component in determining the price (cost) of goods.

Here, we are trying to solve the distribution problem as facility location problem in the context of food grain distribution. Facility location problems can be capacitated, where the supplying or storing capacity of any warehouse is predetermined which is also very unlikely in the practical scenario. This leads to the Uncapacitated Facility Location (UFL), where the capacity of any warehouse is not fixed. The idea

behind is that the delivery centers (depots, warehouses, etc.) can be established keeping the demand into consideration. This also leads to the concept of dynamic demand. It simply signifies that the future demand can't be forecasted exactly. To resolve this kind of scenarios two primary models have been suggested by the researchers. One suggests that the increased capacity can be installed at the planning period considering the recent trend of demand (Erlenkotter 1981, Shulman 1991, Drezner 1995, Canel & Khumawala 1996, 1997, 2001). The other is the concept of chasing demand (Roodman & Schwarz 1975), considers that the facilities can be modified (increased or decreased) if there is a high change in demand. The Uncapacitated Facility Location (UFL), comes under the NP-hard category which is hard to solve in polynomial time (Francis et al. 1983, Krarup & Pruzan 1983, Mirchandani, 1990, Drezner 1995). Adding the dynamic demand concept and the newly introduced product mix concept (Vecihi et al., 2006) further increased the complexity of the problem.

In the context of changing customer demand conditions, facility location plays a significant role in minimizing the cost and increasing efficiency. In the fast-changing economic scenario, the price change happens to very frequently changing the demand location simultaneously. So, the warehouse in proper location can lead to less transportation cost as well as availability in lesser time. The interrelation of logistics costs, service, and customer satisfaction with intelligent facility location can be seen from (Slabinac, 2013). The criterion considered in facility location are (1) identify those customers presently located in the supply point or the existing delivery route, (2) specify the potential facility location, (3) the space where the facilities to be delivered to the customers, (4) the distance matrix between customer and facility and the expected time corresponding to it. These concepts are discussed in detail in (ReVelle & Eiselt, 2005).

Sharma (1996) considered the food grains distribution problems as faced by FCI, a public-sector corporation with the food grain distribution in India. They presented a mix integer linear programming formulation of the problem as faced by the FCI which turns out to be a single stage ware location problem (SSWLP), and suggest a branch and bound based procedure for a solution. At each node of the branch and bound procedure results a minimum cost flow problem which can be solved O (n2 ln(n)) time by using best known procedures, but they presented Lagrangian Relaxation method which obtains a good bound for the associated minimum cost flow problem in O (n2) time, thus offering substantial savings in computations which become particularly significant while solving large sized real-life problem as faced by the FCI. In this research, they gave a few computational results for the representative problems.

Sharma (1996) took the set of potential trans-shipment locations are known and each point has an associated fixed cost with it. The problem is to choose a sufficient number of trans-shipment points such that the sum total of fixed location cost and transportation cost of shipping food grains to markets in (from plant to warehouses and from warehouses to market) minimized while satisfying the demand at each market point. It is assumed that sufficient warehouse capacity is available at the trans-shipment points and that only a single commodity is considered for distribution. The assumption of a single commodity is justified in this case because various commodities made available to fair price shops are in accordance with a predetermined ratio. The solution method applied in the work is the Lagrangian relaxation. In this method instead of solving the entire solution, we solve a relaxed problem and update the Lagrange multipliers in each iteration to get the close to optimal solution.

At the industrial level, the management of logistics is not an independent quantity. To maintain lower operation cost we need to store products at lower inventory costs which helps the firm to provide the product at lower cost at varying demand scenario when we have excess produced quantity. To ease transportation and storage cost good facility location is to be identified as well as proper vehicle routing algorithm is required. When the precision making is done for one period it is known as single period transportation model. But in practice, the decisions are extended more than one period (days/weeks/months) which is known as multi-period transportation model. Considering the multiperiod model, the demand is dynamic in most of the cases. This cause the shortage of supply over the period which is terminated as shortage or backorders. The shortage of any period is usually transmitted as backorder of the next period. All this decision-making for multiple periods is coming under management science.

