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Modeling the Effects of Transmission Dynamics of Malaria: A Mathematical Approach on Healthcare

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Abstract

A mathematical model is proposed and analyzed to study the malaria transmission and how to reduce the transmission by using specific electromagnetic frequency. The model is governed by four compartments for human population and three compartments for mosquito population. The model is analyzed by using the stability theory of non-linear differential equations and numerical simulations. We calculate the basic reproduction number by the method of next generation matrix. We observe that the disease free equilibrium is stable only when the basic reproduction number is less than one and unstable when the basic reproduction number is greater than one. We also investigate that the malaria transmission is reduced by using mosquito net and specific electromagnetic frequency. Numerical simulations are shown to support the presented analytical results.

Keywords: Malaria Transmission, Human population, Mosquito population, Specific electromagnetic frequency, Basic reproduction number.

1. Introduction

Malaria is a life-threatening disease. It's typically transmitted through the bite of an infected *Anopheles* mosquito. Infected mosquitoes carry the *Plasmodium* parasite. When this mosquito bites, the parasite is released into bloodstream. Once the parasites are inside the body, they travel to the liver, where they mature. After several days, the mature parasites enter the bloodstream and begin to infect red blood cells. Within 48 to 72 hours, the parasites inside the red blood cells multiply, causing the infected cells to burst open (Ducrot et al. 2009). Symptoms of malaria include fever and flu-like illness, including shaking chills, headache, muscle aches, and tiredness. Nausea, vomiting and diarrhea may also occur. Malaria may cause anemia and jaundice (yellow coloring of the skin and eyes) because of the loss of red blood cells. If not promptly treated, the infection can become severe and may cause kidney failure, seizures, mental confusion, coma and death (WHO 2018).

Morbidity and mortality from malaria are at almost unprecedented levels, particularly in Africa south of the Sahara, where the disease claims more than 1 million lives per year. Bangladesh has a history of endemic malaria transmission in 13 of 64 districts (Alam et al. 2016). Up to 70,000 laboratory-confirmed and 9,00,000 clinical cases, with more than 500 deaths per year, were reported in the late 1990s. A cross-sectional survey in 2007 reported a crude prevalence of 4% in the 13 malaria-endemic districts. More than 90% of cases were *Plasmodium falciparum* (Bounomo et al. 2013). The highest prevalence (>10%) was identified in three districts of the Chittagong Hill Tracts in southeastern Bangladesh. Prevention and control relies on reducing mosquitoes through source reduction (removal and modification of breeding sites), and reducing contacts between mosquitoes and people.

Mathematics has always benefited from its involvement with developing sciences. Mathematical biology is a fast growing, well recognized subject and is the most exciting modern application of mathematics (Dym 2004 and Murray 1989). It is a branch of biology which employs theoretical analysis, mathematical models and abstractions of the living organisms to investigate the principles that govern the structure, development and behavior of the systems, as opposed to experimental biology which deals with the conduction of experiments to prove and validate the scientific theories (Bubniakova 2007 and Kapur 1985). The field is sometimes called mathematical biology or biomathematics to stress the mathematical side, or theoretical biology to stress the biological side ((Biswas 2012 and Biswas 2014). Theoretical biology focuses more on the development of theoretical principles for biology while mathematical biology focuses on

the use of mathematical tools to study biological systems, even though the two terms are sometimes interchanged (Brauer and Castillo 2010). Mathematical biology aims at the mathematical representation and modeling of biological processes, using techniques and tools of applied mathematics. It has both theoretical and practical applications in biological, biomedical and biotechnology research (Cai 2013). Describing systems in a quantitative manner means their behavior can be better simulated, and hence properties can be predicted that might not be evident to the experimenter. This requires precise mathematical models. Mathematical biology employs many components of mathematics, and has contributed to the development of new techniques.

2. Formulation of the model

The mathematical model of malaria transmission is made up to the vector and host population. The host population consists of four compartments. The susceptible, who can acquire the infection; exposed, when the virus exposed itself into human bodies; infected, who can transmit infection to susceptible; recovered, who is immune from infection. Let, the susceptible, exposed, infected and recovered human are denoted by S_h, E_h, I_h and R_h respectively. The mosquito population consists of three compartments. The susceptible, exposed and infected. Let, the susceptible, exposed and infected mosquito are denoted by S_v, E_v and I_v respectively. There is no recovery or immune in mosquito population.

