

# **Stochastic Modeling of Rail Under-head Failure Risks**

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## **Abstract**

Rail failure is one of the serious problems in the railway industry. Rail failures can significantly impact rail safety as well as foster numerous maintenance challenges. A better understanding of the parameters that lead to rail failures and the ability to predict their uncertainties would result in the facilitation of a more focused maintenance procedure. Rail under-head failure (RUF) occurrence is common in heavy haul rail operations and hence chosen for the modelling and discussions presented in this paper. Stochastic risk analysis of RUF was conducted to determine the probability of its occurrence. The parameters included in the analysis are: vertical elastic foundation of the rail, daily and/or seasonal temperature variances, headwear, a contact patch offset (CPO) and the lateral to vertical force ratio ( $L/V$ ) acting on the rail head. The methodology involves both qualitative and quantitative risk analyses of the above failure parameters. The qualitative analysis was performed by using fault tree analysis and Boolean algebra to determine minimum cut sets. Further, the failure parameter probabilities for the developed quantitative model are assigned after consulting expert opinion and previous research. A simple quantitative model using triangular distributions was created to serve as a template to perform more complex quantitative modelling with higher level probabilistic distributions. The results were analysed and compared with existing research. The results indicated 14-23.5% probability for RUF occurrence in 90% of the cases. Both the simple and complex models had very similar results and displayed skewness's that match previous infrastructural studies on risk analysis. The results conclude that appropriate actions should be taken in maintenance planning to prevent the parameters that can lead to RUF.

## **Keywords**

Failure analysis, rail under-head failure, stochastic modelling, fault tree analysis

## **1. Introduction**

Rail wear and cracks are major maintenance challenges in the railway systems. In general, inevitable issues such as severe head wear combined with transverse defects are among the leading factors for rail failures and safety concerns. The root causes and consequences of rail failures are mostly analysed as deterministic, although many can be deemed as probabilistic. A better understanding of the parameters that lead to railway failures and the ability to predict their uncertainties would result in the facilitation of a more focused maintenance procedure. Only few such as Liu *et al.* (2008), Zhu *et al.* (2013) and Mohammadzadeh *et al.* (2013) discussed specific probabilistic perspectives of railway failures. Liu *et al.* [1] assessed the multiaxial fatigue reliability analysis of railroad wheels. Mohammadzadeh *et al.* (2013) conducted reliability analysis of fatigue crack initiation of rail head in bolted rail joints. Zhu *et al.* (2013) performed the probabilistic modelling of damage accumulation for time dependent fatigue reliability analysis of railway axle steels.

The current approach to mitigate rail failure involves continuous grinding programs for the railhead and periodical track section replacement. This is an expensive approach that may prevent rolling contact fatigue cracks (RCF) but does not prevent rail underhead radius failure (RUF) (as shown in Figure 1). Rail underhead-radius failure (RUF) occurs when the underside of the rail head is subjected to high tensile stresses on the gauge side. These high stresses can cause a fatigue crack initiation at the rail underhead radius as was discussed in (Ranjha *et al.* 2014, Ranjha *et al.* 2015, Ranjha *et al.* 2016). Unlike RCF, which can be maintained through grinding, RUF can cause an unexpected and catastrophic breakage of the rail track. Ranjha *et al.* (2011) has found that several parameters can contribute to RUF, two major factors are the wheel-rail contact eccentricity and a high lateral to vertical force ratio (L/V ratio). Other parameters such as temperature effects and vertical elastic foundation of the rail can also lead to RUF. Head wear (HW) on a track can greatly increase the probability of RUF. Studies have found that a worn rail head suffers from higher tensile stresses in the rail underhead radius under 40 tonne axle loading conditions. A contact patch offset (CPO) due to natural turning and hunting movements of the train can lead to a high L/V ratio; this can eventually lead to failure due to increased tensile stresses on the rail underhead radius (Ranjha *et al.* 2011, Ranjha *et al.* 2012, Ranjha *et al.* 2014, Ranjha *et al.* 2015, Ranjha *et al.* 2016).

This paper will focus on RUF as this is an aspect that has a contributing impact on overall rail failure and constitutes a break of the rail head. This failure method has not been explored as in-depth as other failure methods and therefore represents a gap in the industry's understanding of track failure. Transverse defects and rolling contact fatigue have been explored thoroughly and their links to rail failure are well known. Stochastic risk analysis presents a possible method to determine these probabilities. Stochastic analysis has been successfully used in risk-based inspection planning (Straub and Faber 2006), but its application to rail failure has been limited. The objective of this paper is to stochastically analyse the risk of RUF. The parameters that can lead to RUF will be determined through literature research; fault tree analysis will then be used to qualitatively determine how these parameters may lead to RUF. Quantitative risk analysis was conducted using Palisade's @ Risk software and Decision Tools Suite (Palisade 2018). This will be done to produce a probabilistic model to predict the uncertainties surrounding the risk of RUF.

