

A New Formulation of Multi item Lot Sizing Problem with Set up, Inventory and Shortage Costs

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Abstract

In this article, we propose a novel formulation of Multi Item Capacitated lot sizing problem considering set up, Inventory and shortage cost. This Formulation is motivated from the work by Vimal (2012) and Sharma (2018). We extend the results developed by Sharma (2018) to multi item lot sizing problem. The new formulation has less number of variables as shortage variables are removed and hence offers computational advantage.

Keywords

Multi item CLSP; Reformulation; MILP

1. Introduction

The standard formulation of the CLSP, which is available in the literature, is extended to include the variables and the situations which we propose to study, and model. Capacitated single item lot sizing problem (CLSP) with setup, production, and inventory is a well-studied problem (see Miller et al. (2000) for a detailed literature review. They give several valid inequalities of CLSP which resulted in a reformulation that can be solved much more easily (compared to effort required to solve the 0-1 mixed integer programming formulation of CLSP). The assumed model is a single level, multi-item, multi-period case of a capacitated lot size problem, with backorders and setup time considerations included. These considerations have received little attention in the literature as noted by (Karimi et. al 2003). The formulations shown below is an extension of the classical model given in our recent work (Verma and Sharma, 2009).

2. Problem Formulation

Indices Used

t : Set of Time period from $1..T$

i : Product type

Constants:

CP_{it} : Unit cost of producing item 'i' in period 't'.

CS_{it} : Unit cost of setup, for item 'i' in period 't'.

$CINV_{it}$: Unit cost of holding inventory of item 'i' for 1 period.

CBO_{it} : Unit cost of backordering item 'i', which was demanded during the period 't'.

CAP_{it} : Capacity available to produce item 'i' during the period 't'.

$CAPT_t$: Capacity available in time units, in a period 't'.

D_{it} : Demand of item 'i' during period 't'.
 PT_{it} : Time required to process the item 'i'.
 ST_i : Time required to setup the production for item 'i'.

Definition of Variables:

XP_{it} : Number of items 'i' to be produced during the period 't'.
 $XINV_{it}$: Number of items 'i' carried as inventory to be produced during the period 't'.
 XBO_{it} : Number of items 'i' that will be backordered from period 't'.
 YS_{it} : Binary variable for setup of the resource for item 'i' during the period 't' = 0 (if there is no setup required), 1 otherwise. Entering inventory in time period 't'.

The classical Multi Item Capacitated lot sizing problem with backorder and set up cost (CLSP_BS) can be modeled as below. Some additional constraints are added to original model proposed by Mayank (2010) and we divide it into two problems to reduce the number of variables for obtaining a computational advantage.

Problem A1

$$\text{Minimize } Z = \sum_{i=1}^I \sum_{t=1}^T [CP_{it} * XP_{it} + CS_{it} * YS_{it} + CINV_{it} * XINV_{it} + CBO_{it} * XBO_{it}] \quad (1)$$

$$XP_{it} + XINV_{i,t-1} + XBO_{it} = D_{it} + XINV_{it} + XBO_{i,t-1} \quad \forall i, t \quad (2)$$

$$\sum_{i=1}^I PT_{it} * XP_{it} + ST_i * YS_{it} \leq CAPT_t \quad \forall t \quad (3)$$

$$\sum_{i=1}^I (PT_i * D_{it} + ST_i * YS_{it}) \leq CAPT_t \quad \forall t \quad (4)$$

$$XP_{it} \leq CAP_{it} YS_{it} \quad \forall i, t \quad (5)$$

$$\sum_{i=1}^I \sum_{t=1}^T (PT_{it} * D_{it} + ST_i * YS_{it}) \leq \sum_{t=1}^T CAPT_t \quad (6)$$

$$\sum_{i=1}^I \sum_{t=1}^T (PT_{it} * XP_{it} + ST_i * YS_{it}) \leq \sum_{t=1}^T CAPT_t \quad (7)$$

