

# **A Multi-objective Generalized Intuitionistic Fuzzy Linear Fractional Inventory Model with Possibility and Necessity Constraints**

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## **Abstract**

Generalized intuitionistic fuzzy numbers (GIFNs) a special kind intuitionistic fuzzy set on the real number set are useful to deal with ill-known quantities in fuzzy optimization problems. In this paper, we have considered a multi-objective linear fractional inventory problem with generalized intuitionistic fuzzy environment. By employing the possibility measures and expected value, the multi-objective generalized intuitionistic fuzzy linear fractional inventory model is transformed into an equivalent deterministic linear fractional programming problem. Finally, the model is illustrated with the help of numerical example, and to validate the proposed model few sensitivity analysis are also presented under different parameters.

## **Keywords**

Possibility and Necessity measures, Generalized intuitionistic fuzzy number, Linear fractional inventory model.

## **1. Introduction**

Fuzzy set theory has been well exhibited and applied in a wide variety of real problems since it was proposed in 1965 by Zadeh (1965). Zadeh (1965) expresses the degree of membership of a given element in a fuzzy set, but very often does not express the corresponding degree of non-membership as the complement to 1. Thus, Atanassov (1986) introduced the intuitionistic fuzzy set (IFS) by adding an additional non-membership function; this may express more enormous and supple information as compared with the fuzzy set. The IFS has an eminent characteristic which allots to each element a membership degree and a non-membership degree. Intuitionistic fuzzy numbers are a special instance of fuzzy numbers. As a generalization of fuzzy Set, the intuitionistic fuzzy number (IFN) is a special kind of IFS defined on the real number set, which appears to properly describe an ill-known quantity (2006). Recently, Garai et al (2017) proposed the generalized intuitionistic fuzzy number.

Generally speaking, uncertainty is usual to all real life problems for example fuzziness. Since Zadeh (1965) introduced the fuzzy set theory and it has been well developed and applied in a wide variety of real life problems. Possibility theory was proposed by Zadeh (1988) and developed by many researchers, e.g. Dubois and Prade (1988) Klir (1992 a) Yager (1992 b) and others. A self dual measure called credibility measure was introduced by Liu and Liu [9]. The mean value of a fuzzy number was introduced by Dubois and Prade (1988). Thereafter, Carlsson and Fuller (1992 c) defined a possibilistic mean and variance of fuzzy numbers. The expected value of fuzzy variable using possibility theory was proposed by Hilpern (2001) and application of expected value operator called expected value model introduced by Liu and Liu (2002). Recently Garai et al. (2018) introduced the possibility mean of a generalized intuitionistic fuzzy number and its application multi-item inventory model.

Rao et al. (1993a) developed a set theoretic approach to solve a multiple objective linear fractional programming. Furthermore, Dutta, Tiwari, and Rao (1994) commented over the fuzzy approaches for multiple criteria linear fractional optimization. A possibility programming approach for stochastic fuzzy multi-objective linear fractional is considered by Iskander (2004). Sadjadi et al. (2005) developed a multi-objective linear fractional inventory model with fuzzy approach. Recently, Dutta and kumar (2015) proposed a multi-objective linear fractional inventory model with fuzzy goal programming approach.

In many cases, the parameters in inventory problems may not be crisp and be somewhat vague in nature. For examples, the holding cost for an item is supposed to be dependent on the amount of storage. Similarly, the replenishment cost depends upon the total quantity to be produced in a scheduling period. Moreover, because the inventory system, the total profit in a scheduling period may be uncertain, and uncertainties may be associated with these variables and the above goals and parameters are normally vague and imprecise, i.e., fuzzy in nature. Maximum total average profit is imprecise in practical inventory problem. In these situations, fuzzy set theory can be used for the formulation of inventory models. However, no attempt has been made that includes all purchasing cost, holding cost, selling price, demand and ordering cost, which are generalized intuitionistic fuzzy variables, are considered within the multi-objective inventory model. Therefore, we have developed and solved a multi-objective inventory model with generalized intuitionistic fuzzy coefficients via expected value and possibility measures approaches.

The rest of the paper organized as follows: In Section 2, we present some basic knowledge of GIFNs, and gives also the concept of possibility and necessity. In Section 3, we introduce the notations and assumption which are used throughout the paper and formulated multi-objective generalized intuitionistic fuzzy linear fractional inventory model. In Section 4, we have discussed the solutions methodology of the proposed model. The numerical example is illustrated in Section 5. Finally, the conclusion and scope of future work plan given in Section 6.