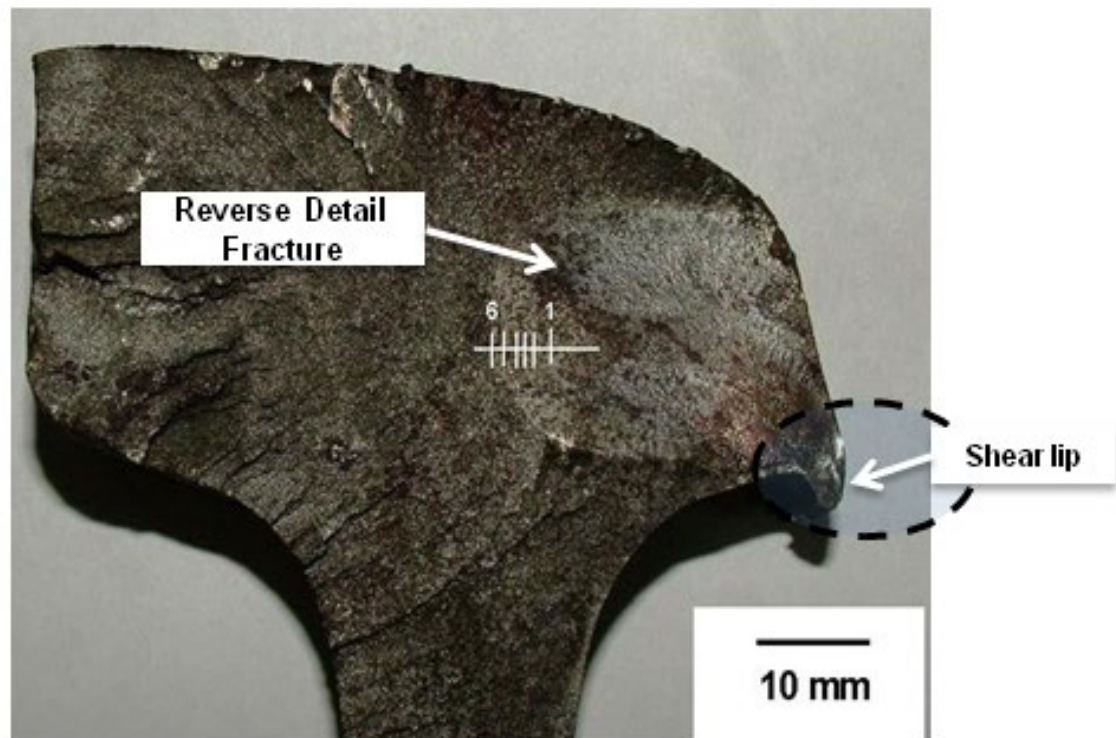


Figure 1 Under-head radius failure, Reverse transverse fractures, RTD's {Ranjha *et al.* 2011, Ranjha *et al.* 2015)

## 2. Methodology

The main contributors to RUF were identified by reviewing previous literature and case studies. These contributors form the parameters that have been used to construct the fault tree, developed by following the process outlined in the industry standard, AS IEC 61025-2008. Boolean algebra will then be used to determine minimum cut sets in the fault tree. Through analysis of industry data, probabilities can be assigned to these basic events and cut sets in order to quantitatively analyse the fault tree and produce probabilistic distributions. Quantitative analysis will be conducted using @Risk stochastic modelling software. Basic events in the fault tree will be inserted into the software as unknown inputs. Probabilistic distributions will be assigned to these inputs to try and assess the chance of RUF occurring if one or more of these basic events were to occur. Many trials and iterations will be completed in order to produce a probabilistic model of RUF, the results will then be analyzed.

### 2.1 Stochastic risk analysis

Stochastic analysis is the study of a completely random system, in order to predict uncertainties in that system. Fault tree analysis is a method of stochastic modelling that can be used for risk analysis of infrastructure systems. The fault tree is analysed and minimum cut sets are created, this process is done using Boolean algebra. Probabilities can then be incorporated into the fault tree in order to produce a probabilistic model. Probability distributions are a much more realistic way of describing uncertainty in variables of a risk analysis. This is most often done with the aid of computing software. @Risk is a good example of software that can model a probabilistic system using iteration.

