Estimation of Conditional Value-at-Risk under Assets Liability Model with Non Constant Volatility

Sukono, Alit Kartiwa, Betty Subartini
Department of Mathematics, Faculty of Mathematics and Natural Sciences,
Universitas Padjadjaran, Indonesia
sukono@unpad.ac.id, alit.kartiwa@unpad.ac.id, betty.subartini@unpad.ac.id

Yuyun Hidayat
Department of Mathematics, Faculty of Mathematics and Natural Sciences,
Universitas Padjadjaran, Indonesia
yuyunhidayat@unpad.ac.id

Abdul Talib Bon
Department of Production and Operations,
University Tun Hussein Onn Malaysia, Johor, Malaysia
talibon@gmail.com

Abstract

This paper analyzes the risk measurement using CVaR when return follows the liability asset model with an unstable volatility approach. An unstable flat model is estimated using ARMA models, and the volatility model is not constantly estimated using GARCH models. The surplus return is estimated using the asset liability model. The prediction value using flat models and unstable volatility, is then used to determine the mean and variance of surplus returns that follow the asset liability model. Next, the average value and the variance of surplus returns are used to determine the magnitude of VaR and CVaR of each surplus return. The results of VaR and CVaR calculation of surplus return can be used as information in considering investment risk policy on each assets and liabilities analyzed.

Keywords:
ARMA model, GARCH model, asset liability model, VaR and CVaR.

1. Introduction

Investing in financial assets is an activity that contains uncertainty. Investors cannot know exactly how much the expected rate of return will be earned, and cannot take into account how much of the deviation that occurred on the investment (Jaaman et al., 2013). The uncertainty that can lead to such deviation is a risk, which is usually faced by investors. It should be realized, however, that it is difficult to separate between returns as investment benefits and possible risks (Sukono et al., 2016a; 2016b). There is a correlation between the return and risk, as generally, the greater the return offered by an investment asset, the higher the risk content in the investment asset. That is, any future expectations of an investment, there must be a potential risk of investment. Moreover, the future contains full of uncertainty, while uncertainty is a risk (Boudt et al., 2013; Sukono et al., 2017a).

Risk measurement is very important in the analysis of investments in financial assets, considering this is related to investment with substantial allocation of funds (Tokpavi and Vaucher, 2012). Risk is usually measured by using standard deviation or variance. Note, however, that standard deviation or variance is a measure of average risk. So it cannot accommodate any risk events that may occur. Given that, it develops that risk is measured using quantitative, or better known as Value-at-Risk (Sukono et al., 2017b; 2017c). Despite its wide use, Value-at-Risk is also known to have unattractive properties, among them that Value-at-Risk is a measure of risk that is not strong and unconventional (Sawik, 2011). It thus encourages the use of the Conditional Value-at-Risk. Using Conditional Value-at-Risk allows

investors to analyze the probability of a greater risk of loss event than Value-at-Risk (Sawik, 2011; Boudt, 2013; Xiong and Idzorek, 2010).

There are some investors who are able to invest in financial assets, but on the other hand the investor also has an obligation to pay from the investment. Thus, the profit earned by investors is a surplus between investment returns with obligations (liabilities) to be borne (Romanyuk, 2010; Sukono et al., 2017a). Therefore, in this paper we will analyze Conditional Value-at-Risk under the asset-liability model. In the analysis is done by using the approach of volatility is not constant, namely that the return of assets and return liability is considered time series data. The objective is to analyze Conditional Value-at-Risk on a data return that has uneven mean and volatility. As a numerical illustration, discussed several assets that are traded on the Indonesia Stock Exchange.

2. Methodology

Analysis of Conditional Value-at-Risk under the asset liability models with non constant volatility approach covering the steps that are outlined below.

2.1 Determination of Return

Suppose P_t price or value of an asset-liability at a time t ($t = 1,...,T$ and T number of observation data), and r_t return asset-liability at time t. The amount of asset-liability return can be determined by equation (Sukono et al., 2017b; Tsay, 2005):

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

Return data r_t then used in the estimation of the average model as follows.