## 2. Basic Preliminaries

**Definition 2.1 (Generalized generalized intuitionistic fuzzy number)** Let  $w_{\tilde{a}} \in [0,1]$  and  $u_{\tilde{a}} \in [0,1]$  be any two real number  $\mathbb{R}$ , which satisfy that  $0 \leq w_{\tilde{a}} + u_{\tilde{a}} \leq 1$ . A generalized intuitionistic fuzzy number (GIFN)  $\tilde{a}^I$  is a special type [4] intuitionistic fuzzy set (Garai et al. 2017) on the real number  $\mathbb{R}$ , whose member ship function and non membership function are  $\mu_{\tilde{a}^I} : \mathbb{R} \rightarrow [0, w_{\tilde{a}}]$  and  $\nu_{\tilde{a}^I} : \mathbb{R} \rightarrow [u_{\tilde{a}}, 1]$  respectively, which are satisfies the following conditions (1) -(4).

- There exit at least two real number  $x_1$  and  $x_2$  such that  $\mu_{\tilde{a}^I}(x_1) = w_{\tilde{a}}$  and  $\nu_{\tilde{a}^I}(x_1) = u_{\tilde{a}}$ .
- $\mu_{\tilde{a}^I}$  is a upper semi continuous and quasi concave on the real number  $\mathbb{R}$ .
- $\nu_{\tilde{a}^I}$  is a lower semi continuous and quasi convex on the real number  $\mathbb{R}$ .
- The support of  $\tilde{a}^I$  (i.e.,  $\tilde{a}^I_{>0,1} = \{x \in \mathbb{R} : \mu_{\tilde{a}^I}(x) > 0, \nu_{\tilde{a}^I}(x) < 1\}$ ) is bounded.

From the above definition of the generalized intuitionistic fuzzy number, we can easily construct a generalized intuitionistic fuzzy number  $\tilde{a}^I = \{(\underline{a}_1, a_{1l}, a_{1r}, \bar{a}_1); w_{\tilde{a}}, (\underline{a}_2, a_{2l}, a_{2r}, \bar{a}_2)\}$  whose membership and non membership function are given below.

$$\mu_{\tilde{a}^I}(x) = \begin{cases} 0, & \text{if } x < \underline{a}_1 \\ g_{\mu l}(x), & \text{if } \underline{a}_1 \leq x < a_{1l} \\ w_{\tilde{a}}, & \text{if } a_{1l} \leq x \leq a_{1r} \\ g_{\mu r}(x) & \text{if } a_{1r} < x \leq \bar{a}_1 \\ 0, & \text{if } x > \bar{a}_1 \end{cases}$$

and

$$\nu_{\tilde{a}^I}(x) = \begin{cases} 1, & \text{if } x < \underline{a}_2 \\ g_{\nu l}(x), & \text{if } \underline{a}_2 \leq x < a_{2l}, \\ u_{\tilde{a}}, & \text{if } a_{2l} \leq x \leq a_{2r} \\ g_{\nu r}(x) & \text{if } a_{2r} < x \leq \bar{a}_2 \\ 1, & \text{if } x > \bar{a}_2 \end{cases}$$

respectively. Where the function  $g_{\mu l} : [\underline{a}_1, a_{1l}] \rightarrow [0, w_{\tilde{a}}]$  and  $g_{\nu r} : [a_{2r}, \bar{a}_2] \rightarrow [u_{\tilde{a}}, 1]$  are continuous, non-decreasing and satisfies the conditions  $g_{\mu l}(\underline{a}_1) = 0, g_{\mu l}(a_{1l}) = w_{\tilde{a}}, g_{\nu r}(a_{2r}) = u_{\tilde{a}}$  and  $g_{\nu r}(\bar{a}_2) = 1$ , the functions

$g_{\mu r} : [a_{1r}, \bar{a}_1] \rightarrow [0, w_{\bar{a}}]$  and  $g_{\nu l} : [\underline{a}_2, a_{2l}] \rightarrow [u_{\bar{a}}, 1]$  are continuous, non-increasing and satisfies the conditions  $g_{\mu r}(a_{1r}) = w_{\bar{a}}$ ,  $g_{\mu r}(\bar{a}_1) = 0$ ,  $g_{\nu l}(\underline{a}_2) = 1$  and  $g_{\nu l}(a_{2l}) = u_{\bar{a}}$ .  $\underline{a}_1$  and  $\bar{a}_1$  are called lower and upper limits and  $[a_{1l}, a_{1r}]$  be called the mean interval of the generalized intuitionistic fuzzy number  $\tilde{a}^I$  for the membership function respectively.  $\underline{a}_2$  and  $\bar{a}_2$  are called the upper and lower limits and  $[a_{2l}, a_{2r}]$  be called the mean interval of the generalized intuitionistic fuzzy number  $\tilde{a}^I$  for the non-membership function respectively.  $w_{\bar{a}}$  and  $u_{\bar{a}}$  are called the maximum membership and minimum non-membership degree respectively. Then further we can construct some specific forms of generalized intuitionistic fuzzy numbers such as generalized trapezoidal intuitionistic fuzzy number and generalized triangular intuitionistic fuzzy number. For some particular values  $\acute{a}_1, a_1, a_2, a_3, a_4$  and  $\acute{a}_4$  of the parameters  $\underline{a}_1, a_{1l}, a_{1r}, \bar{a}_1, \underline{a}_2, a_{2l}, a_{2r}$  and  $\bar{a}_2$  ( $a_{1l} = a_{2l}, a_{1r} = a_{2r}$ ) respectively.

