

Supplier Selection Using Two Phases Model

**Gamal Mohamed Nowara,
Mohammad Abdel Halim Mansour and Ghada Mohamed Abdel Salam**
Industrial Engineering Department
Zagazig University
Cairo, Egypt
Schultraeger@europaschulekairo.com, Mamansour68@yahoo.com, and
ghadayousf22@yahoo.com

Abstract

Supply chain management interested in the selection of suppliers against multiple criteria rather than the single criterion as a cost only. These criteria may be quantitative, qualitative or both. To choose the best supplier, the criteria are being measured or weighted for ranking. The supplier selection problem is considered as a multi-decision problem and it can be solved by the Analytical Hierarchical Process. It allows the decision maker to structure such complex problems in the form of a hierarchy, or a set of integrated levels. In this paper, a mathematical model is combined with an analytical hierarchical process in two phases, the first phase starts by the weighted mathematical model from an exciting example, and the second phase makes the check and refines the result from the first phase by checking the consistency. And finally, a new ranking of the suppliers are obtained.

Keywords

Analytical Hierarchical Process (AHP); Consistency Test; Multi-Criteria-Decision-Making (MCDM); Multi-Objective Problem (MOP).

1. Introduction

As the role of purchasing in supply management is important, the purchasing process consolidates the efficiency of the supply chain and it is an essential issue to select the best suppliers. There are many factors that affect the selection process as a firm's ability to choose the right supplier. There is a need to understanding of the supplier selection criteria. Firms consider some of the factors; such as a commitment and trust, quality, adequate finance, reliable delivery times, adequate logistic technological capabilities (Cox, 1999). However, the other criteria are ISO certification, credibility, reliability, good references and product development. So the focus isn't only on the quantitative factors but also to include the qualitative criteria (Harps, 2000). (Vera and Pullman, 1998) coincided that the supplier selection is a complex decision-making problem with the following factors (a) multiple criteria, (b) Conflicting among criteria, the (c) involvement of many alternatives (d) internal and external constraints imposed on the buying process.

2. Literature Review

During recent years supply chain management and supplier selection process have received considerable attention in the literature. The objective of supplier selection is to choose suppliers with the respect to a firm's needs and an acceptable cost. Comparison of suppliers to select the best one is by using a common set of measures and criteria. Supplier selection is a multi-criteria problem and there are not a lot of efficient techniques or algorithms that address this problem. There are two major groups of methods in the literature; mathematical programming models cost-based models and categorical models. Since supplier selection problems always have many objectives such as maximization of quality or minimization of cost, the mathematical programming can be used to model the problem. (Weber and Current, 1993) proposed a multi-objective approach to supplier selection which minimizes the price, maximize the quality and on-time delivery using systems' constraints and policy constraints in a mixed integer model.

(Ghodsypour and O'Brien, 1998) integrated AHP with linear programming to behold both tangible and intangible factors in selecting the best suppliers and placing the optimum order quantities among them such that the total value of purchasing is maximizing.

(Cebi and Bayraktar, 2003) structured the problem of supplier selection as an integrated lexicographic goal programming and AHP model including both qualitative and quantitative conflicting factors.

(Wang et al., 2004) proposed AHP and preemptive goal programming based multi-criteria decision-making (

(Wang and Yang, 2009) searched the supplier selection for a quantity discount environment using multi-objective linear programming, fuzzy compromise programming, and AHP.

Because of the importance of the price, it has been a leading factor to select suppliers and the model based on cost has been a common approach. A popular application of the cost approach has been calculating the total cost for each purchase. The total cost of working with each supplier is calculated and the cheapest one is selected. (Timmerman,1986) proposed a cost-ratio method which collects all costs related to delivery, quality and services and shows them as a benefit or penalty percentage on unit price.

3. Two Phases Model

The mathematical model (phase 1) is combined with AHP method (phase 2), first to have a first selection from the given suppliers, then go to the next step to check the consistency for the given criteria and its importance and also making a new ranking for the selected suppliers.

4. Mathematical Model (Phase 1)

4.1 notation of model

I : The number of suppliers ($i \in I$)

J : The number of criteria ($j \in J$)

x_{ij} : The measure of supplier i under criterion j

w_{ij} : The weight of criterion j of supplier i

y_{ij} : The linear transformation of x_{ij}

S_i : The score of supplier i

4.2 Assumption of model

The mathematical model considers a situation in which a set of I suppliers are available for a company. The purchasing manager would like to evaluate these suppliers based on J criteria.