We present a compartmental model of malaria transmission which has been taken from (Chitnis et al. 2008 and Chiyaka et al. 2008). The model equations are given by

The Host population

$$\frac{dS_h}{dt} = r_h + \rho R_h - \alpha_4 I_v S_h - \beta_h S_h - \mu_h S_h \tag{1}$$

$$\frac{dE_h}{dt} = \beta_h S_h - \alpha_1 E_h - \mu_h E_h \tag{2}$$

$$\frac{dI_h}{dt} = \alpha_1 E_h + \alpha_4 I_v S_h - \alpha_2 I_h - \mu_h I_h \tag{3}$$

$$\frac{dR_h}{dt} = \alpha_2 I_h - \rho R_h - \mu_h R_h \tag{4}$$

The vector population

$$\frac{dS_v}{dt} = r_v - \alpha_5 I_h S_v - \beta_v S_v - \mu_v S_v \tag{5}$$

$$\frac{dEv}{dt} = \beta_v S_v + -\alpha_3 E_v - \mu_v E_v \tag{6}$$

$$\frac{dI_v}{dt} = \alpha_3 E_v + \alpha_5 I_h S_v - \mu_V I_V \tag{7}$$

where, susceptible individuals are recruited into the human population either by birth or immigration at a rate r_h, and that of mosquito population at a rate r_v. When a mosquito carry malaria virus bite a susceptible human, the virus is passed into a human and the person moves to the infected class at a rate α_4. Human moves from susceptible to exposed compartments with progression rate β_h. Then human moves from exposed to infectious class with progression rate α_1. The rate of removal of infectious to recovered compartment is proportional to the number of infectious only and so the progression rate is α_2. A human can die at any stage by natural causes at a rate μ_h and mosquito at a rate μ_v. Recovered humans can enter susceptible compartment at a rate ρ_h. Similarly, when an uninfected mosquito bites an infectious human, it carries the virus and so the number of infected mosquitoes increase

at a rate α_5 proportional to both the number of infectious human and the number of susceptible mosquito. Mosquitoes moves from exposed to infectious compartments with progression rate α_3.

2.1. Modified Model of Malaria Transmission

We have modified the model (1)-(7) to interfere the life cycle of malaria parasite using specific electromagnetic frequency, f (Cosic 2015). To reduce the transmission of malaria parasite it is necessary to diminish the contact between human and mosquito. One of the way is the use of mosquito net which is denoted by ψ. With this basic description, we illustrate the flow diagram in Figure 1.

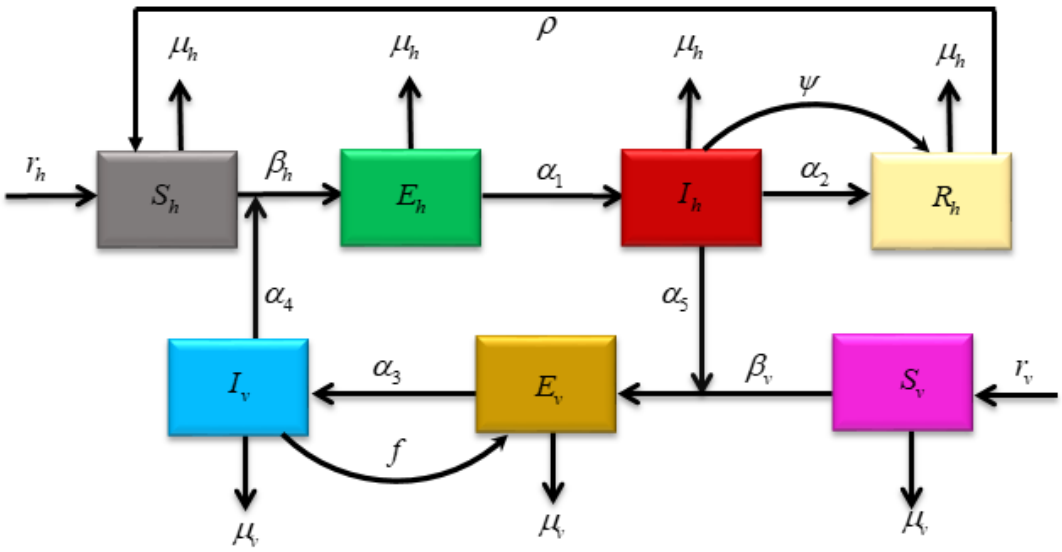

Figure 1: Flow diagram of malaria transmission model including humans and mosquito.