**Definition 2.2 (Generalized trapezoidal intuitionistic fuzzy number)** Let  $\acute{a}_1 \leq a_1 \leq a_2 \leq a_3 \leq a_4 \leq \acute{a}_4$  and  $0 \leq w_{\bar{a}} + u_{\bar{a}} \leq 1$ . A generalized trapezoidal intuitionistic fuzzy number (GTIFN)  $\tilde{a}^I$  in  $\mathbb{R}$  can be written (Garai et al. 2017) as  $\tilde{a}^I = ((a_1, a_2, a_3, a_4; w_{\bar{a}}) (\acute{a}_1, a_2, a_3, \acute{a}_4; u_{\bar{a}}))$  whose membership (cf. Fig. 1) and non-membership function (cf. Fig. 1) can be written as

$$\mu_{\tilde{a}^I}(x) = \begin{cases} w_{\bar{a}} \frac{x - a_1}{a_2 - a_1}, & \text{if } a_1 \leq x \leq a_2 \\ 1, & \text{if } a_2 \leq x \leq a_3 \\ w_{\bar{a}} \frac{a_4 - x}{a_4 - a_3}, & \text{if } a_3 \leq x \leq a_4 \\ 0 & \text{otherwise} \end{cases}$$

and

$$\nu_{\tilde{a}^I}(x) = \begin{cases} u_{\bar{a}} \frac{x - \acute{a}_1}{a_2 - \acute{a}_1}, & \text{if } \acute{a}_1 \leq x \leq a_2 \\ 1, & \text{if } a_2 \leq x \leq a_3 \\ u_{\bar{a}} \frac{\acute{a}_4 - x}{\acute{a}_4 - a_3}, & \text{if } a_3 \leq x \leq \acute{a}_4 \\ 0 & \text{otherwise} \end{cases}$$

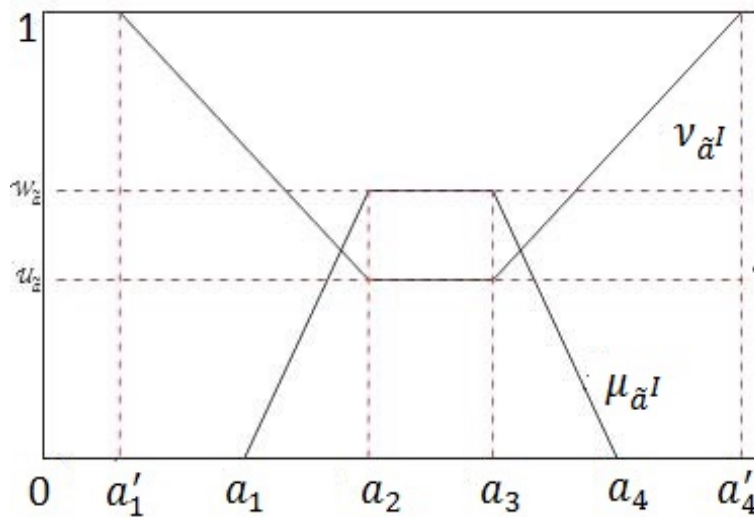


Figure 1: Membership and non-membership function of GTIFN ( $\tilde{a}^I$ )

**Definition 2.3 (Possibility and Necessity of GIFNs)** Let  $\tilde{a}^I$  and  $\tilde{b}^I$  be two GIFN with membership functions  $\mu_{\tilde{a}^I}, \mu_{\tilde{b}^I}$  and non-membership (Garai et al. 2017) functions  $\nu_{\tilde{a}^I}, \nu_{\tilde{b}^I}$  respectively, and  $\mathbb{R}$  be the set of real number. Then the possibility and necessity of  $\tilde{a}^I$  and  $\tilde{b}^I$  are given by

$$Pos_{\mu}(\tilde{a}^I * \tilde{b}^I) = \sup\{\mu_{\tilde{a}^I}(x) \wedge \mu_{\tilde{b}^I}(y) : x, y \in \mathbb{R}, x * y\} \quad (1)$$