The mathematical model evaluates a supplier ($i = 1,2,3, \dots, I$) by converting multiple measures under all criteria into a single score as (Wang, 2008).

The measure of supplier i under criteria j is denoted as x_{ij} ($i = 1,2, 3, \dots, I, j = 1,2, 3, \dots, J$). the assumption of the model as following:

- 1- All measures are positively related to the score of a supplier. If there is a negatively related criterion, the transformation of negativity or taking reciprocal can be applied for conversions. A common scale for all measures is also an important issue. A particular criteria measure, in a large scale, may always dominate the score.
- 2- The normalizing of all measures x_{ij} into a 0- 1 scale.
- 3- All transformed measures denoted as y_{ij} .
- 4- The criteria are arranged in the descending order of importance such as ($w_{i1} > w_{i2} > w_{i3} > \dots > w_{ij}$). Also the weights w_{ij} 's are non-negative and are normalized.
- 5- The sum of weights, $\sum_{j=1}^J w_{ij} = 1$, after normalization, and all scores S_i ($i = 1, 2, 3, \dots, I$) are always within a 0-1 scale. The value of the weight of particular criteria is equal to the proportion of contribution of the criteria in the total contribution of all criteria.

4.3 The model

The score of a supplier is expressed as the weighted sum of transformed measures,

$$S_i = \sum_{j=1}^J w_{ij} y_{ij}$$

$$y_{ij} = \frac{x_{ij} - \min_{i=1,2,\dots,I}\{x_{ij}\}}{\max_{i=1,2,\dots,I}\{x_{ij}\} - \min_{i=1,2,\dots,I}\{x_{ij}\}}$$

5. Analytical Hierarchical Process Method (Phase 2)

AHP is a subjective decision-making tool developed by (Satty, 1990) that evaluates and weights a number of criteria and sub-criteria which affect the final decision. Unlike the subjective and prior weight assignments in a number of multi-attribute decision models, AHP calculates criteria weights systematically throughout the process itself which reduces subjectivity significantly. Using the same hierarchy and the criteria, one might end up with different criteria weights depending on the strategies or preferences of the decision makers.

It is based on three principles such as (a) constructing the hierarchy, (b) priority setting, and (c) logical consistency (checking consistency).

The problem is placed at the top of the hierarchy while the middle layer contains all the criteria and sub-criteria needed in the selection process and the alternatives are placed at the lowest layer of the hierarchy. After completion of the hierarchy, a prioritization procedure is followed to assign the priority of the elements of the hierarchical structure. Prioritization is surely a type of judgment that identifies the dominance of one element over another. This kind of judicial procedure is constructed by using pair-wise comparison among the elements.

AHP uses a standardized comparison scale at nine levels to make the pair-wise comparison that may be equally important, moderately more important, strongly more important, very strongly more important, and extremely more important. As these are the verbal form of the levels, it has been translated into numerical values of 1, 3, 5, 7 and 9 respectively while 2, 4, 6 and 8 are translated as the intermediate values for pair-wise comparison between two successive elements to achieve the final AHP Pair-wise comparison as shown in Table 1 below as suggested by (Miller, 1965).

Table 1. The AHP pair-wise comparison

Preferences	Numerical rating
Equally preferred	1
Equally to moderate preferred	2
Moderately preferred	3
Moderately to strongly preferred	4
Strongly preferred	5
Strongly to very strongly preferred	6
Very strongly preferred	7
Very strongly to extremely preferred	8
Extremely preferred	9

6. Numerical Example

For the example obtained from (Wang, 2008), there are 18 suppliers available and the measures of each supplier under the five criteria are listed in Table 2.

6.1 Phase 1 (Mathematical Model)

According to the mathematical model, the scores of 18 suppliers are obtained in Table 3. And the 5 best suppliers (Bold numbers) are 10, 17, 15, 5, and 11, with the high score values.