2.2 Estimation of Mean Model

The estimation of the mean model in time series data is done using the autoregressive moving average (ARMA) model approach. Suppose r_t is a data return that follows the time series pattern, ARMA degree model p and q written as ARMA(p, q),which generally have in common:

$$r_t = \phi_0 + \sum_{i=1}^{p} \phi_i r_{t-i} + a_t - \sum_{j=1}^{q} \theta_j a_{t-1}, \tag{2}$$

Where $\{a_t\}$ residual rows of white noise, as well as p and q non-negative integers Sukono et al., 2016a).

The process of estimating the average model in time series data is done by the following stages: (i) Stationarity test of data return; (ii) Identification of a flat model; (iii) Estimation of average model parameters; (iv) Diagnostic test of the average model; and (v) Prediction using an average model (Sukono et al., 2016b;Tsay, 2005).

2.3 Estimation of Volatility Model

The estimation of volatility is not constant using the generalized autoregressive conditional heteroscedastic (GARCH) model approach. The estimation of the volatility model is not constant by using the residual data from the average model. Suppose residual a_t where $a_t = r_t - \hat{r}_t$, model GARCH degrees u and v written as GARCH(u , v),and has a volatility equation σ_t^2 as:

$$a_t = \sigma_t \varepsilon_t, \ \sigma_t^2 = \alpha_0 + \sum_{i=1}^{u} \alpha_i a_{t-i}^2 + \sum_{j=1}^{v} \beta_j \sigma_{t-j}^2 + \varepsilon_t, \tag{3}$$

where $\{\varepsilon_t\}$ the residual row of volatility models which are randomly distributed and iid variables with a mean of 0 and variance 1. The parameters must meet the following constraints: $\alpha_0 > 0$, $\alpha_i \geq 0$, $i = 1,...,u$, and $\beta_j \geq 0$, $j = 1,...,v$ (Johansson and Sowa, 2013; Tsay, 2005).

The process of estimating the volatility model is carried out by the following steps: (i) ARCH element testing in average residuals; (ii) Identification of volatility models; (iii) Estimation of volatility model parameters; (iv) Diagnostic testing of volatility models; and (v) Predictions using volatility models (Sukono et al., 2016b; Tsay, 2005).

2.4 Modeling Mean and Variance Surplus of Asset-Liability

The model of surplus return on assets is described briefly as follows. Suppose A_t assets on time t, L_t liability at the time t, and S_t surplus on time t. At the beginning t =0, the initial surplus is given by:

$$S_0 = A_0 - L_0 .$$

The surplus obtained after one period is (Imaduddin, 2010; Romanyuk, 2010):

$$S_1 = A_1 - L_1 = A_0[1 + r_{A_1}] - L_0[1 + r_{L_1}] ,$$

Suppose r_S return surplus expressed as:

$$r_S = \frac{S_1 - S_0}{A_0} = \frac{A_0 r_A}{A_0} - \frac{L_0 r_L}{A_0} = r_A - \frac{1}{f_0} r_L , \tag{4}$$

with $f_0 = \frac{L_0}{A_0}$ (Imaduddin, 2010; Romanyuk, 2010).

Based on equation (4) the average of the surplus return can be determined by the formula:

$$\mu_S = E[r_S] = \mu_A - \frac{1}{f_0}\mu_L . \tag{5}$$

Where μ_S, μ_A and μ_L respectively are the average of surplus returns, assets, and liabilities. Also based on (4), the surplus variance can be determined by the formula:

$$\sigma_S^2 = \sigma_A^2 - \frac{2}{f_0}\sigma_{AL} + \frac{1}{f_0^2}\sigma_L^2 . \tag{6}$$

Where σ_S^2, σ_A^2 and σ_L^2 respectively are variance of surplus returns, assets, and liabilities. While σ_{AL} covariance between asset return and liability return (Imaduddin, 2010; Romanyuk, 2010).