$$Pos_v(\tilde{a}^I * \tilde{b}^I) = \sup\{v_{\tilde{a}^I}(x) \wedge v_{\tilde{b}^I}(x) : x, y \in \mathbb{R}, x * y\} \quad (2)$$

and

$$Nec_\mu(\tilde{a}^I * \tilde{b}^I) = \inf\{\mu_{\tilde{a}^I}(x) \vee \mu_{\tilde{b}^I}(x) : x, y \in \mathbb{R}, x * y\} \quad (3)$$

$$Nec_v(\tilde{a}^I * \tilde{b}^I) = \inf\{v_{\tilde{a}^I}(x) \vee v_{\tilde{b}^I}(x) : x, y \in \mathbb{R}, x * y\} \quad (4)$$

Where abbreviation  $Pos_\mu$ ,  $Pos_v$  represents the possibility of membership and non-membership functions and  $Nec_\mu$ ,  $Nec_v$  represents the necessity of membership and non-membership functions for GIFN, “\*” denote the any relations of  $\leq, \geq, >, <, =$  and  $\vee = \max$   $\wedge = \min$ .

**Definition 2.4 (Measure of a GIFN)** Let  $\tilde{a}^I$  be a GIFN. Then the measure (Garai et al. 2017) of  $\tilde{a}^I$  for membership and non-membership function are defined as follows

$$Me_\mu(\tilde{a}^I) = \theta Pos_\mu(\tilde{a}^I) + (1 - \theta) Nec_\mu(\tilde{a}^I) \quad (5)$$

and

$$Me_v(\tilde{a}^I) = \theta Pos_v(\tilde{a}^I) + (1 - \theta) Nec_v(\tilde{a}^I) \quad (6)$$

Where  $\theta$  ( $0 \leq \theta \leq 1$ ) is the optimistic pessimistic parameter, we determine the combined attitude of decision maker  $\theta$ , as follows

If  $\theta = 1$ , then  $Me_\mu = Pos_\mu$ ,  $Me_v = Pos_v$  it means the decision maker is optimistic and maximum chance of  $\tilde{a}^I$  holds.

If  $\theta = 0$ , then  $Me_\mu = Nec_\mu$ ,  $Me_v = Nec_v$  it means the decision maker is pessimistic and minimal chance of  $\tilde{a}^I$  holds.

If  $\theta = 1/2$ , then  $Me_\mu = Cr_\mu$ ,  $Me_v = Cr_v$  it means the decision maker is compromise attitude.

Where the abbreviation  $Cr$  is the credibility measure and defined by

$$Cr_\mu = \frac{Pos_\mu + Nec_\mu}{2}$$

and

$$Cr_v = \frac{Pos_v + Nec_v}{2}$$

**Example 2.1** Let  $\tilde{a}^I = ((a_1, a_2, a_3, a_4; w_{\tilde{a}}) (\acute{a}_1, \acute{a}_2, \acute{a}_3, \acute{a}_4; u_{\tilde{a}}))$  and Let  $\tilde{b}^I = ((b_1, b_2, b_3, b_4; w_{\tilde{b}}) (\acute{b}_1, \acute{b}_2, \acute{b}_3, \acute{b}_4; u_{\tilde{b}}))$  be two GTIFNs. From (Garai et al. 2017) Equations (1) and (2), the possibility of the event  $(\tilde{a}^I \leq \tilde{b}^I)$  for membership (cf. Fig. 2) and non-membership (cf. Fig. 3) functions are given by

$$Pos_\mu(\tilde{a}^I \leq \tilde{b}^I) = \begin{cases} w_{\tilde{a}} \wedge w_{\tilde{b}}, & \text{if } a_2 \leq b_3 \\ \frac{(b_4 - a_1)w_{\tilde{a}}w_{\tilde{b}}}{(b_4 - b_3)w_{\tilde{a}} + (a_2 - a_1)w_{\tilde{b}}}, & \text{if } a_1 < b_4, a_2 > b_3 \\ 0, & \text{if } b_4 \leq a_1 \end{cases} \quad (7)$$