#### 2.1.1 Qualitative risk analysis

The first step of qualitative analysis is to develop the fault tree diagram (Figure 2) that can be modeled in later quantitative analysis. From the current model, three main secondary events (“X” level) can lead to the top event, RUF. These are: boundary conditions acting on the rail system, the geometry of the rail and the loading that the rail head is subject to. From these three intermediate events a group of third tier (“Y” level) and basic events were developed. All these events can lead to RUF, as was determined in our literature review.

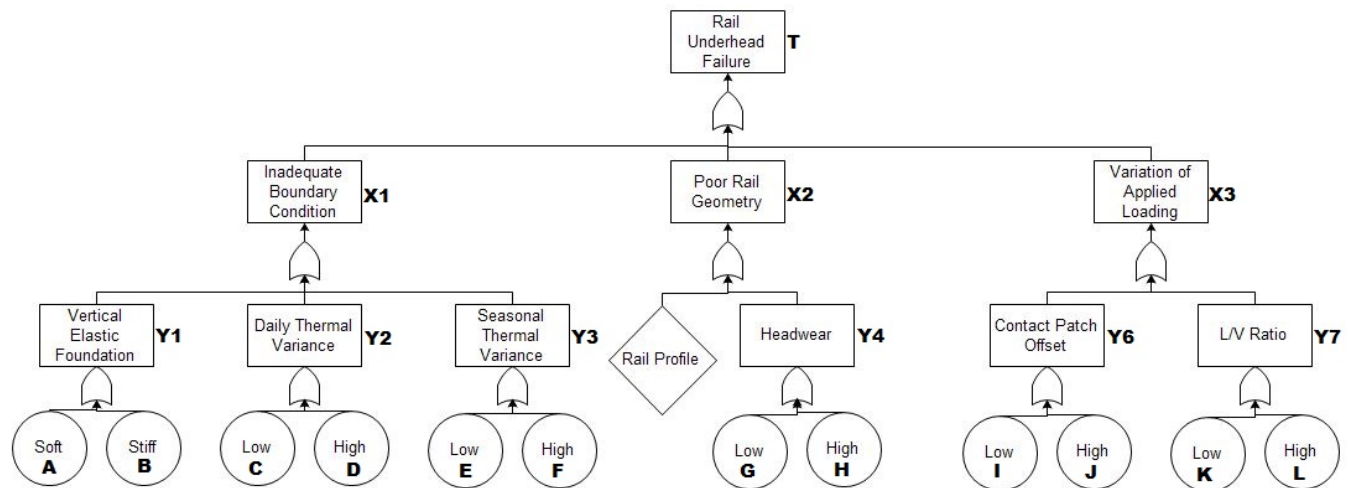


Figure 2 Qualitative fault tree

Step two of the qualitative analysis was to determine minimum cut sets from the fault tree using Boolean algebra. Referring to the labelled fault tree in Figure 2 the following equations show how the cut sets were determined.

$$T = X1 + X2 + X3$$

$$T = (Y1 + Y2 + Y3) + Y4 + (Y6 + Y7)$$

$$T = ([A+B] + [C+D] + [E+F]) + (G+H) + ([I+J] + [K+L])$$

$$T = A+B+C+D+E+F+G+H+I+J+K+L$$

Due to all events of the fault tree having the ability to act independently of each other, all gates were “OR” gates. Therefore, when analysing the system using Boolean algebra, all parameters must be totalled leading to a simplified fault tree diagram as shown in Figure 3.