III. Research framework

In the state-sponsored distribution system, FCI procures the food grains from locations called "Mandis" where farmers deliver their product. The procurement prices are fixed by the government in advance, and hence the quantities available at the "Mandis" for procurement can be assumed to be estimated with reasonable accuracy. The food grains are then moved to the large-sized warehouse owned by the FCI, and it may be assumed for the practical purpose that warehouse sizes are unlimited. This is so because the food grains get the topmost priority for storage space available and then the remaining space is allotted to other commodities. From these largely sized warehouses, the food grains are moved to smaller sized warehouses which supply the food grains to the government-owned Fair Price shops that are attached to it. Thus the quantity to be supplied to the smaller sized warehouses can be known by the summing quantities of food grains to be supplied to Fair Price shops that are attached to them. The operational problem of the FCI is to choose a sufficient number of large-sized and small-sized warehouses and managing quantities available there so as to minimize the sum of location and transportation costs. In this paper we focus on operational problem whereby we seek to minimize transportation cost plus cost of inventory plus cost of back order plus cost of space that is the first objective and back order minimization will be the second objective which represents the comprehensive research work review in the area of facility location problems; it covers different techniques and relative strength of Lagrangian relaxation.

A. Distribution model of food grain from Punjab to U.P. for single/composite commodity:

In India, FCI distributes mainly Wheat and Rice. As we know Punjab is a surplus state of food grain and has 50 % contribution of procurement of food grain by FCI in PDS (public distribution system), U.P. produces sufficient quantity of food grain, but huge amount of food grains are consumed by the people so has little bit contribution of food grain in PDS by FCI. Here we took Punjab as a overproduce state of food grain and food grain distribution from Punjab to U.P. with different mode (Rail/Road). There are 75 districts in U.P. and out of the 45 districts are connected with Broad Gauge and other 30 districts are connected with Meter Gauge. Here we are using aggregate planning for the single/multi-commodity. Aggregate planning is used to reduce a multi-commodity problem to a single composite commodity. Its approach is predicated on an existence of an aggregate unit of production. When the types of items produced are similar, an aggregate production unit can correspond to an "average" item, but if many different types of items are produced, it would be more appropriate to consider aggregate units in terms of weight (tons of steel), volume (gallons of gasoline), amount of work required (worker-year of programming time), or dollar value (value of inventory in dollars). It depends on the context of the particular planning problem and the level of aggregation required (Zhibin, 2009).

Backorder is an order for a good or service that cannot be filled at the current time due to a lack of available supply. The higher the number of items backorder, it means the demand is higher for the item. It is an important factor in inventory management analysis. If a company consistently sees items in backorder then this could be taken as a signal that it is running too lean - and that it is losing out on business by not providing the products demanded by its customers. $XPWS1_{itj}$ is quantity moved from Punjab to warehouse stage 1 which is directly connected with Broad gauge connection and $XWS1WS2_{itjk}$ is Quantity moved from warehouse stage 1 to warehouse stage 2 which is connected with Meter gauge connection. We have developed a Mathematical Model for food grain distribution.

We have taken some assumptions which are following:

1. Quantities available at the "Mandis" for procurement can be assumed to be estimated with reasonable accuracy.
2. Warehouse sizes are unlimited.
3. Aggregate planning is used to reduce a multi-commodity problem to a single composite commodity.

We are considering the supply points, warehouse points 1 and warehouse points 2. In my formulation I have taken "i" is the mode of transportation, the supply point Punjab is represented by "p", warehouse stage 1 point by "j" and warehouse stage 2 point by "k".