2.5 Modeling of VaR and CVaR

Suppose VaR_S stated Value-at-Risk surplus return. When given μ_S mean and σ_S the standard deviation from the surplus return, then VaR_S expressed as:

$$VaR_S = -W_0\{\mu_S + z_c\sigma_S\}.$$

Where W_0 initial funds invested, and z_c percentile of standard normal distribution with significance level c (Sawik, 2011; Sukono et al., 2017c). Using the mean and variance of surplus returns in equations (5) and (6), VaR_S under the asset-liability model can be expressed as:

$$VaR_S = -W_0\{(\mu_A - \frac{1}{f_0}\mu_L) + z_c(\sigma_A^2 - \frac{2}{f_0}\sigma_{AL} + \frac{1}{f_0^2}\sigma_L^2)^{\frac{1}{2}}\} . \tag{7}$$

Furthermore, suppose $CVaR_S$ states Conditional Value-at-Risk surplus return, and $L(r_S)$ loss rate (Loss) of surplus return. Conditional Value-at-Risk is basically (Sawik, 2011; Xiong and Idzorek, 2010):

$$CVaR_S = E[L(r_S) \mid L(r_S) \geq VaR_S] . \tag{8}$$

Conceptually $CVaR_S$ can be shown in Figure-1 below.

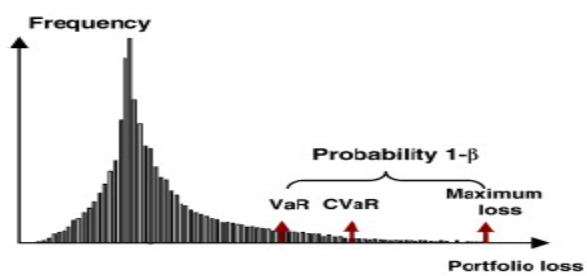

Figure 1. The concept of VaR_t and $CVaR_t$

As a numerical illustration, analyzes of several financial assets traded on the Indonesia Stock Exchange are as follows.

3. Illustration Analysis

In this section numerical illustrative analysis includes: illustrative data, uneven constant model estimation and volatility, average estimation and variance of surplus returns, as well as the estimated Conditional Value-at-Risk return surplus, as follows.

3.1 Illustration Data

The data analyzed is accessed through the website http://www.finance.go.id. The data consists of five selected assets, for the period of January 2, 2014 up to June 4, 2017, covering the closing prices of INDF, LSIP, HDMT, UNTR and BBRI assets. Next, in sequence is called by A_1, A_2, A_3, A_4, and A_5. The data of each liability are generated simulation. The data of the liability is subsequently referred to as L_1, L_2, L_3, L_4, and L_5.

Both the asset price and the liability data are then determined by the return of each use (1). The asset and liability return data will be used to estimate the following unchanged average and volatility models.

3.2 Estimation Model of Mean and Non Constant Volatility

In this section an average model estimation and unstable volatility with time series approach are used. First, stationarity test is done to asset return data A_1, A_2, A_3, A_4, and A_5 using unit root test statistics. The stationarity test is done with the help of Eviews 7 software, and the results show that all asset return data is stationary. Secondly, each stationary data return is then estimated by the average model. Estimates are performed using ARMA models referring to equation (2). Estimates carried out include stages: identification of the average model, model parameter estimation, parameter verification test, and diagnostic test. All stages were performed using the help of Eviews 7 software, and the result of the average model estimation of all shows was significant.

Third, *using residuals from each model of average return on assets* A_1, A_2, A_3, A_4, and A_5, do non constant volatility model estimation. The non constant volatility estimated using GARCH models refers to equation (3). Estimation phase is non constant volatility models include: ARCH element test, identification of the model, the model parameter estimation, verification test parameters, and diagnostic testing. All phases conducted with the help of software Eviews 7, and the estimation results indicate that all non constant volatility models has been significant. The results of the models estimates the mean and non-constant volatility are in the outline given in Table-1 in the Model column. Estimator mean and non constant volatility models, then used for prediction one period ahead, that is $\hat{r}_{A_i}(1)$ and $\hat{\sigma}^2_{A_i}(1)$. Predictions are done iteratively with the help of Excel 2007 software, and the results are summarized in Table-1.