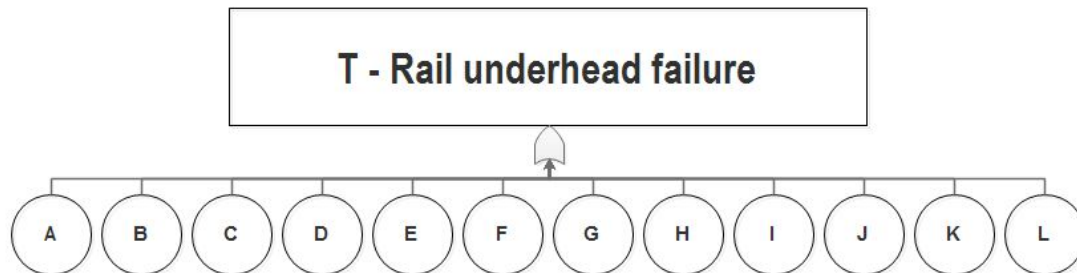


Figure 3: Simplified fault tree

## 2.1.2 Quantitative risk analysis

To conduct the quantitative risk analysis, the probability of system failure (RUF) must be assigned to each basic event. Table 1 demonstrates that there are essentially 6 basic events that lead to the top event, as can be seen in the Figure 2. Each of these events has two states such as: “low” and “high”. Figure 4 shows the input distributions for basic events used in @Risk, they are all triangular with minimum, maximum and “most common” values. Note that vertical elastic foundation and headwear, as well as L/V ratio and CPO have exactly the same distribution as labelled on Figure 4. The y-axis in Figure 4 is irrelevant because the program takes one random value from the x-axis during each iteration of the model. The x-axis is the probability that the system would fail (RUF), if that particular parameter was to occur. The justification for probability assignments and triangular model shapes, for each input parameter, is explained in the following section.

It is much simpler using triangular distributions to assign probabilities to failure parameters, and the distributions develop a distinctive shape due to the “most common” values. The second stage of quantitative analysis is to take the median, standard deviation, skewness and kurtosis values from the triangular distributions and place them into more complex distributions that are more commonly used for this kind of risk analysis as shown in Table 3. The justification of the chosen complex distribution and assigned probability for each failure parameter is given in the section below.

### 2.1.2.1 Vertical elastic foundation

Previous sensitivity studies conducted by Jeong *et al.* (1998) show that a decrease in foundation stiffness of 54 Mpa can decrease the safe crack growth life of rail by 40%. Therefore, considering that the foundation stiffness may further reduce in real world cases, it was assumed that a soft vertical elastic foundation may cause a 50% chance of system failure. A stiff vertical elastic foundation, the least critical scenario in this case, can cause a 2% chance of system failure. The “most common” value input into the triangular distribution was 35%, this means the distribution skews positively towards soft vertical elastic foundation and a higher failure rate as shown in Figure 5. The positive shape of this distribution means that a Johnson moments distribution, with median, standard deviation, skewness and kurtosis values input from the triangular distribution, was used for the complex analysis.

### 2.1.2.2 Daily and seasonal temperature variance

The weather data sourced from the Australian Bureau of Meteorology for typical heavy haul mining rail areas such as Port Headland shows that seasonal changes of approximately 20° C occur regularly. It also shows that daily variations can be in excess of 30° C in extreme cases. Since seasonal change is common and does not often deviate from averages, it is unlikely to greatly affect the system hence low min and max values (2-15%, Table 1) have been assigned. High daily variation may have a greater likelihood of causing RUF, therefore the maximum value assigned to its triangular distribution was 30%. The shape of these two triangular distributions was adjusted to mimic a normal distribution, the

model used most commonly when predicting temperature as shown in Table 2. A normal distribution was then used for the complex modeling.

Table 1. Summary of the statistics of input random variables for triangular distribution

Basic Event	Description	Probability of T (RUF) (System Failure) if basic event occurs	Most common probability
A	Soft vertical elastic foundation	0.02	0.35
B	Stiff vertical elastic foundation	0.50	
C	Low daily thermal variance	0.02	0.20
D	High daily thermal variance	0.30	
E	Low seasonal thermal variance	0.02	0.06
F	High seasonal thermal variance	0.15	
G	Low headwear	0.02	0.35
H	High headwear	0.50	
I	Low contact patch offset	0.02	0.12
J	High contact patch offset	0.30	
K	Low L/V ratio	0.02	0.12
L	High L/V ratio	0.30	

### 2.1.2.3 Head wear

High headwear is assumed to have the greatest effect on system failure because it has a good chance of weakening of structural strength of the railhead. Only 10-15 mm of the outer layer of the railhead is usually hardened therefore a rail head with a worn exterior will not be as strong. *Jeong et al.* (1998) clearly stated that as the railhead is worn away (measured as a percentage of railhead area), the safe crack growth life (Million gross tonnes) rapidly decreases. The maximum value assigned for headwear is 50%. The distribution is positively skewed, as can be seen in Figure 4. This is because RUF is more likely to occur on a worn railhead. This positive bias means that a Johnson moments distribution was used for the complex modelling as shown in Table 2.

### 2.1.2.4 Contact patch offset (CPO) and L/V ratio

Both of these phenomena are affected by track loading, occurring naturally when there is turning and hunting movements of the train. If a track is already worn or susceptible to faults, these loading patterns can be the final cause of failure. The safe crack growth life on rail in tracks with a curve of 8° is 73% less than that of straight track (Orringer 1988). After considering the amount of curved track on railway systems, and the sharpness of these curves, it was assumed that these two loading parameters may have between a 2-30% chance of causing RUF. Both these parameters were skewed negatively with a most likely value of 12% as shown in Table 1. This skewed triangular distribution resembles a lognormal model. Lognormal models are often used in papers related to rail system risk analysis such as

Lyons et al. (2009). This is because stresses are never negative, yet have the ability to be infinitely positive therefore a lognormal model is most suitable as shown in Table 2.