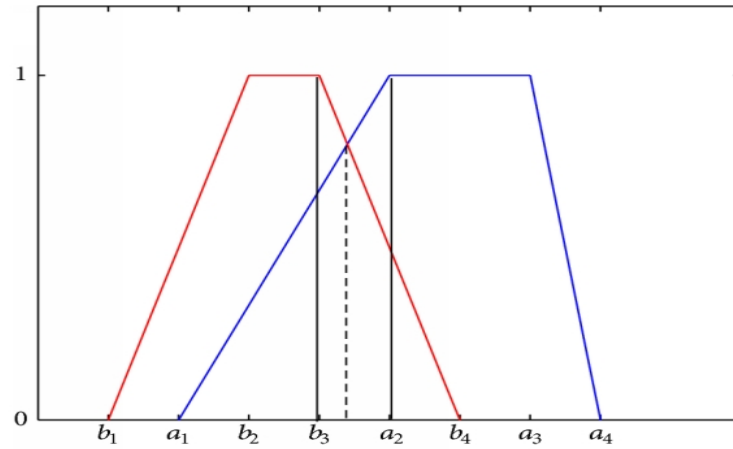
and

$$Pos_v(\tilde{a}^I \leq \tilde{b}^I) = \begin{cases} 0, & \text{if } a_2 \leq b_3 \\ \frac{(a_2 - b_3)u_{\tilde{a}}u_{\tilde{b}}}{(\acute{b}_4 - \acute{b}_3)u_{\tilde{a}} + (a_2 - \acute{a}_1)u_{\tilde{b}}}, & \text{if } a_2 > b_3, \acute{b}_4 > \acute{a}_1 \\ u_{\tilde{a}} \wedge u_{\tilde{b}}, & \text{if } \acute{b}_4 \leq \acute{a}_1 \end{cases} \quad (8)$$

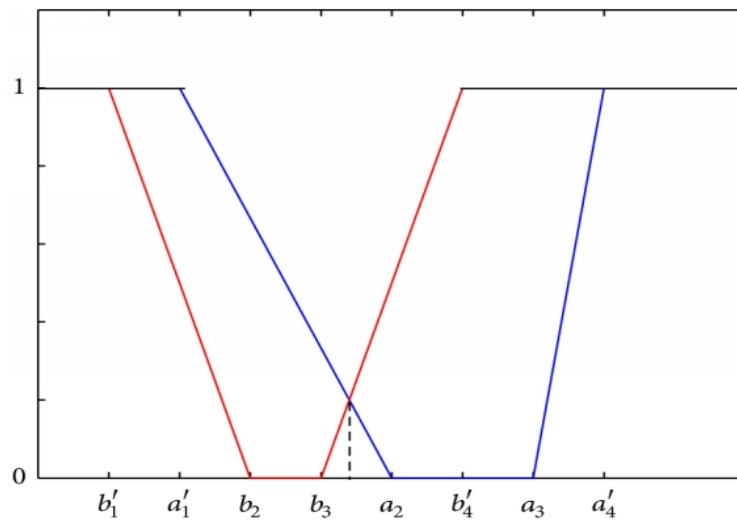
Similarly, we also calculate necessity measures

$$Nec_\mu(\tilde{a}^I \leq \tilde{b}^I) = 1 - Pos_\mu(\tilde{a}^I > \tilde{b}^I)$$

$$Nec_v(\tilde{a}^I \leq \tilde{b}^I) = 1 - Pos_v(\tilde{a}^I > \tilde{b}^I)$$



**Figure 2:** Membership function of GTIFN  $\tilde{a}^I, \tilde{b}^I$  and  $Pos_\mu(\tilde{a}^I \leq \tilde{b}^I)$



**Figure 3:** Non-membership function of GTIFN  $\tilde{a}^I, \tilde{b}^I$  and  $Pos_\nu(\tilde{a}^I \leq \tilde{b}^I)$

**Lemma 2.1** Let  $\tilde{a}^I = ((a_1, a_2, a_3, a_4; w_{\tilde{a}}) (\acute{a}_1, \acute{a}_2, \acute{a}_3, \acute{a}_4; u_{\tilde{a}}))$  and  $\tilde{b}^I = ((b_1, b_2, b_3, b_4; w_{\tilde{b}}) (\acute{b}_1, \acute{b}_2, \acute{b}_3, \acute{b}_4; u_{\tilde{b}}))$  be two GTIFNs, then

$$Pos_\mu(\tilde{a}^I \leq \tilde{b}^I) \geq \alpha \Leftrightarrow \frac{(b_4 - a_1)w_{\tilde{a}}w_{\tilde{b}}}{(b_4 - b_3)w_{\tilde{a}} + (a_2 - a_1)w_{\tilde{b}}} \geq \alpha;$$

$$Pos_\nu(\tilde{a}^I \leq \tilde{b}^I) \leq \beta \Leftrightarrow \frac{(a_2 - b_3)u_{\tilde{a}}u_{\tilde{b}}}{(\acute{b}_4 - b_3)u_{\tilde{a}} + (a_2 - \acute{a}_1)u_{\tilde{b}}} \leq \beta;$$

**Proof:** Let us consider  $Pos_\mu(\tilde{a}^I \leq \tilde{b}^I) \geq \alpha$  and  $Pos_\nu(\tilde{a}^I \leq \tilde{b}^I) \leq \beta$ .