Table 2.Measures of Suppliers under Criteria

Supplier Number	Supply variety (Unit)	Quality %	Distance (Mile)	Delivery %	Price Index %
1	2	100	249	90	100
2	13	99.79	643	80	100
3	3	100	714	90	100
4	3	100	1809	90	100
5	24	99.83	238	90	100
6	28	96.59	241	90	100
7	1	100	1404	85	100
8	24	100	984	97	100
9	11	99.91	641	90	100
10	53	97.54	588	100	100
11	10	99.95	241	95	100
12	7	99.85	567	98	100
13	19	99.97	567	90	100
14	12	91.89	967	90	100
15	33	99.99	635	95	80
16	2	100	795	95	100
17	34	99.99	689	95	80
18	9	99.36	913	85	100

Table 3.Scores of Suppliers

Suppliers	Scores
1	0.66
2	0.60
3	0.52
4	0.52
5	0.81
6	0.69
7	0.50
8	0.72
9	0.59
10	1.00
11	0.73
12	0.58
13	0.67
14	0.21
15	0.81
16	0.51
17	0.82
18	0.54

6.2 Phase 2 (AHP)

As the steps of AHP method, the comparison between criteria is obtained below in Table 4, as the criteria for the previous example is ranked according to $w_{i1} > w_{i2} > w_{i3} > w_{i4} > w_{i5}$. The importance of the criteria starts from the supply variety to reach to the price index.

6.2.1 Comparison Matrix

There are five criteria; the comparison between them is obtained using the AHP pair-wise comparison Table 1. The importance of different criterion j is determined according to the purchasing manager as shown in Table 4.

Table 4. Pair-wise Comparison Matrix between Criteria

Criteria	Supply variety	Quality	Distance	Delivery	Price Index
Supply variety	1	3	5	7	9
Quality	1/3	1	3	5	7
Distance	1/5	1/3	1	3	5
Delivery	1/7	1/5	1/3	1	3
Price	1/9	1/7	1/5	1/3	1

Table 5. Pair-wise Comparison Matrix and Column Sums

Criteria	Supply variety	Quality	Distance	Delivery	Price Index
Supply variety	1	3	5	7	9
Quality	1/3	1	3	5	7
Distance	1/5	1/3	1	3	5
Delivery	1/7	1/5	1/3	1	3
Price	1/9	1/7	1/5	1/3	1
Sum	1.7873	4.6762	9.5333	16.3333	25

6.2.2 Normalizing Matrix

The normalizing is done, to have a weight for each criterion, so the value of the comparison Table 4 is divided by the column's sum in Table 5.

Table 6. Normalizing step

Criteria	Supply Variety	Quality	Distance	Delivery	Price Index	Row Average
Supply Variety	0.5595	0.6416	0.5245	0.4286	0.36	0.5028
Quality	0.1865	0.2166	0.3147	0.3061	0.28	0.2608
Distance	0.1119	0.0713	0.1049	0.1837	0.2	0.1344
Delivery	0.0799	0.0428	0.035	0.0612	0.12	0.0678
Price Index	0.0622	0.0306	0.021	0.0204	0.04	0.0348
Sum	--	--	--	--	--	1

6.2.3 Ranking Matrix (Criteria and its Weight)

The new ranking of the criteria is made according to the raw average from Table 6, by selecting the highest value and so on.

Table 7. The final ranking of criteria

Criteria	Row Average(weight)	Ranking
Supply variety	0.5028	1
Quality	0.2608	2
Distance	0.1344	3
Delivery	0.0678	4
Price index	0.0348	5
Sum	1	--

6.2.4 Checking For Consistency

After the pair-wise comparisons are determined from the purchasing or supply chain expert in the firm under study, the next stage is to calculate a Consistency Ratio (CR) to measure how consistent the judgments have been relative to large samples of purely random judgments. In practice, a CR of 0.1 or below is considered acceptable; any higher value at any level indicates that the judgments warrant reexamination.

The Consistency Ratio(CR) = CI/RI,

Where;

CI: The Consistency Index

RI: The Random Index for the corresponding random matrix (Satty, 1980).

By considering $[A * X = \lambda_{\max} * X]$,

Where;

A: The pair-wise comparison matrix as depicted in Table 5

X : The eigenvector or row averages from Table 6.