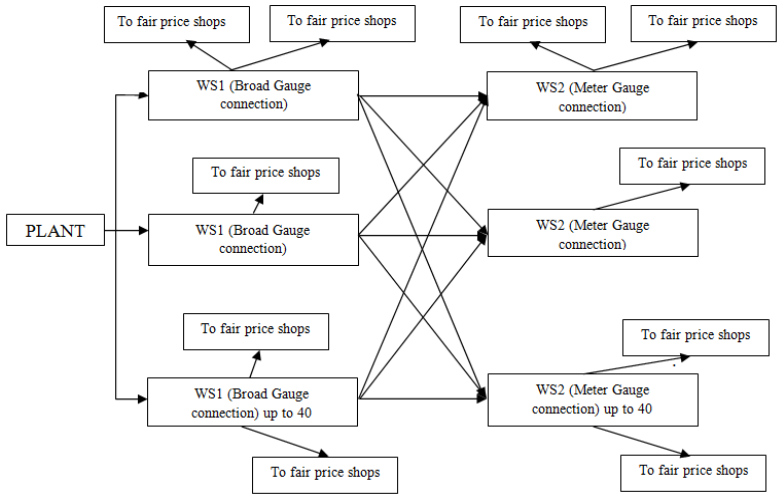

Figure 1: Food Grain Distribution through the two stages

B. Formulation for Distribution

We have taken t for four time periods t1, t2, t3, t4 and there are twenty warehouses at both stages in the first case and forty warehouses at both stages in the second case, so finally we have

i=2: Mode of transportation (Rail/Road)

t: time period (t=1…..4) as t1,t2,t3,t4

j: number 20 warehouses at stage 1 (in the first case of sample problems)

k: number 20 warehouses at stage 2 (in the first case of sample problems)

j: number 40 warehouses at stage 1 (in the second case of sample problems)

k: number 40 warehouses at stage 2 (in the second case of sample problems)

C. Parameters

$CPWS1_{itj}$: Cost of transportation from Punjab to ws1 with mode 'i' in time period't' at location 'j'.

$CWS1WS2_{itjk}$: Cost of transportation from ws1 to ws2 with mode 'i' in time period't' from location 'j' to location 'k'.

$CInv\_WS1_{tj}$: unit cost of inventory at ws1 in time period 't' at location 'j'.

$CInv\_WS2_{tk}$: unit cost of inventory at ws2 in time period't' at location 'k'.

$CWS1\_BO_{tj}$: the cost of backorder at ws1 in time period't' at location 'j'.

$CWS2\_BO_{tk}$: the cost of backorder at ws2 in time period't' at location 'k'.

$C_{Space}WS1_{j}$: the cost of warehouse space in warehouse stage 1 at location 'j'.

$C_{Space}WS2_{k}$: the cost of warehouse space in warehouse stage 2 location 'k'.

$DWS1_{tj}$: demand at Warehouse stage1 in time period't' at location 'j'.

$DWS2_{tk}$: demand at Warehouse stage2 in time period't' at location 'k'.

$Supply\_PWS1_{itj}$: upper limit on supply from plant to warehouse stage 1 by mode 'i' time period 't' at location 'j'.

D. Decision Variables

$XPWS1_{itj}$: Quantity moved from Punjab to warehouse stage 1 with mode 'i' in time period't' at location 'j'.

$XWS1WS2_{itjk}$: Quantity moved from warehouse stage 1 to warehouse stage 2 with mode 'i' in time period't' from location 'j' to location 'k'.

$XWS1\_BO_{tj}$: Backorder quantity at ws1 in time period 't' at location 'j'.

$XWS2\_BO_{tk}$: Backorder quantity at ws2 in time period 't' at location 'k'.

Inv_WS1_{tj}: Inventory at warehouse stage 1 at time period't' at location 'j'.

Inv_WS2_{tk}: Inventory at warehouse stage 2 in time period't' at location 'k'.

Inv_WS1 $_{(t-1)j}$: is the number of units of inventory minus the number of units of back orders at the beginning of the month at warehouse stage 1 at location 'j'.
Inv_WS2 $_{(t-1)k}$: is the number of units of inventory minus the number of units of back orders at the beginning of the month at warehouse stage 2 at location 'k'.
$XWS1\_Qty\_moved_{tj}$: quantity moved at warehouse stage one in time period't' at location 'j'.
$XWS2\_Qty\_moved_{tk}$: quantity moved at warehouse stage two in time period't' at location 'k'.
$XWS1\_Qty\_Supplied_{tj}$: quantity supplied at warehouse stage one in time period't' at location 'j'.
$XWS2\_Qty\_Supplied_{tk}$: quantity supplied at warehouse stage two in time period't' at location 'k'.