Now from Equations (7) and (8)

$$Pos_\mu(\tilde{a}^I \leq \tilde{b}^I) \geq \alpha \Leftrightarrow \frac{(b_4 - a_1)w_{\tilde{a}}w_{\tilde{b}}}{(b_4 - b_3)w_{\tilde{a}} + (a_2 - a_1)w_{\tilde{b}}} \geq \alpha;$$

$$Pos_v(\tilde{a}^I \leq \tilde{b}^I) \leq \beta \Leftrightarrow \frac{(a_2 - b_3)u_{\tilde{a}}u_{\tilde{b}}}{(\tilde{b}_4 - b_3)u_{\tilde{a}} + (a_2 - \tilde{a}_1)u_{\tilde{b}}} \leq \beta;$$

The proof is complete.

## 2.1 Expected value of GIFN

Let  $\tilde{a}^I$  be a GIFN, then the expected (Garai et al. 2017) value of  $\tilde{a}^I$  for membership ( $E_{\mu}^{Me}$ ) and non-membership ( $E_{\nu}^{Me}$ ) function can be defined as follows

$$E_{\mu}^{Me}(\tilde{a}^I) = \int_0^{\infty} Me\{\tilde{a}^I \geq x\}dx - \int_{-\infty}^0 Me\{\tilde{a}^I \geq x\}dx \quad (9)$$

and

$$E_{\nu}^{Me}(\tilde{a}^I) = \int_0^{\infty} Me\{\tilde{a}^I \geq x\}dx - \int_{-\infty}^0 Me\{\tilde{a}^I \geq x\}dx \quad (10)$$

**Definition 2.5** Let  $\tilde{a}^I = ((a_1, a_2, a_3, a_4; w_{\tilde{a}}) (\tilde{a}_1, a_2, a_3, \tilde{a}_4; u_{\tilde{a}}))$  be a GTIFN (Garai et al. 2017), then the expected value of  $\tilde{a}^I$  is defined as follows

$$E(\tilde{a}^I) = \frac{E_{\mu}^{Cr}(\tilde{a}^I) + E_{\nu}^{Cr}(\tilde{a}^I)}{2} \\ = \frac{(a_1 + a_2 + a_3 + a_4)w_{\tilde{a}} + (\tilde{a}_1 + a_2 + a_3 + \tilde{a}_4)u_{\tilde{a}}}{8}$$

**Remark 2.1** For particular value of  $w_{\tilde{a}} = 1/2$  and  $u_{\tilde{a}} = 1/2$  (with respect to the condition  $0 \leq w_{\tilde{a}} + u_{\tilde{a}} \leq 1$ ), the expected value of  $\tilde{a}^I$  is given by

$$E(\tilde{a}^I) = \frac{(\tilde{a}_1 + a_1 + 2a_2 + 2a_3 + a_4 + \tilde{a}_4)}{16}$$

## 3. Mutli-objective generalized intuitionistic fuzzy linear fractional inventory model

The following notations and assumptions which have used in the formulation of the proposed linear fractional inventory problem.

### 3.1 Notations and Assumptions

#### Notations:

- n= Number of items
- k= fixed cost per order
- F=Maximum available space for all item
- B= Maximum available budget for all item
- $Q_i$ =Ordering quantity for *ith* item
- $h_i$ =Holding cost per item per unit time for *ith* item
- $P_i$ = Purchasing cost for *ith* item
- $S_i$ = Selling price of *ith* item
- $f_i$ = Space required per unit for *ith* item
- $D_i$ = Demand quantity per unit for *ith* item
- $OC_i$ = Ordering cost for *ith* item

#### Assumptions:

- (i) A multi-objective inventory is considered.
- (ii) The time horizon is infinite and there is only one period in the cycle time.
- (iii) Lead time is zero.
- (iv) Demand rate is constant over time for each item
- (v) Purchase price of the item is constant for each item
- (vi) Holding cost is known and constant for each item
- (vii) No deterioration allowed

(viii) Shortages are not allowed

A multi-objective model under resources constraints is proposed as linear fraction programmer. This model refers to a multi-objective inventory problem, with limited capacity of warehouse and constraints on investment in inventories. Demand of this inventory for each item is known and constant and it must be met over an infinite horizon without shortages or backlogging. We assume a zero lead time and replenishments are instantaneous. In real life, we observe multi-objective inventory problem very useful than the single objective inventory problem. These objectives may be in duel with each other, or may not be. In such type of inventory models, the decision maker is interested to maximize or minimize two or more objectives together on decision variables. We call this type inventory model as multi-objective linear fractional inventory model.

Now, we propose a multi-objective inventory model with fractional objective functions.