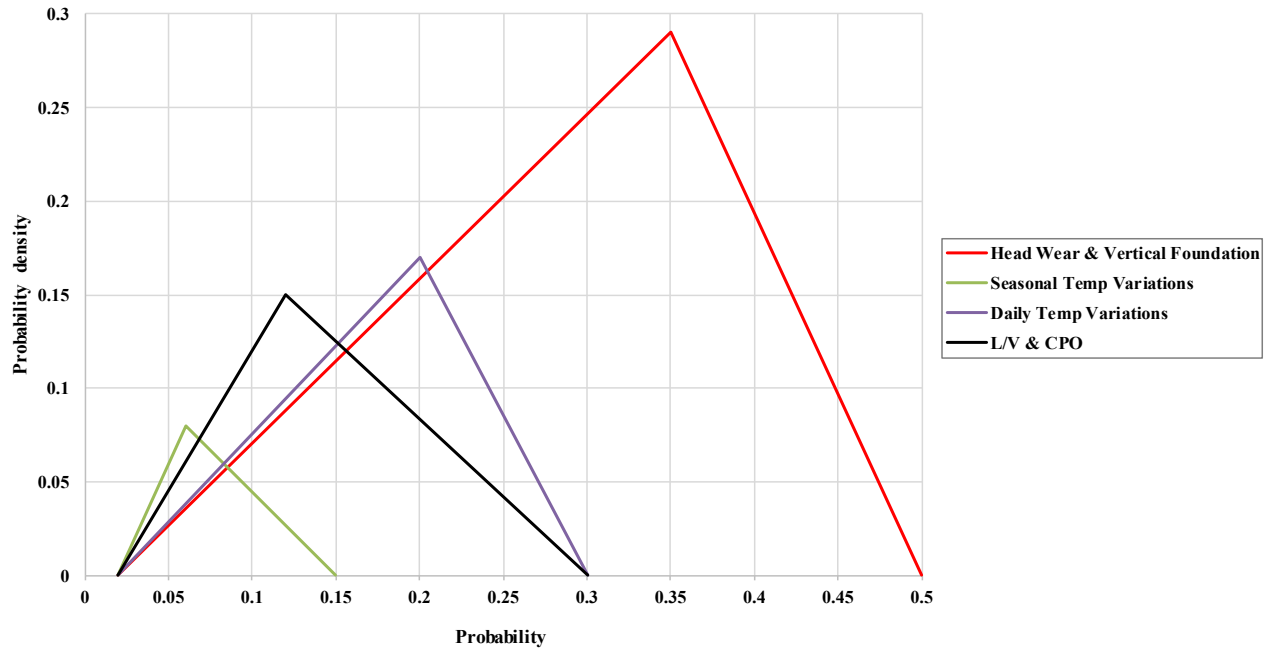


Figure 4 Triangular distribution models for input probabilities

Table 2. Summary of the statistics of input random variables for complex distribution

Uncertain Inputs	Median	Standard Deviation	Distribution type
Vertical Elastic Foundation	0.30142	0.10025	Right - Johnson
Daily thermal variance	0.17875	0.05793	Middle - Normal
Seasonal thermal variance	0.07351	0.02718	Middle - Normal
Headwear	0.30142	0.10025	Right - Johnson
Contact patch offset	0.14125	0.05793	Left - Lognorm
L/V ratio	0.14125	0.05793	Left - Lognorm

## 2.2 Comparison of failure parameters

When assigning the probabilities of system failure to each of the failure parameters, the affect they have on the system must be considered. Figure 5 shows that each failure parameter is rated in three main areas. The dimensions are stated including controllability, predictability and the overall effect on the RUF. When analysing the Figure 5, it can be seen that the temperature changes cannot be controlled, yet they can be easily predicted and don't have a major effect on RUF. Vertical stiffness cannot be controlled or predicted well, yet it has a major effect on the system. This plot is another way to consider the failure parameters in the system, and is a tool to help assign accurate probabilities in quantitative analysis.