E. Mathematical formulation

Objective function 1

Min of (Cost of transportation + Inv. Carrying cost + B.O. cost + Cost of warehouse space)

$$\sum_{i,t,j} (CPWS1_{itj})\,(XPWS1_{itj}) + \sum_{i,t,j,k} (CWS1WS2_{itjk})\,(XWS1WS2_{itjk}) +$$
$$\sum_{t,j} (CInv\_WS1_{tj}.Inv\_WS1_{tj}) + \sum_{t,k} (CInv\_WS2_{tk}.Inv\_WS2_{tk}) +$$
$$\sum_{t,j} (CWS1\_BO_{tj})\,(XWS1\_BO_{tj}) + \sum_{t,k} (CWS2\_BO_{tk})\,(XWS2\_BO_{tk}) +$$
$$\sum_{j} (C_{Space}WS1_{j}.Max_{t}\,(Inv\_WS1_{tj}) + \sum_{k} (C_{Space}WS2_{k}.Max_{t}\,(Inv\_WS2_{tk})$$

Objective function 2

We have taken a uniform distribution of back orders at all points. If rail capacity is low then it will produce a plan that has a uniform distribution of backorder across warehouse points.

$$\text{Min. } \sum_{t,j} (XWS1\_BO_{tj})^2 + \sum_{t,k} (XWS2\_BO_{tk})^2$$

Subjected to

$$(Inv\_WS1_{(t-1)\,j}) - (XWS1\_Qty\_Supplied_{tj}) + \sum_{i} XPWS1_{itj} - \sum_{i,k} XWS1WS2_{itjk} = Inv\_WS1_{tj} \quad \text{for all t \& j} \;\ldots..(1)$$

$$(Inv\_WS2_{(t-1)\,k}) - (XWS2\_Qty\_Supplied_{tk}) + \sum_{i,k} XWS1WS2_{itjk} = Inv\_WS2_{tk} \quad \text{for all t \& k} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots.....(2)$$

$$\sum_{t=1}^{t_1} XWS1\_Qty\_Supplied_{tj} + XWS1\_BO_{t1j} = \sum_{t=1}^{t_1} D_tWS1_{tj} + Inv\_WS1_{t1j} \quad \forall_{\,j} \;\&\; t_1 = 1\ldots\ldots T \;\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

$$\sum_{t=1}^{t_1} XWS2\_Qty\_Supplied_{tk} + XWS2\_BO_{t1k} = \sum_{t=1}^{t_1} D_tWS2_{tk} + Inv\_WS2_{t1k} \quad \forall\; k \;\&\; t_1=1\ldots..T \;\ldots...\ldots\ldots\ldots\ldots.(4)$$

$$XWS1\_Qty\_moved_{tj} = \sum_{i} XPWS1_{itj} \quad \forall\; t, j \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots....(5)$$

$$XWS2\_Qty\_moved_{tk} = \sum_{i} \sum_{j} XWS1WS2_{itjk} \quad \forall\; t, k \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.....(6)$$

$$XPWS1_{itj} \le Supply\_PWS1_{itj} \quad \forall\; i, t, j \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(7)$$
$$XWS1WS2_{itjk} \le Supply\_\,(WS1WS2_{itjk}) \quad \forall\; i, t, j, k \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.....(8)$$
$$XPWS1_{itj} \ge 0 \text{ for all i and j} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.....(9)$$
$$XWS1WS2_{itjk} \ge 0 \text{ for all i, j and k} \;\ldots\ldots\ldots\ldots.....\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(10)$$

In our solution approach $Inv\_WS1_{(t-1)\ j} = 0$ at the initial stage in the first month and in next month inventory will have some value, which will come due to the difference between supply and demand and it will be adjusted with back orders. It will be same for ($Inv\_WS2_{(t-1)\ k}$). Equation (1) indicates that LHS is equal to RHS, here quantity distributed from location 'j' is equal to quantity supplied to location 'k' and quantities supplied to different PDS connected to 'j'. Equation (2) indicates that that total inventory at the warehouse at stage 2 is equal to the difference between the sum of previous inventory and quantity moved at k and the supply of food grain. Equation (3) and (4) indicate that the sum of quantity supplied and backorder is equal to the sum of demand and inventory from p to j and j to k. Equation (5) and (6) indicate that the quantity moved is always equal to XPWS1 from p to j and XWS1WS2 from j to k. Equation (7) and (8) indicate that the quantity supplied is always greater than quantity moved from p to j and j to k.