The profit of inventory =  $\sum_{i=1}^n (S_i - P_i) Q_i$

The holding cost =  $\sum_{i=1}^n \frac{h_i Q_i}{2}$

Back ordered quantity =  $\sum_{i=1}^n (D_i - Q_i)$

The total ordering quantity =  $\sum_{i=1}^n Q_i$

The ordering cost =  $\sum_{i=1}^n \frac{k D_i}{Q_i}$

Hence the multi-objective linear fractional inventory model with budgetary, space and investment constraints is given by

$$\begin{aligned} \text{Maximize } Z_1 &= \frac{\sum_{i=1}^n (S_i - P_i) Q_i}{\sum_{i=1}^n (D_i - Q_i)} \\ \text{Minimize } Z_2 &= \frac{\sum_{i=1}^n \frac{h_i Q_i}{2}}{\sum_{i=1}^n Q_i} \\ \text{subject to the constraint} & \\ k D_n - (O C_n) Q_n &\leq 0 \quad (Q_n \geq 0, O C_n \geq 0); \\ \sum_{i=1}^n f_i Q_i &\leq F; \\ \sum_{i=1}^n P_i Q_i &\leq B; \end{aligned} \quad (11)$$

Here, Constraint-I denotes the budgetary constraint on ordering cost. Ordering cost for the  $n$ th item can express as:

For 1<sup>st</sup> item  $\frac{k D_1}{Q_1} \leq O C_1 \Rightarrow k D_1 - (O C_1) Q_1 \leq 0$ ;

For 2<sup>nd</sup> item  $\frac{k D_2}{Q_2} \leq O C_2 \Rightarrow k D_2 - (O C_2) Q_2 \leq 0$ ;

.....  
For  $n$ th item  $\frac{k D_n}{Q_n} \leq O C_n \Rightarrow k D_n - (O C_n) Q_n \leq 0$ ;

Constraint-II denotes restriction on the warehouse spacing.

Constraint-III denotes the upper limit of the total investment.

In the inventory systems different costs are imprecise natures. For example (i) the holding cost cannot be fixed in advance, and it may vary over time period. (ii) The ordering cost cannot also be prefixed. It is also an independent factor of human behavior. (iii) The demand for the items cannot prefix in advanced. It may always vary due to the behaviors' of customers' requirements. (iv) Purchasing price and selling price cannot be uniform respectively for the different zones and time periods. So, all these parameters that are holding cost, purchasing cost, selling price, ordering cost and demand are may be uncertain. This uncertainty may consider in fuzzy environment. In these situations, generalized intuitionistic fuzzy environment can be used for the formulation of inventory problems. Therefore all parameters holding cost, purchasing cost, selling price, ordering cost and demand of the inventory are considered as generalized intuitionistic fuzzy numbers (GIFNs). Hence model (11), can be rewritten after amendments all these above assumptions as follows:

$$\begin{aligned} \text{Maximize } \tilde{Z}_1^I &= \frac{\sum_{i=1}^n (\tilde{S}_i^I - \tilde{P}_i^I) Q_i}{\sum_{i=1}^n (\tilde{D}_i^I - Q_i)} \\ \text{Minimize } \tilde{Z}_2^I &= \frac{\sum_{i=1}^n \frac{\tilde{h}_i^I Q_i}{2}}{\sum_{i=1}^n Q_i} \end{aligned}$$

subject to the constraint (12)

$$\begin{aligned} kD_n - (\widetilde{OC_n})^I Q_n &\leq 0 \quad (Q_n \geq 0, OC_n \geq 0); \\ \sum_{i=1}^n f_i Q_i &\leq \widetilde{F}^I; \\ \sum_{i=1}^n \widetilde{P}_i^I Q_i &\leq \widetilde{B}^I; \end{aligned}$$

We assume that  $\widetilde{S}_i^I, \widetilde{P}_i^I, \widetilde{h}_i^I, \widetilde{D}_i^I, \widetilde{F}^I, \widetilde{B}^I$  and  $(\widetilde{OC_n})^I$  are GIFNs for each  $i = 1, 2, \dots, n$ .