So that:

$$A * X = \begin{matrix} & \begin{matrix} 1 & 3 & 5 & 7 & 9 \end{matrix} \\ \begin{matrix} 1 \\ 3 \\ 5 \\ 7 \\ 9 \end{matrix} & \begin{matrix} 1 & 3 & 5 & 7 & 9 \\ \frac{1}{3} & 1 & 3 & 5 & 7 \\ \frac{1}{5} & \frac{1}{3} & 1 & 3 & 5 \\ \frac{1}{7} & \frac{1}{5} & \frac{1}{3} & 1 & 3 \\ \frac{1}{9} & \frac{1}{7} & \frac{1}{5} & \frac{1}{3} & 1 \end{matrix} \end{matrix} * \begin{matrix} 0.5028 \\ 0.2608 \\ 0.1344 \\ 0.0678 \\ 0.0348 \end{matrix} = \begin{matrix} 2.745 \\ 1.413 \\ 0.698 \\ 0.339 \\ 0.176 \end{matrix} \quad (1) \text{ then,}$$

$$\lambda_{\max} = \text{average} \left(\frac{2.745}{0.5028}, \frac{1.413}{0.2608}, \frac{0.698}{0.1344}, \frac{0.339}{0.06780}, \frac{0.176}{0.0348} \right) = 5.2254 \quad (2)$$

Consistency Index (CI) is found by:

$$CI = (\lambda_{\max} - n)/(n - 1) = (5.2254 - 5)/(5 - 1) = 0.0564 \quad (3)$$

The value of RI is taken from Saaty's Table as depicted in Table 8.

Table 8. The values of RI according to n

(n)	2	3	4	5	6	7	8	9	10
RI	0	0.52	0.89	1.11	1.25	1.35	1.4	1.45	1.49

Where; n : the number of criteria compared.

Thus, Consistency Ratio(CR) = $CI / RI = 0.0564 / 1.11 = 0.0508 < 0.1$, so the evaluations for the criteria pair-wise comparisons are consistent.

6.2.5 Ranking of Suppliers

Here a second ranking is doing between the obtained best suppliers from the mathematical model (phase 1), adding another two suppliers with a high score from Table 3, to find the new ranking for each supplier with each criterion, and then calculate the final score of each supplier.

Table 9. Comparison between Suppliers according to Supply Variety

Supply Variety	Supplier						
	1	5	10	11	12	15	17
Supplier							
1	1	1/8	1/9	1/3	1/3	1/7	1/7
5	8	1	1/5	5	5	1/2	1/2
10	9	5	1	7	7	3	3
11	3	1/5	1/7	1	1	1/6	1/6
12	3	1/5	1/7	1	1	1/6	1/6
15	7	2	1/3	6	6	1	1
17	7	2	1/3	6	6	1	1
Sum	38	10.525	2.263	26.333	26.333	5.976	5.9760

Table 10. Normalized Matrix of Supply Variety

Supply Variety	Supplier							Row Average
	1	5	10	11	12	15	17	
Supplier								
1	0.026	0.012	0.049	0.013	0.0127	0.024	0.024	0.023
5	0.211	0.095	0.088	0.2	0.2	0.084	0.084	0.137
10	0.237	0.475	0.442	0.266	0.266	0.502	0.502	0.384
11	0.08	0.019	0.063	0.04	0.04	0.03	0.03	0.043
12	0.08	0.019	0.063	0.04	0.04	0.03	0.03	0.043
15	0.184	0.19	0.147	0.23	0.23	0.167	0.167	0.188
17	0.184	0.19	0.02	0.23	0.23	0.167	0.167	0.17
Sum	--	--	--	--	--	--	--	1

Table 11. Comparison between Suppliers according to Quality

Quality	Supplier						
Supplier	1	5	10	11	12	15	17
1	1	2	5	1	2	1	1
5	½	1	3	1/2	1	1/3	1/3
10	1/5	1/3	1	1/3	1/4	1/5	1/5
11	1	2	3	1	1	1/2	1/2
12	½	1	4	1	1	1/2	1/2
15	1	3	5	2	2	1	1
17	1	3	5	2	2	1	1
Sum	5.2	12.333	26	7.8333	9.2	4.533	4.533