F. Procedure for Lagrangian relaxation:

LR is techniques well suited for problems where the constraints can be difficult when difficult constraints are relaxed, and then reduced problem is easily solvable. Here equation (2) is bad constraints so Lagrangian relaxation is used in solution procedure using GAMS code only at k. The main idea is to relax the problem by removing the difficult constraints and putting them into the objective function, assigned with weights (the Lagrangian multiplier). We assume Lagrangian multiplier U_k. Each weight represents a penalty which is added to a solution that does not satisfy the particular constraint.

Step 1. Obtain an upper bound from the combined formulation above non linear equation. Objective function 2 is ((t,j),xws1_bo(t,j)*xws1_bo(t,j))+ sum((t,k),xws2_bo (t,k) * xws2_bo (t,k)) and Axkminusb_k.l(t,k) = inv_ws2.l(t-1,k) - xws2_qty_supplied.l(t,k) + xws2_qty_moved.l(t,k) - inv_ws2.l(t,k) is bad constraint.

Step 2. Obtain the value of Lagrangian coefficient as shown below.

LR.. bound_bo = sum((t,j),xws1_bo(t,j)*xws1_bo(t,j))+ sum((t,k), xws2_bo(t,k)* xws2_bo (t,k)) - sum((t,k),U_k (t,k)* (inv_ws2(t-1,k) - xws2_qty_supplied (t,k) + xws2_qty_moved (t,k) - inv_ws2(t,k))).