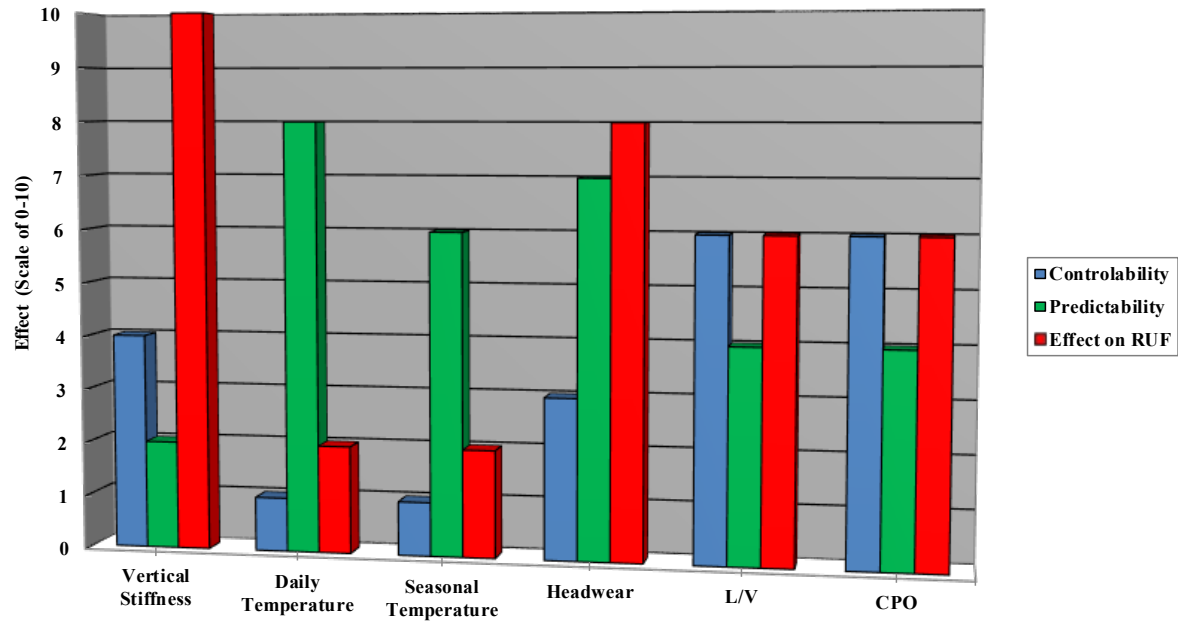


Figure 5 Comparison of failure parameters with field data from Orringer et al. (1988)

### 3. Results

By randomly modelling the unknown inputs (fault tree basic events) using simple triangular distributions, @Risk produces a model that is similar to a normal distribution yet slightly negatively skewed (Figure 6). In 90% of the cases, RUF has between a 13.8-23.4% probability of occurring.

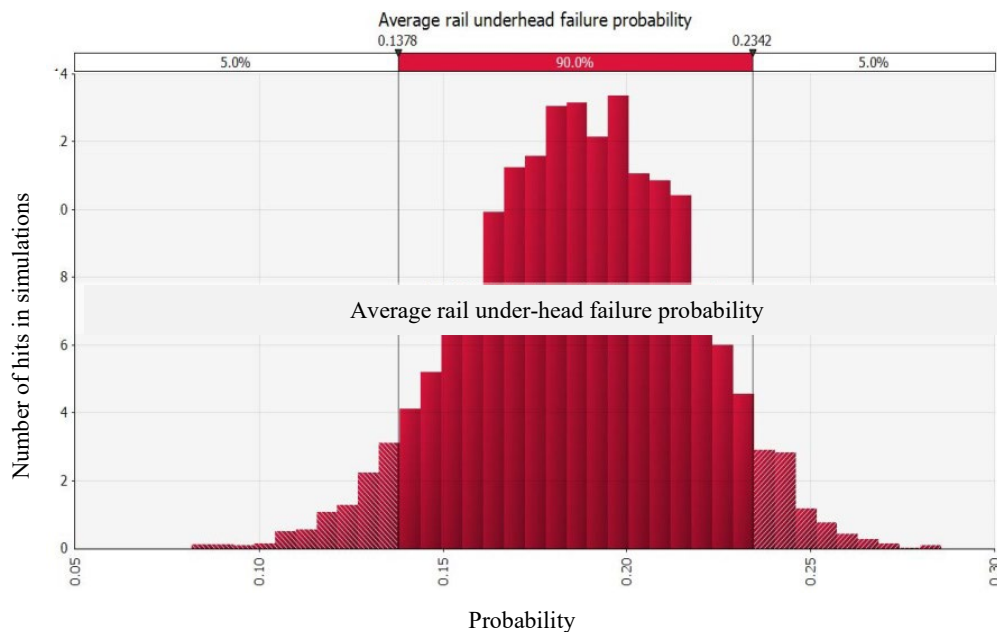


Figure 6 Probability distribution for triangular distributions

When complex input distributions were used, the output probability density function shows that in 90% of cases of an input failure parameter occurring, RUF has between a 14.2-23.7% chance of occurrence (Figure 7).