Table-1. Estimator of Mean and Volatility of Asset Return

Asset (A_i)	**Models**	**Mean** $\hat{r}_{A_i}(1)$	**Volatility** $\hat{\sigma}^2_{A_i}(1)$
A_1	ARMA(1,0)-GARCH(1,1)	0.045389	0.002653
A_2	ARMA(1,1)-GARCH(1,1)	0.008673	0.001931
A_3	ARMA(1,1)-GARCH(1,1)	0.003575	0.001172
A_4	ARMA(1,1)-TGARCH(1,1)	0.020707	0.013462
A_5	ARMA(0,1)-GARCH(1,1)	-0.000675	0.001246

The same way as above, used for the estimation of the mean and non contant volatility models on the five data return liabilities. The result, both the mean and the non constant volatility models return liability L_1, L_2, L_3, L_4, and L_5 shows have been significant. The result of the mean and the non constant volatility models estimation of return liability is given in Table-2 of the models column.

Table-2. Estimator of Mean and Volatility Return Liabilities

Liability (L_i)	**Models**	**Mean** $\hat{r}_{L_i}(1)$	**Volatility** $\hat{\sigma}^2_{L_i}(1)$
L_1	ARMA(2,2)-ARCH(1)-M	0.039018	0.002787
L_2	ARMA(0,1)-GARCH(3,3)	0.003326	0.001333
L_3	ARMA(0,1)-GARCH(1,1)	-0.000273	0.001773
L_4	ARMA(0,1)-EGARCH(1,1)	0.001595	0.001413
L_5	ARMA(1,1)-GARCH(1,1)	0.022184	0.001191

The values of the mean estimator $\hat{r}_{A_i}(1)$ and volatility $\hat{\sigma}^2_{L_i}(1)$ return of assets in Table-1, as well as the mean values $\hat{r}_{L_i}(1)$ and variance $\hat{\sigma}^2_{L_i}(1)$ in Table-2, is then used to estimate the mean value μ_S and variance σ^2_S surplus return, the following.

3.3 Estimated Values of Mean and Variance Return Surplus

In this section to estimate the mean value μ_S and variance σ^2_S return of surplus S_1, S_2, S_3, S_4, and S_5, using an asset-liability model. Here it is assumed that the ratio between liabilities and assets at the time of the beginning $f_0 = 1$. Furthermore, using estimator of mean return of assets in Table-1 and estimator of mean return of liability in Table-2 can be estimated the mean value of surplus return. The estimated average value of surplus return is done using equation (5). Here also an estimate of the value of covariance between asset returns with the corresponding return liability. This is because to estimate the variance of surplus return using equation (6), requires the value of covariance between asset return and return liability σ_{AL}. The estimated value of the variance of surplus return is done using equation (6). From the estimator the value of variance can be used to calculate the standard deviation of surplus return. The result of estimation of mean value, covariance, variance, and standard deviation of surplus return is given in Table-3.

Table-3. The mean estimator, covariance and variance Return Susrplus

Surplus (S_i)	**Mean** (μ_{S_i})	**Covariance** (σ_{AL_i})	**Variance** ($\sigma^2_{S_i}$)	**Standard Dev.** (σ_{S_i})
S_1	0.006371	0.000212	0.004892	0.069943
S_2	0.005347	0.000740	0.001308	0.036166
S_3	0.003848	0.000780	0.001861	0.043139
S_4	0.019112	0.000421	0.014413	0.120054
S_5	-0.022859	0.000311	0.002437	0.049366

Estimator of mean value and standard deviation of surplus return in Table-3, then used to estimate Conditional Value-at-Risk value from the following surplus return.

3.4 Estimation of Conditional Value-at-Risk Return Surplus

In this section, the Conditional Value-at-Risk is estimated ($CVaR_S$) return of surplus S_1, S_2, S_3, S_4, and S_5, for some conditions the value of significance level c, which cover: c=10%; c=7.5%; c=5%; c=2.5% and c=0.5%. Estimates are done by referring to equations (7) and (8), the results given in Table-4.

Table-4. Estimator of Conditional Value-at-Risk

	$CVaR_S$				
Surplus	**10%**	**7.5%**	**5%**	**2.5%**	**0.5%**
S_1	0.044445	0.050710	0.058851	0.071346	0.095766
S_2	0.047555	0.054076	0.062552	0.075559	0.100982
S_3	0.054950	0.062198	0.071618	0.086076	0.114332
S_4	0.133898	0.152760	0.177274	0.214897	0.288427
S_5	0.086124	0.093923	0.104059	0.119615	0.150017

Estimator value $CVaR_S$ for some conditions the value of significance level c in Table-4, for each surplus return S_1, S_2, S_3, S_4, and S_5 can be shown as a graphic form as in Figure-1.