Step 3. Update the values of elements as shown below.

```
Iter_count /iter_count1*iter_count20/
We assume that U_k (t,k) = 0.0005 and upper bound = 3
Loop (iter_count$continue)
norm2 = sum ((t,k),sqr (Axkminusb_k.l (t,k)))
Norm = norm2 + 5
Results (iter_count,'temp_counter') = temp_counter
Results (iter_count,'theta') = theta
Step size = theta*(upper bound-bound_bo.l)/norm
Results (iter_count,'norm') = norm
Results (iter_count,'step') = step size
U_k(t,k) = U_k(t,k)+stepsize*Axkminusb_k.l(t,k)
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Step 4. Repeat the above steps for 20 iterations and obtain the bound as max.

IV. Computational results

In this section, we are going to present the results based on empirical investigations in order to verify the closeness and accuracy of the formulation in terms of time and lower bound obtained. The first part of the empirical investigation is on 2x4x20x20 matrix and the second empirical investigation is on 2x4x40x40 matrix. The problem size of 2x4x20x20 means that there is 2 modes of transportation, the time duration is 4 months where cycle time is fixed at the start of the month, a number of warehouses at stage one is 20 and number of warehouses at stage two is 20. The problem size of 2x4x40x40 means that there is 2 modes of transportation, the time duration is 4 months where cycle time is fixed at the start of the month, a number of warehouses at stage one is 40 and number of warehouses at stage two is 40.

Sample problems were created randomly for the different values. These problems were then written and solved using optimization software GAMS (Generalize Algebraic Modeling Structure). The values of input parameters along with the solution of both sets of problems are given below:

Computational result for size 2 X 4 X 20 X 20: [i = 2, t = 4, j = 20, k = 20]

SN	Z1	Z2	Lagrangian value	Z1	Z2	Lagrangian value	CPU Time (in sec.)
1	5476739	83322600	68937000	35609030	109881	564100	17.09
2	366985	6725136	4118942	2212705	26721	26889	14.36
3	1292154	19234290	15107000	7923368	541	275380	15.98
4	69215910	526319000	354875500	290629300	2488741	7639464	14.72
5	8410631	178488200	92861080	156975000	44522	7745170	15.81
6	3458400	42968550	33030000	3263600	96200	481870	15.82
7	7745520	62168000	51023130	125880800	221	313476	14.77
8	3341101	141240400	41558210	48561920	15241	1113673	15.74
9	2666165	64875800	47019120	37956000	221	208046	15.12
10	3608641	149175000	29543360	34283640	36941	250566	16.79
11	19073370	167364500	104697400	7978499	153170	4099005	16.17
12	14284190	149766400	101090000	8856423	591592	5509800	15.24
13	2329740	17452050	13367870	1673296	15109	78825	16.69
14	911939	8839354	3626837	733476	8945	126578	14.76
15	906984	9202363	3950407	838705	12637	170657	15.64
16	1023213	7411163	3985542	582643	8314	128031	14.24
17	913743	5433973	4068050	468692	7450	127092	15.08
18	1029933	8812434	4137234	780004	39208	180260	15.85
19	1142870	8205474	4621499	758516	43072	202023	15.36
20	1058631	8927751	3782870	457787	19010	162499	15.94

Table 1: Result table for size 2 X 4 X 20 X 20

Computational result for size 2 X 4 X 40 X 40: [i = 2, t = 4, j = 40, k = 40]

SN	Z1	Z2	Lagrangian value	Z1	Z2	Lagrangian value	CPU Time (in sec.)
1	3419300	14019000	6044200	2421200	9061	284760	33.94
2	3352700	22379790	5812200	1966600	10760	233270	32.31
3	3354100	11945920	5969500	2261000	6970	257360	28.31
4	3289500	12382790	5929400	2259900	15850	223260	29.86
5	3290300	11731570	5752800	1903300	6266	229070	32.32
6	3545800	11611170	6425000	2697300	139816	487880	30.55
7	12372000	45190000	30055000	3728200	119170	3266900	38.63