Considering the modification and the flow diagram in Figure 1, the model is formulated as a system of ordinary differential equation

The Host population

$$\frac{dS_h}{dt} = r_h + \rho R_h - \alpha_4 I_v S_h - \beta_h S_h - \mu_h S_h \tag{8}$$

$$\frac{dE_h}{dt} = \beta_h S_h - \alpha_1 E_h - \mu_h E_h \tag{9}$$

$$\frac{dI_h}{dt} = \alpha_1 E_h + \alpha_4 I_v S_h - \alpha_2 I_h - \psi I_h - \mu_h I_h \tag{10}$$

$$\frac{dR_h}{dt} = \alpha_2 I_h + \psi I_h - \rho R_h - \mu_h R_h \tag{11}$$

The Vector population

$$\frac{dS_v}{dt} = r_v - \alpha_5 I_h S_v - \beta_v S_v - \mu_v S_v \tag{12}$$

$$\frac{dEv}{dt} = \beta_v S_v + fI_v - \alpha_3 E_v - \mu_v E_v \tag{13}$$

$$\frac{dI_v}{dt} = \alpha_3 E_v + \alpha_5 I_h S_v - fI_v - \mu_V I_V \tag{14}$$

with initial conditions,

$$S_h(0) = S_{h_0},\ E_h(0) = E_{h_0},\ I_h(0) = I_{h_0},\ R_h(0) = R_{h_0}$$
$$\text{and}\quad S_v(0) = S_{v_0},\ E_v(0) = E_{v_0},\ I_v(0) = I_{v_0}$$

3. Mathematical Analysis of the Malaria Model

3.1 Positivity of the solutions:

Theorem 1: If $S_{h_0},\ I_{h_0},\ E_{h_0},\ R_{h_0}, S_{v_0},\ E_{v_0}$ and I_{v_0} are non-negative then the solution of $S_h(t),\ E_h(t),\ I_h(t),\ R_h(t),\ S_v(t),\ E_v(t)$ and $I_v(t)$ of the system of differential equations (8)-(14) are positive for all time $t > 0$.

3.2 Existence and Uniqueness Analysis for the Model

Theorem 2. We consider the system of differential equations

$$\frac{dx}{dt} = f(t,x)$$
$$\text{where, } x = \{S_h, E_h, I_h, R_h, S_v, E_v, I_v\} \tag{15}$$
$$\text{and}\quad f = \left\{\dot{S}_h, \dot{E}_h, \dot{I}_h, \dot{R}_h, \dot{S}_v, \dot{E}_v, \dot{I}_v\right\}$$

Which are taken from the Equations (8)-(14).

$$\text{If the region, } D = \left\{(S_h, E_h, I_h, R_h, S_v, E_v, I_v) : \left|S_h - S_{h_0}\right| \le a, \left|E_h - E_{h_0}\right| \le b, \left|I_h - I_{h_0}\right| \le c, \left|R_h - R_{h_0}\right| \le d,\right.$$
$$\left.\left|S_v - S_{v_0}\right| \le e, \left|E_v - E_{v_0}\right| \le f, \left|I_v - I_{v_0}\right| \le g, \left|t - t_0\right| \le h\right\}$$

Then the system have a unique solution (Khalil 1996 and Ross 2004).

Proof: Since, $\left|\frac{\partial f_i}{\partial x_j}\right|, i, j = 1,2,...7$ all are continuous and bounded. Hence the system of differential equation (8)-(14) have a unique solution and the model is both epidemiologically feasible and mathematically well posed.