Table 12. Normalized Matrix according to Quality

Quality	Supplier							Row Average
Supplier	1	5	10	11	12	15	17	
1	0.192	0.162	0.192	0.128	0.217	0.221	0.221	0.19
5	0.096	0.081	0.115	0.064	0.109	0.074	0.074	0.088
10	0.038	0.027	0.038	0.043	0.027	0.044	0.044	0.037
11	0.192	0.162	0.115	0.128	0.109	0.11	0.11	0.132
12	0.096	0.081	0.154	0.128	0.109	0.11	0.11	0.113
15	0.192	0.243	0.192	0.255	0.217	0.221	0.221	0.22
17	0.192	0.243	0.192	0.255	0.217	0.221	0.221	0.22
Sum	--	--	--	--	--	--	--	1

Table 13. Comparison between Suppliers according to Distance

Distance	Supplier						
Supplier	1	5	10	11	12	15	17
1	1	1	3	1	3	5	6
5	1	1	3	1	3	5	6
10	1/3	1/3	1	1/3	1	2	2
11	1	1	3	1	1/5	5	6
12	1/3	1/3	1	5	1	3	2
15	1/5	1/5	1/2	1/5	1/3	1	2
17	1/6	1/6	1/2	1/6	1/2	½	1
Sum	4.033	4.033	12	8.7	9.033	21.5	25

Table 14. Normalized matrix of Distance

Distance	Supplier							Row Average
Supplier	1	5	10	11	12	15	17	
1	0.25	0.25	0.25	0.115	0.332	0.233	0.24	0.239
5	0.25	0.25	0.25	0.115	0.332	0.233	0.24	0.239
10	0.083	0.083	0.083	0.038	0.111	0.093	0.08	0.082
11	0.25	0.25	0.25	0.115	0.022	0.233	0.24	0.194
12	0.083	0.083	0.083	0.575	0.111	0.14	0.08	0.165
15	0.05	0.05	0.042	0.023	0.037	0.047	0.08	0.047
17	0.041	0.041	0.042	0.019	0.055	0.023	0.04	0.037
Sum	--	--	--	--	--	--	--	1

Table 15. Comparison between suppliers according to Delivery

Delivery	Supplier						
Supplier	1	5	10	11	12	15	17
1	1	1	1/7	1/5	1/6	1/5	1/5
5	1	1	1/7	1/5	1/6	1/5	1/5
10	7	7	1	5	2	3	3
11	5	5	1/5	1	1/4	1	1
12	6	6	1/2	4	1	2	2
15	5	5	1/3	1	1/2	1	1
17	5	5	1/3	1	1/2	1	1
Sum	30	30	2.652	12.4	4.583	8.4	8.4

Table 16. Normalized matrix according to Delivery

Delivery	Supplier							Row Average
Supplier	1	5	10	11	12	15	17	
1	0.033	0.033	0.054	0.016	0.036	0.024	0.031	0.032
5	0.033	0.033	0.054	0.016	0.036	0.024	0.031	0.032
10	0.233	0.233	0.377	0.403	0.436	0.357	0.342	0.34
11	0.167	0.167	0.075	0.081	0.055	0.123	0.113	0.112
12	0.2	0.2	0.189	0.323	0.218	0.238	0.229	0.228
15	0.167	0.167	0.126	0.081	0.11	0.119	0.127	0.128
17	0.167	0.167	0.126	0.081	0.11	0.119	0.127	0.128
Sum	--	--	--	--	--	--	--	1

Table 17. Comparison between Suppliers according to Price index

<u>Price index</u>	Supplier						
Supplier	1	5	10	11	12	15	17
1	1	1	1	1	1	5	5
5	1	1	1	1	1	5	5
10	1	1	1	1	1	5	5
11	1	1	1	1	1	5	5
12	1	1	1	1	1	5	5
15	1/5	1/5	1/5	1/5	1/5	1	1
17	1/5	1/5	1/5	1/5	1/5	1	1
Sum	5.4	5.4	5.4	5.4	5.4	27	27

Table 18. Normalized matrix according to Price index

<u>Price index</u>	Supplier							Row Average
Supplier	1	5	10	11	12	15	17	
1	0.185	0.185	0.185	0.185	0.185	0.185	0.185	0.185
5	0.185	0.185	0.185	0.185	0.185	0.185	0.185	0.185
10	0.185	0.185	0.185	0.185	0.185	0.185	0.185	0.185
11	0.185	0.185	0.185	0.185	0.185	0.185	0.185	0.185
12	0.185	0.185	0.185	0.185	0.185	0.185	0.185	0.185
15	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037
17	0.037	0.037	0.037	0.037	0.037	0.037	0.037	0.037
Sum	--	--	--	--	--	--	--	1

After all the comparisons and normalized matrices for all suppliers with the corresponding criteria, now the final scores of the suppliers have to be calculated, and then the ranking is doing finally.