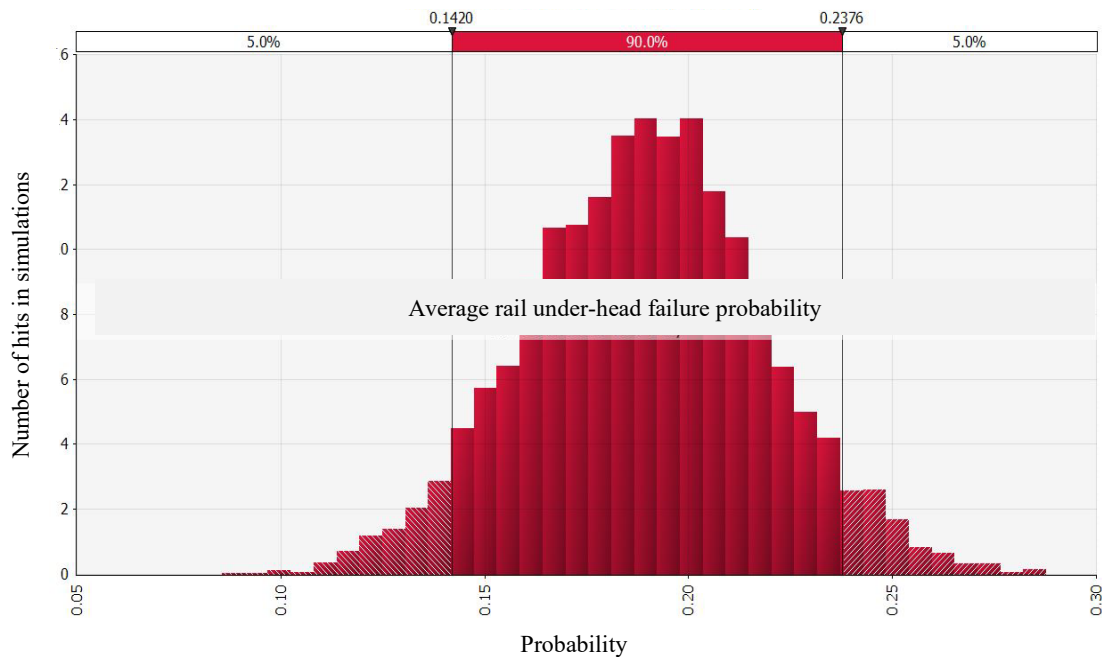


Figure 7 probability distribution for complex input distributions

As expected, the parameters that have the greatest effect on system failure are: the vertical elastic foundation and headwear, CPO and L/V ratio have a moderate effect, while the temperature variations have the least effect. The average system failure probability of these inputs, or the baseline, is 18.7% as shown in Figure 8.