3.3 Disease Free Equilibrium (DFE) Analysis:

For the disease free equilibrium point of the model (8)-(14), we have to solve, $\frac{dx}{dt} = f(t,x) = 0$

The disease free equilibrium point of the model (8)-(14) is

$$E_0(S_h, E_h, I_h, R_h, S_v, I_v, R_v) = E_0\left(\frac{r_h}{\beta_h + \mu_h}, 0, 0, 0, \frac{r_v}{\beta_v + \mu_v}, 0, 0\right) \tag{16}$$

3.4 Derivation of Basic Reproduction Number:

An important notion in epidemic logical models is the basic reproduction number (R_0). It is a threshold valve that is often used to measure the spread of a disease. It is defined as the number of secondary infections in humans in a wholly susceptible population. It is evaluated as

$$R_0 = \frac{\alpha_5 r_v}{(\beta_v + \mu_v)(\alpha_2 + \psi + \mu_h)} \times \frac{\alpha_4 r_h}{(\beta_h + \mu_h)(f + \mu_v)} = \frac{\alpha_4 \alpha_5 r_h r_v}{(\beta_h + \mu_h)(\beta_v + \mu_v)(\alpha_2 + \psi + \mu_h)(f + \mu_v)}$$

(5.67)

3.5 Stability Analysis at Disease Free Equilibrium Point

Theorem 3: The disease free equilibrium (DFE) point of the differential equations (8)-(14) is locally asymptotically stable if $R_0 < 1$, otherwise it is unstable (Biswas et al. 2014).

Proof: The Jacobian matrix of the system (16)-(22) can be written as

$$J = \frac{\partial(f_1, f_2, f_3, f_4, f_5, f_6, f_7)}{\partial(S_h, E_h, I_h, R_{h,} S_v, E_v, I_v)} = \begin{bmatrix} \frac{\partial f_1}{\partial S_h} & \frac{\partial f_1}{\partial E_h} & \frac{\partial f_1}{\partial I_h} & \frac{\partial f_1}{\partial R_h} & \frac{\partial f_1}{\partial S_v} & \frac{\partial f_1}{\partial E_v} & \frac{\partial f_1}{\partial I_v} \\ \frac{\partial f_2}{\partial S_h} & \frac{\partial f_2}{\partial E_h} & \frac{\partial f_2}{\partial I_h} & \frac{\partial f_2}{\partial R_h} & \frac{\partial f_2}{\partial S_v} & \frac{\partial f_2}{\partial E_v} & \frac{\partial f_2}{\partial I_v} \\ \frac{\partial f_3}{\partial S_h} & \frac{\partial f_3}{\partial E_h} & \frac{\partial f_3}{\partial I_h} & \frac{\partial f_3}{\partial R_h} & \frac{\partial f_3}{\partial S_v} & \frac{\partial f_3}{\partial E_v} & \frac{\partial f_3}{\partial I_v} \\ \frac{\partial f_4}{\partial S_h} & \frac{\partial f_4}{\partial E_h} & \frac{\partial f_4}{\partial I_h} & \frac{\partial f_4}{\partial R_h} & \frac{\partial f_4}{\partial S_v} & \frac{\partial f_4}{\partial E_v} & \frac{\partial f_4}{\partial I_v} \\ \frac{\partial f_5}{\partial S_h} & \frac{\partial f_5}{\partial E_h} & \frac{\partial f_5}{\partial I_h} & \frac{\partial f_5}{\partial R_h} & \frac{\partial f_5}{\partial S_v} & \frac{\partial f_5}{\partial E_v} & \frac{\partial f_5}{\partial I_v} \\ \frac{\partial f_6}{\partial S_h} & \frac{\partial f_6}{\partial E_h} & \frac{\partial f_6}{\partial I_h} & \frac{\partial f_6}{\partial R_h} & \frac{\partial f_6}{\partial S_v} & \frac{\partial f_6}{\partial E_v} & \frac{\partial f_6}{\partial I_v} \\ \frac{\partial f_7}{\partial S_h} & \frac{\partial f_7}{\partial E_h} & \frac{\partial f_7}{\partial I_h} & \frac{\partial f_7}{\partial R_h} & \frac{\partial f_7}{\partial S_v} & \frac{\partial f_7}{\partial E_v} & \frac{\partial f_7}{\partial I_v} \end{bmatrix}$$