$$\sum_{t=1}^T XP_{it} \geq \sum_{t=1}^T D_{it} \quad \forall i \quad (8)$$

$$XINV_{i0} = 0 \quad \forall i \quad (9)$$

$$XINV_{iT} = 0 \quad \forall i \quad (10)$$

$$XBO_{i0} = 0 \quad \forall i \quad (11)$$

$$XBO_{iT} = 0 \quad \forall i \quad (12)$$

$$YS_{it} \in \{0,1\} \quad \forall i,t \quad (13)$$

$$XINV_{it}, XP_{it}, XBO_{it} \geq 0 \quad \forall i,t \quad (14)$$

Problem A2

Minimize

$$Z = \sum_{i=1}^I \sum_{t=1}^T [CP_{it} * XP_{it} + CS_{it} * YS_{it} + CINV_{it} * XINV_{it}] + \sum_{t_1=1}^T \sum_{i=1}^I [CBO_{it_1} * (\sum_{t=1}^{t_1} D_{it} + XINV_{it_1} - \sum_{t=1}^{t_1} XP_{it})] \quad (15)$$

$$\sum_{t=1}^{t_1} D_{it} + XINV_{it_1} - \sum_{t=1}^{t_1} XP_{it} \geq 0 \quad \forall i, t_1 = 1, \dots, T. \quad (16)$$

$$\sum_{i=1}^I PT_{it} * XP_{it} + ST_i * YS_{it} \leq CAPT_t \quad \forall t \quad (17)$$

$$XP_{it} \leq CAP_{it} YS_{it} \quad \forall i,t \quad (18)$$

$$\sum_{i=1}^I \sum_{t=1}^T (PT_{it} * D_{it} + ST_i * YS_{it}) \leq \sum_{t=1}^T CAPT_t \quad (19)$$

$$\sum_{i=1}^I \sum_{t=1}^T (PT_{it} * XP_{it} + ST_i * YS_{it}) \leq \sum_{t=1}^T CAPT_t \quad (20)$$

$$\sum_{t=1}^T XP_{it} \geq \sum_{t=1}^T D_{it} \quad \forall i \quad (21)$$

$$XINV_{i0} = 0 \quad \forall i \quad (22)$$

$$XINV_{iT} = 0 \quad \forall i \quad (23)$$

$$YS_{it} \in \{0,1\} \quad \forall i,t \quad (24)$$

$$XINV_{it}, XP_{it} \geq 0 \quad \forall i,t \quad (25)$$

Equation (1) and equation (15) gives the objective function of the problem, which intends to minimize the production cost, setup cost, inventory cost and the backlogging cost, summed over all items and time periods. (2) is the inventory balance constraint which is for each item and for each period considering backorder quantity carried over next period. (3) is the time capacity limit constraint, which ensures that the total time utilized in doing a production of all items added to the time used to do the setup is always less than or equal the maximum time available in any period. (4) is quite similar to (3), it ensures that the demand of particular item in a period can be fully satisfied by the available processing time. (5) is the physical capacity constraint of all the plants and restricts the production from a facility with its stated capacity. (6) and (7) are different from (3) and (4) in a sense that while (3) and (4) restricts the production quantity of all items with available processing time in a particular period, (6) and (7) restricts the production of all items with total processing time over all the periods. (8) ensures that demand is satisfied at least by production in all the periods. (9) – (12) sets the initial and final conditions by substituting inventory and backordered quantity to be 0. (13) and (14) are integer (0,1) and non-negativity constraints for the variables. Problem A2 has similar constraints with similar implications with the elimination of backordered quantities. It is to be noted that left hand side of equation (16) is the value of XBO_{it} .

3. Conclusion

We have proposed a novel model for multi item lot sizing problem. This model has less number of variables and extra constraints which provides us with computational advantage. Therefore it enables us to develop computationally attractive methods for solving CSLP_BS which is known to be a NP complete problem (Maes et al. 1991)

4. References

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