8	12950000	56009000	3202800	5695600	201600	3255500	33.84
9	515301	3632900	2500400	1392534	1801	37157	31.07
10	537360	4048100	2594200	1532200	340	34935	33.02
11	135030	2351803	594170	69238	245	20488	30.91
12	158090	4019851	692750	145760	1469	24735	31.91
13	146210	2486345	673110	98882	2986	39595	35.84
14	325650	5416509	1398600	155280	9769	122330	31.36
15	326130	3728368	1547300	240490	6101	138410	28.9
16	311380	4158503	1488500	179880	1370	119290	33.92
17	305600	6390172	1559100	206790	9077	157230	31.36
18	372650	3826079	1471000	144600	4777	83651	31.86
19	356800	4584327	1638700	336740	9025	160060	33.66
20	402670	3657630	1616900	210420	8488	108850	31.78

Table 2: Result table for size 2 X 4 X 40 X 40

2x4x20x20 for Costs:

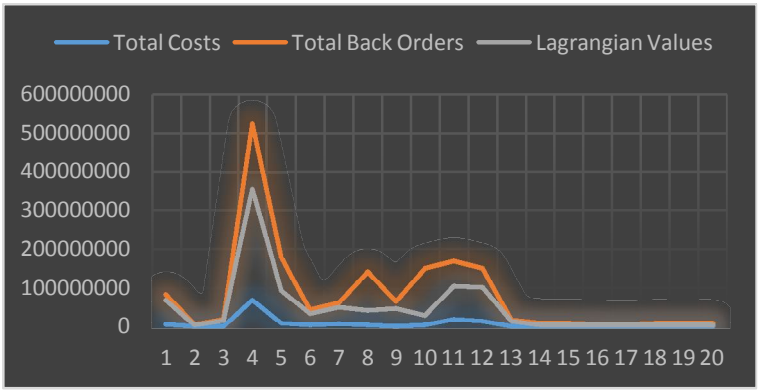

Figure 2: Overall cost for problem size 2X4X20X20

2x4x20x20 for BO:

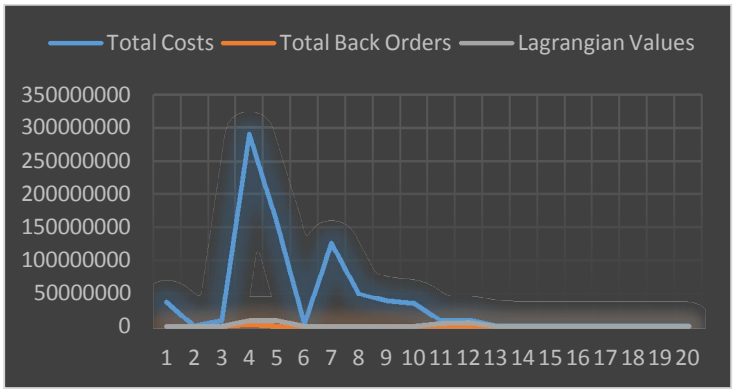

Figure 3: Total backorders for problem size 2X4X20X20

2x4x40x40 for Cost:

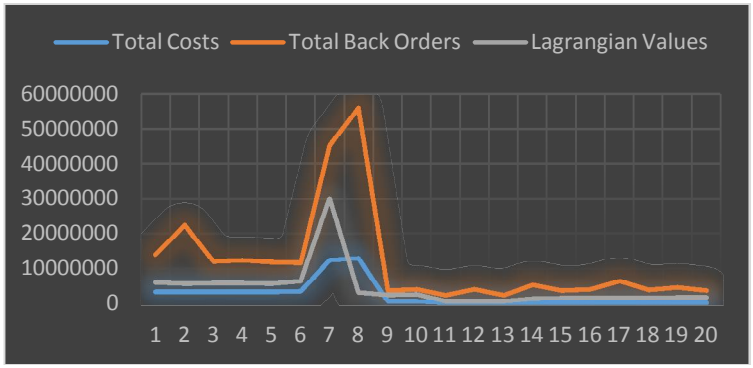

Figure 4: Overall cost for problem size 2X4X40X40

2x4x40x40 for BO:

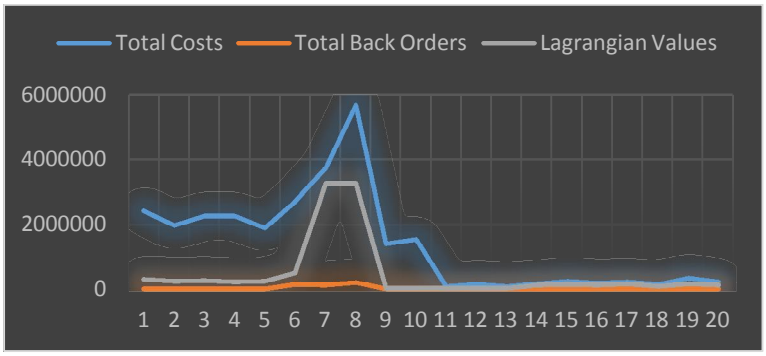

Figure 5: Total backorders for problem size 2X4X40X40

From the results, we can see that quantity supplied to various big warehouses can be calculated. These values have been calculated considering the given constraints and they minimize the backorder cost. As it can be inferred from the result and subsequent example that the resulting quantities supplied to any big warehouse come in the form of demand functions for the various time period. So with the demand estimates for various big warehouses for different time periods, total backorder cost can be minimized. When the Lagrangian values of cost (Z1) and BO (Z2) solving in the GAMS problem there are two parameters to solve the new values of Z1 and Z2:

1. When taking the Lagrangian value of Z1 is add as a constraint and solve for the value of Z2.
2. When taking the Lagrangian value of Z2 is add as a constraint and solve for the value of Z1.

The values of input parameters of Lagrangian values of Z1 and Z2 along with the new solution values if Z1 and Z2 both sets of problems and solutions are given below:

LR is successful when the reduced problem is in P (polynomial solvable). The way in which LR is attempted, the reduced problem is again solved as NLP (quadratic objective function s.t. linear constraints), hence LR is NOT successful. Moreover, LR gives a feasible solution, and GAMS gives the optimal solution. Hence, GAMS type of approach is preferable. Maybe later GAMS performance is to be compared to LDR type of approach developed by Singh (2012) which just requires a matrix inversion for a solution. It is expected that LR will be promising if used in conjunction with methodology due to Singh (2012) as then we have to invert a matrix of smaller size.

Statistical Analysis for Problem 2*4*20*20 and 2*4*40*40

Hypothesis tests are conducted as follows:

1) To check whether the Lagrangian results are significantly related to overall cost and total back orders for both type problems. We propose that different constraints of the supply chain problem are relaxed at Z1 and Z2 to generate different solutions. Therefore, we check the Lagrangian result is relaxed at Z1 is higher than the Lagrangian result is relaxed at Z2 for a given set of problems where their mean values are μ_1 and μ_2 respectively.
   So, Null hypothesis, H0: $\mu_1 < \mu_2$
   Alternate hypothesis, Ha: $\mu_1 > \mu_2$
   Paired sample t-test is used to test the hypotheses for the Lagrangian values are given below.