The multiple of the two matrices, the first matrix is the columns of average form the normalized matrixes corresponding to the five criteria, the last columns of the normalized matrices in tables above and the second matrix is (1).

0.021 0.19 0.24 0.031 0.185
 0.125 0.087 0.24 0.031 0.185 2.745
 0.384 0.037 0.08 0.342 0.185 1.413
 0.043 0.132 0.24 0.113 0.185* 0.698 =
 0.043 0.113 0.08 0.229 0.185 0.339
 0.188 0.22 0.08 0.127 0.037 0.176
 0.188 0.22 0.04 0.127 0.037

[0.5367 0.6767 1.31 0.3454 0.4437 0.9323 0.9044]

According to AHP method, the scores of suppliers are given as the final matrix obtained and the final result are shown in Table 19.

Table 19. The Final Score of Suppliers

Rank	Supplier
1	10
2	15
3	17
4	5
5	1
6	12
7	11

So the best suppliers are supplier10, supplier15, supplier17, supplier5, supplier1, supplier12, supplier 11.

Conclusion

In this paper, both quantitative and qualitative criteria are considered and an approach is introduced, composed of two phases, phase 1 is a weighted mathematical model at which the criteria are ranked in descending order and with these weights, the final score for each supplier is obtained and from it the order of best suppliers, then the advantage of AHP in phase 2, a strategic approach to evaluate alternatives. AHP is a very useful for managerial decision making because it is flexible enough to accommodate a large set of evaluation criteria. The selection of suppliers is improved by composing the two phases that give the final ranking of suppliers as supplier 10 is the best one, and the rest of suppliers are supplier 15, supplier 17, supplier 5, supplier 1, supplier 12, and finally, the worst one is supplier 11.

References

- Cox A. "Power, Value and SCM", *Supply Chain Management: An International Journal*, (vol.) 4, (pp.)167-175,1999.
- Harp L. H., "The Haves and the Have Nots", *Supply Chain Practices for the New Millennium: Inbound Logistics Journal*, (pp.) 75-114,2000.
- Vera R, and Pullman ME., "An analysis of the supplier selection process", *Omega International Journal of Management Science*, (vol.) 26, (pp.) 739–50, 1998.
- Weber, C. A. and J. R. Current, "A multi-objective approach to vendor selection", *European Journal of Operational Research*, (vol.) 68 , (pp.) 173–184,1993.
- Ghodsypour, S. H. and O'Brien, C., "A decision support system for supplier selection using an integrated analytic hierarchy process and linear programming", *International Journal of Production Economics*, (vol.) 56, (no.) 57, (pp.) 199–212, 1998.
- Cebi, F. and D. Bayraktar, "An integrated approach for supplier selection", *Logistics Information Management*, (vo.)16, (no.) 6 , (pp.) 395–400,2003.
- Wang, G., Huang S. H., and Dismukes J. P., "Product-driven supply chain selection using integrated multi-criteria decision-making methodology", *International Journal of Production Economics*, (vol.) 91, (pp.) 1–15, 2004.
- Wang, T.Y. and Yang, Y. H. " A fuzzy model for supplier selection in quantity discount environments", *Expert Systems with Applications* , (vol.) 36, (no.) 10, (pp.) 12179–12187,2009.
- Timmerman, E., "An approach to vendor performance evaluation", *Journal of Purchasing and Materials Management* , (vol.) 22, (no.) 4, (pp.) 2–9,1986.
- Wang, Ng, "An efficient and simple model for multiple criteria supplier selection problem", *European journal of operational research*, (vol.)186, (pp.) 1059-1067, 2008.
- Saaty, T. L., "An Exposition of the AHP in Reply to the Paper: Remarks on the Analytic Hierarchy Process", *Management Science journal*, (vol.) 36, (pp.) 259-268,1990.
- Miller, G. A., "The Magical Number Seven, Plus or Minus Two: Some Limits on Our Capacity for Processing Information", *Psychological Review* , (vol.) 63, (pp.) 81-97,1965.
- Saaty, T.L., "The analytic hierarchy process", *McGraw-Hill*, New York, 1980.