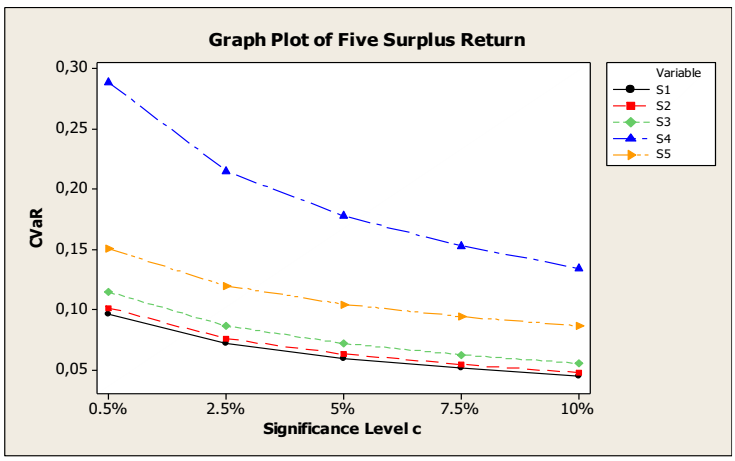

Figure 1. *CVaR* Graph For Five Return Surplus

In Figure-1, on the horizontal axis, respectively indicates the value of significance level c=0.5%; c=2.5%; c=5%; c=7.5% and c=10%. While on the vertical axis shows the value of $CVaR_S$.

Noting the graph in Figure 1 appears that for the value of the level of significance c=0.5%; c=2.5%; c =5%; c=7.5% and c=10%; shows that the estimator graph $CVaR_S$ for surplus return S_1 is the lowest. This gives a signal that the risk of loss on the surplus return S_1 is the least likely. Next, followed by the graphs respectively $CVaR_S$ for surplus return S_2, S_3 and S_5. While the estimator graph $CVaR_S$ for return S_4 is much higher than the other four surplus returns. This gives a signal that the risk of loss for a surplus return S_4 far more likely.

Furthermore, if note the change in estimator graph $CVaR_S$ for each value of significance level c=0.5%; c=2.5%; c=5%; c=7.5% and c=10%; then it is seen that the surplus return S_1, S_2, S_3 and S_5 the trend of lowering is relatively low. The downward trend is somewhat diminished after reaching the level of significance c=2.5%. While the estimator graph $CVaR_S$ for surplus return S_4 the downward trend is relatively sharp, and the downward trend is decreasing lower after reaching the value of significance level c=2.5%. The high decreasing trend of estimator graph $CVaR_S$ it clearly illustrates the change in risk level, for any change in the condition of the value of significance level c specified by the investor.

4. Conclusions

In this paper we have performed an analysis of Conditional Value-at-Risk (*CVaR*) under the asset liability model with unstable volatility. The data used in the analysis covered the prices of INDF, LSIP, HDMT, UNTR, and BBRI assets. While the liability price data used is generated simulation, this is because the data liability used in the analysis here is not obtained. From result of analysis, surplus return S_1 has an estimator $CVaR_S$ the lowest for any condition value of significance level c=10%; c=7,5%; c=5%; c=2,5% and c =0,5%, followed subsequently by surplus returns S_2, S_3 and S_5. While the surplus return S_4 has an estimated value $CVaR_S$ much higher. Such an analysis is expected to provide an understanding, and be taken into consideration in decision making for the investor, to establish the condition of the level of significance in the measurement of risk.

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References

Boudt, K., Carl, P. & Peterson, B.G. Asset Allocation with Conditional Value-at-Risk Budgets. *Journal of Risk.* Volume 15/Number 3, Spring 2013 (39-68). 2013.

Imaduddin, M., 2010, Manajemen Asset dan Liabilitas Dalam Bank Syariah (*Asset Management and Liabilities in Sharia Bank*), *Jurnal Ekonomi Islam Al-Infaq*, Vol. 1, No. 1, September 2010

Jaaman, S.H., Lam, W.H. & Isa, Z. Risk Measures and Portfolio Construction in Different Economic Scenarios. (Pengukuran Risiko dan Penjanaan Portfolio dalam Senario Ekonomi Berbeza). *Sain Malaysiana,* 42(6)(2013): 875-880. 2013.