Result: From problem size 2*4*20*20, the statistical t-tables we have the critical value for t-stats at α = 0.05 as 2.086 for dof = 19 and for this t-test, t = 2.749 thus we can easily reject the null hypothesis. μ_1 = 49270052.55 and μ_2 = 1470170.20, therefore it supports the alternate hypothesis. From problem size 2*4*40*40, the statistical t-tables we have the critical value for t-stats at α = 0.05 as 2.086 for dof = 19 and for this t-test, t = 2.996 thus we can easily reject the null hypothesis. μ_1 = 4348281.50 and μ_2 = 464236.55, therefore, it supports the alternate hypothesis.

2) To check whether execution timing is significantly related to problem size 2*4*20*20 and 2*4*40*40 are considered as problem 1 and problem 2. Definitely, the execution timing of big complex problem is high. Therefore, we check the execution timing of both the problems is equal for the given set of problems where μ_1: Mean execution timing of problem 1 and μ_2: Mean of execution timing of problem 2.
So, Null hypothesis, H0: $\mu_2 \leq \mu_1$
Alternate hypothesis, Ha: $\mu_2 > \mu_1$
Paired sample t-test is used to test the hypothesis for problem 1 and problem 2, the values are given below.

Result: From the statistical t-tables we have the critical value for t-stats at α = 0.05 as 2.086 for dof = 19 and for this t-test, t = -30.30 thus we can easily reject the null hypothesis. μ_1 = 15.559 and μ_2 = 32.268, therefore, it supports the alternate hypothesis.

V. Conclusion and future research

We have proposed a solution set that optimizes both the values when the overall costs decreased then back orders are increased and vice-versa. This optimal solution is getting by using Lagrangian relaxation method. We decomposed the combined formulation into two different subproblems and solved it for different instances. Here the efficiency of decomposition approach for warehouse stages is shown in terms of the time and lower bound for backorder is obtained. In all the different instances of the problem the proposed formulation with Lagrangian relaxation provides good bound in comparatively less computational time. This method is compared to LDR rules (Holt et al, 1960) that it minimizes the quadratic problem statement into a linear function. From the results, we can see that quantity supplied to various big warehouses can be calculated. These values have been calculated considering the given constraints and they minimize the total backorder.

For future work, different obstacles are still left, as we have mentioned in the research problem, which can be included in the model. Few of them are: (1) no proper number of warehouses to procure food grain; (2) no circulation of food grains; (3) delay of rail; (4) high lead time; (5) no manpower; (6) gap between planned and actual movement and; (7) poor infrastructure which are faced by FCI as well as Indian Railway. For these, we can suggest FCI and Indian Railway develop the new warehouses at certain places to fulfill the demand of PDS without delay and these changes will give a different feasible solution in order to minimize the wastage of food grain. We propose for future work that different constraints of the supply chain problem be relaxed to generate different quality solutions. These may be compared with Linear Decision Rule type of methodology applied to NLP by Singh (2012). Also, we can change different parameters of LR such as step size; starting values of multipliers, halving of a parameter of LR etc that are making good tradeoffs between back orders and operational cost.

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