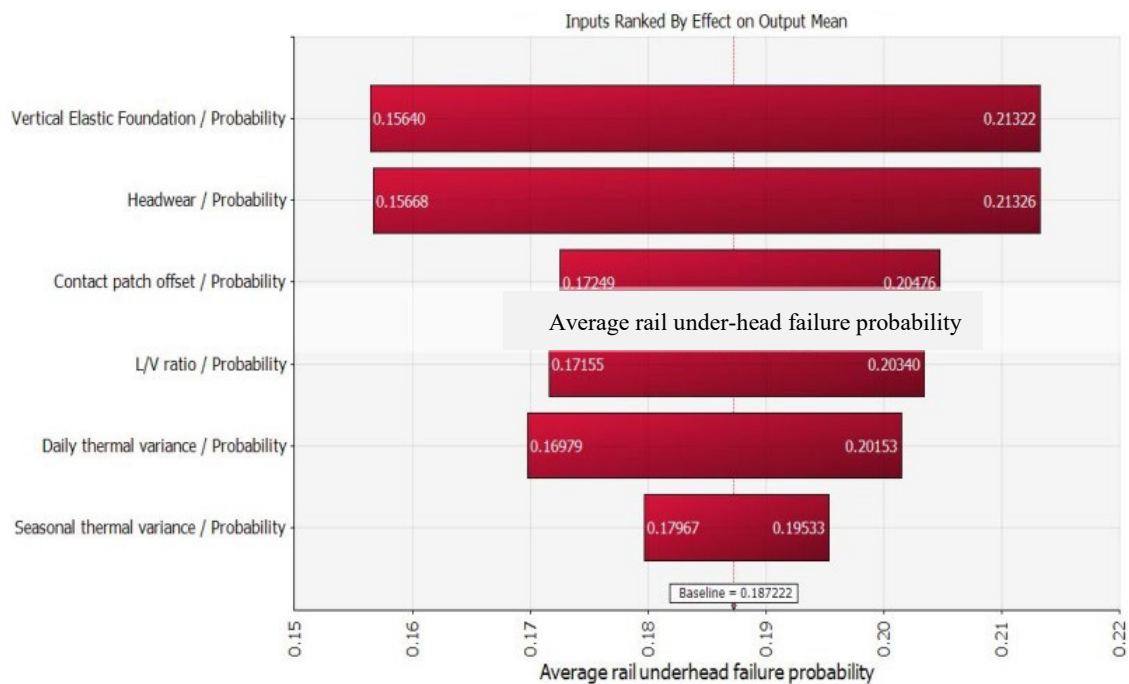


Figure 8 Probability of inputs



#### **4. Discussions**

Stochastic risk analysis of RUF was achieved as outlined in the results. The first part of this stochastic analysis was to qualitatively analyse the parameters that lead to RUF. The six major factors considered have all been thoroughly evaluated in previous research; therefore it is known that they play a major part in causing RUF. The fault tree that was developed incorporated all these factors and was adjusted to cater for the use of software for risk analysis. Determination of the minimum cut sets proved successful, all basic events in the fault tree are independent of each other and are connected with “OR” gates. Because of this, using Boolean algebra to produce a simplified fault tree was a straight forward process.

The Palisade Decision Tools - @Risk software (version 7.5) was used for the stochastic analysis in this research. The first step in the process was to assign probabilities to each basic event in the fault tree. The most important aspect of assigning the probabilities was to justify them using expert opinion and peer reviewed research. This was completed well due to the use of major sensitivity studies into the crack growth life of rail, subject to each failure parameter as was demonstrated by *Jeong et al.* (1998) and *Lyons et al.* (2009). Simple triangular distributions provided the first set of results, they followed a normal distribution and it seemed that increasing the complexity of the distributions may increase the accuracy of the results. A new set of complex input distributions were used and they produced very similar results.

Both sets of results showed similarities. The simple analysis produced a probability density function of a normal shape with RUF having between a 13.7 and 23.4% chance of occurrence in 90% of cases. The complex model was slightly more negatively skewed, like a lognormal distribution, and RUF had between a 14.2 % and 23.7% chance of occurrence, 90% of the time. The only difference between the two sets of results was the complex model having a narrower range and slightly positively skewed probability density function. Because of this, it can be said that this range of RUF probabilities is fairly accurate, despite the need for some obvious improvements.

To refine and improve the results, additional steps with further analysis were taken in this research – e.g. a qualitative analysis for the complexities of the fault tree. Further research is required to expand the datasets and additional parameters to explore the underlying relationships between parameters and refine the Boolean algebra including AND/OR gates and probability values. A notable fix would be to further explore the relationship between CPO and L/V ratio, as they are closely linked. An AND gate, as well as considering factors like train hunting and curving movements, might produce a better model of the system. Improvements of the quantitative analysis would start with improving the accuracy of the assigned probabilities, through experimentation, real data capture and expert input. Of course, comprehensive capturing of real world data in the heavy haul rail industry will provide relevant opportunities for exploring further and developing sound theoretical concepts and practical predictive models of rail asset management.

#### **5. Conclusions**

Rail failures such as rail under-head failure (RUF) are inevitable and hence a major concern for the railway industry. RUF occurrence is significant in heavy haul rail operations. Stochastic risk analysis of RUF was conducted and the parameters considered are: vertical elastic foundation of the rail, daily and/or seasonal temperature variances, headwear, contact patch offset (CPO), and the lateral to vertical force ratio (L/V) acting on the rail head. The results revealed that if one or combination of basic events in the fault tree happens then the RUF has a probability of occurrence is between 14-23.5%, which is significant. The input probabilities have come about from expert opinion and previous research, so it can be concluded that this is a fairly accurate model. Moreover, the results resemble a lognormal distribution, which confirms to the previously verified models in the literature. The parameters that lead to RUF should be thoroughly considered, and suitable maintenance planning strategies should be implemented in order to develop preventative mitigation of these failures.

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