#### 4. Solution methodology

To solve the multi-objective generalized intuitionistic fuzzy inventory problem (12) is based on possibility, necessity and expected value. We transformed the multi-objective generalized intuitionistic fuzzy linear fractional inventory problem (8) into deterministic problem under possibility and necessity measure. Thus we have the following deterministic multi-objective inventory problem.

$$\begin{aligned} &\text{Maximize } Z_1^* = E[\widetilde{Z}_1^I] \\ &\text{Minimize } Z_2^* = E[\widetilde{Z}_2^I] \\ &\text{subject to the constraint} \\ &Pos_\mu(kD_n - (\widetilde{OC_n})^I Q_n \leq 0) \geq \alpha; \\ &Pos_\nu(kD_n - (\widetilde{OC_n})^I Q_n \leq 0) \leq \beta; \\ &Pos_\mu(\sum_{i=1}^n f_i Q_i \leq \widetilde{F}^I) \geq \alpha; \\ &Pos_\nu(\sum_{i=1}^n f_i Q_i \leq \widetilde{F}^I) \leq \beta; \\ &Nec_\mu(\sum_{i=1}^n \widetilde{P}_i^I Q_i \leq \widetilde{B}^I) \geq \alpha; \\ &Nec_\nu(\sum_{i=1}^n \widetilde{P}_i^I Q_i \leq \widetilde{B}^I) \leq \beta; \\ &Q_i \geq 0, i = 1, 2, \dots, n \end{aligned} \quad (13)$$

Where  $\alpha$  &  $\beta$  are the predetermined confidence levels ( $0 \leq \alpha + \beta \leq 1$ ). After the fuzzification of the proposed model (12), the current multi-objective model transformed to single objective using fuzzy interactive satisfied method (Garai et al. 2018). Then this model solve using soft computing technique generalized reduced gradient (GRG) method (Lingo-14.0).

#### 5. Numerical Example:

A manufacturing company produces some items and stocks these items in a warehouse. The company required space for each item are  $f_1 = 2, f_2 = 4$  and warehouse total floor area  $\widetilde{F}^I = ((850, 900, 1050, 1100; 0.6)(900, 1050, 1100, 1220; 0.4))$ , total available budget is  $\widetilde{B}^I = ((900, 950, 960, 1000; 0.6)(800, 850, 980, 1050; 0.4))$  and fixed cost per order is  $k = 8$ . The input generalized trapezoidal intuitionistic fuzzy parameters are given in Table 1. Now, we find the optimal order quantity  $Q_i$  ( $i = 1, 2$ ) and  $Z_i$  ( $i = 1, 2$ ). Then the optimal solution of the proposed problem given in Table 2.

**Table 1.** Input generalized intuitionistic trapezoidal fuzzy parameters

Item	I	II
$\widetilde{S}_i^I$	$((648, 660, 670, 680; 0.6)(647, 665, 685, 690; 0.4))$	$((580, 590, 600, 680; 0.6)(600, 650, 700, 750; 0.4))$
$\widetilde{P}_i^I$	$((623, 624, 628, 629; 0.6)(621, 622, 625, 628; 0.4))$	$((438, 440, 445, 447; 0.6)(438, 440, 442, 444; 0.4))$
$\widetilde{h}_i^I$	$((10, 11, 13, 14; 0.6)(9, 11, 12, 15; 0.4))$	$((15, 16, 18, 20; 0.6)(15, 18, 20, 21; 0.4))$
$\widetilde{D}_i^I$	$((945, 950, 1000, 1050; 0.6)(1250, 1350, 1450, 1500; 0.4))$	$((1600, 1680, 1700, 1800; 0.6)(1700, 1800, 1950, 2000; 0.4))$
$(\widetilde{OC_n})^I$	$((320, 325, 328, 335; 0.6)(325, 330, 340, 360; 0.4))$	$((300, 320, 335, 340; 0.6)(325, 330, 340, 360; 0.4))$

**Table 2.** Optimal Result for  $\alpha = 0.6, \beta = 0.4$

$\mu_1$	$\mu_2$	$Q_1$	$Q_2$	$\rho$	$Z_1$	$Z_2$
1	1	22.8745	42.9630	0.0912	7.6582	8.6312
1	0.98	22.4568	42.1253	0.0982	7.5483	8.6012
1	0.96	22.6345	42.7523	0.1025	7.6531	8.5542

1	0.94	22.4563	42.4125	0.1104	7.4523	8.4523
0.98	1	21.6332	41.5254	0.1145	7.4282	8.4012
0.96	1	21.4521	41.2543	0.1158	7.3520	8.3545
0.94	1	22.4251	41.3201	0.1168	7.2513	8.2512

## 6. Conclusion

As an eminent issue of intuitionistic fuzzy sets, GIFNs are the consistent tools to fleet the fuzzy and uncertain quantities with hesitance degrees. In this paper, we have considered linear fractional inventory modelling GIFNs and maximize the profit and minimize the holding cost related to per-ordered quantity with some realistic set of constraints. The formulated problem solved unless it converted into an equivalent deterministic problem, which has been done by using possibility and necessity measures. This model solve using soft computing technique generalized reduced gradient (GRG) method (Lingo-14.0). The optimal result of the proposed model given in Table 2. The model can be further extended many ways for example inventory production related costs considered as exponential, hyperbolic, etc. fuzzy numbers.

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