Johansson, A. & Sowa, V. A Comparison of GARCH Models for VaR Estimation in Three Different Markets. Department of Statistics, Uppsala University, 2013-06-07. 2013.

Romanyuk, Y. Asset-Liability Management: An Overview**.** Discussion Paper/Document d'analyse. Bank of Canada Discussion Paper 2010-10. August 2010.

Sawik, B. Conditional Value-at-Risk and Value-at-Risk for Weighting Approach. *Automatyka*. 2011. Tom 15. Zeszyt 2. 2011.

Sukono, Bin Bon, A. T., Mamat, M., & Supian, S. The APT with lagged, value-at-risk and asset allocations by using econometric approach. Paper presented at the *Proceedings of the International Conference on Industrial Engineering and Operations Management,* 532-542. 2016a. Retrieved from www.scopus.com

Sukono, Hidayat, Y., Suhartono, Sutijo, B., Bon, A. T. B., & Supian, S. Indonesian financial data modeling and forecasting by using econometrics time series and neural network. *Global Journal of Pure and Applied Mathematics, 12*(4), 3745-3757. 2016b. Retrieved from www.scopus.com

Sukono, Hidayat, Y., Talib Bin Bon, A., & Supian, S. Modelling of capital asset pricing by considering the lagged effects. Paper presented at the *IOP Conference Series: Materials Science and Engineering, 166*(1), 2017a. doi:10.1088/1757-899X/166/1/012001

Sukono, Sidi, P., Bon, A. T. B., & Supian, S. Modeling of mean-VaR portfolio optimization by risk tolerance when the utility function is quadratic. Paper presented at the *AIP Conference Proceedings, 1822-1827.* 2017b. doi:10.1063/1.4979451

Sukono, Sidi, P., Susanti, D., & Supian, S. Quadratic investment portfolio without a risk-free asset based on value-at-risk. *Journal of Engineering and Applied Sciences, 12*(19), 4846-4850. 2017c. doi:10.3923/jeasci.2017.4846.4850

Tokpavi, S. & Vaucher, B. Conditional Value-at-Risk: an alternative measure for low-risk Strategies?. *Unigestion*. 2012.

Tsay, R.S. *Analysis of Financial Time Serie,* Second Edition, USA: John Wiley & Sons, Inc. 2005.

Xiong, J.X. & Idzorek, T. Mean-Variance Versus Mean-Conditional Value-at-Risk Optimization: The Impact of Incoporating Fat Tails and Skewness into the Asset Allocationa Decision. *Ibbotson.* A Morningstar Company. 2010.

Biographies

Sukono is a lecturer in the Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran. Currently serves as Head of Master's Program in Mathematics, the field of applied mathematics, with a field of concentration of financial mathematics and actuarial sciences.

Alit Kartiwa is a lecturer at the Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran. He currently serves as Head of Pure Mathematics Laboratory, a field of pure mathematics, with a field of concentration analysis.

Betty Subartini is a lecturer at the Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran. Currently serves as Secretary of Master Program in Mathematics, the field of pure mathematics, with the field of concentration numerical analysis.

Yuyun Hidayat is a lecturer at the Department of Statistics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran. He currently serves as Deputy of the Head of Quality Assurance Unit, a field of Statistics, with a field of Quality Control Statistics.

Abdul Talib Bon is a professor of Production and Operations Management in the Faculty of Technology Management and Business at the Universiti Tun Hussein Onn Malaysia since 1999. He has a PhD in Computer Science, which he obtained from the Universite de La Rochelle, France in the year 2008. His doctoral thesis was on topic Process Quality Improvement on Beltline Moulding Manufacturing. He studied Business Administration in the Universiti Kebangsaan Malaysia for which he was awarded the MBA in the year 1998. He’s bachelor degree and diploma in Mechanical Engineering which his obtained from the Universiti Teknologi Malaysia. He received his postgraduate certificate in Mechatronics and Robotics from Carlisle, United Kingdom in 1997. He had published more 150 International Proceedings and International Journals and 8 books. He is a member of MSORSM, IIF, IEOM, IIE, INFORMS, TAM and MIM.