At the disease free equilibrium point, $E(S_h, E_h, I_h, R_h, S_v, E_v, I_v) = E\left(\frac{r_h}{\beta_h + \mu_h}, 0, 0, 0, \frac{r_v}{\beta_v + \mu_v}, 0, 0\right)$

$$J(E)=\begin{bmatrix} -\beta_h-\mu_h & 0 & 0 & p & 0 & 0 & \dfrac{-\alpha_4 S_h}{\beta_h+\mu_h} \\ \beta_h & -\alpha_1-\mu_h & 0 & 0 & 0 & 0 & \dfrac{\alpha_4 r_h}{\beta_h+\mu_h} \\ 0 & \alpha_1 & -\alpha_2-\psi-\mu_h & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha_2+\psi & -\beta-\mu_h & 0 & 0 & 0 \\ 0 & 0 & \dfrac{-\alpha_5 S_v}{\beta_v+\mu_v} & 0 & -\beta_v-\mu_v & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta_v & -\alpha_3-\mu_v & f \\ 0 & 0 & \dfrac{\alpha_5 S_v}{\beta_v+\mu_v} & 0 & 0 & \alpha_3 & -f-\mu_v \end{bmatrix}$$

This is the square matrix of order 7.

Now, we consider the three order block matrix that is denoted by A. and $A=\begin{bmatrix} -\beta_v-\mu_v & 0 & 0 \\ \beta_v & -\alpha_3-\mu_v & f \\ 0 & \alpha_3 & -f-\mu_v \end{bmatrix}$

$$B=\begin{bmatrix} -\beta_h-\mu_h & 0 \\ \beta_h & -\alpha_1-\mu_h \end{bmatrix} \text{ and } C=\begin{bmatrix} -\alpha_2-\psi-\mu_h & \dfrac{\alpha_4 r_h}{\beta_h+\mu_h} \\ \dfrac{\alpha_5 r_v}{\beta_v+\mu_v} & -f-\mu_v \end{bmatrix}$$

Hence, it is proved that, if

1) $R_0<1$ then $|C|>1$ (i.e. C is nonsingular) then the DFE is locally asymptotically stable.
2) $R_0>1$ then $|C|<1$ (i.e. C is singular) and so the DFE is unstable.

4. Numerical Results:

We performed numerical simulations of the model (8)-(14) by ode45 solver using MATLAB programming language. We used a set of suitable parameter values. The description of all the parameters with the estimated values used in the simulations (8)-(14) is presented in Table 6.1. We have considered the initial condition $S_{h_0}=600,\ E_{h_0}=0,\ I_{h_0}=0,\ \ R_{h_0}=0,\ S_{v_0}=850,\ E_{v_0}=0$ and $I_{v_0}=35$.

Firstly, we solved the model (5.8)-(5.14) considering the initial values and all other parameters that are shown in Table 6.1. Also we performed the numerical simulations for time $t=0$ to 60 days. The description of all the parameters with the estimated values used in the simulations (8)-(14) is presented in Table 1.

Table 1: Description and Estimation of the parameters

Descriptions	Symbols	Values	References
Recruitment rate of the susceptible humans	r_h	0.028	Chitnis et al. (2008)
Recruitment rate of the susceptible mosquitoes	r_v	0.083	Cai et al.(2013)
Exposed rate of human	β_h	0.00638	Chiyaka et al.(2008)
Exposed rate of mosquitoes	β_v	0.0696	Chitnis et al. (2008)
Effective rate of exposed (humans) becoming infectious	α_1	0.05	Chiyaka et al.(2008)
Recovery rate of human	α_2	0.0035	Cai et al.(2013)
Effective rate of exposed (mosquitoes) becoming infectious	α_3	0.083	Chiyaka et al.(2008)
Probability of transmission of infection from an infectious mosquitoes to human	α_4	0.0225	Chiyaka et al.(2008)
Probability of transmission of infection from infected human to mosquito	α_5	0.0246	Chiyaka et al.(2008)
Loss of immunity of humans	ρ	0.00017	Chitnis et al. (2008)
Rate of recovery from infected human using mosquito nets	ψ	0.03863	assumed
Rate of electromagnetic frequency	f	0.0034	Cosic et al. (2015)
Natural death rate of human	μ_h	0.000386	Chitnis et al. (2008)
Natural death rate of mosquito	μ_v	0.05624	Chitnis et al. (2008)

The result of the simulation of the combined class is presented in Figure 1.

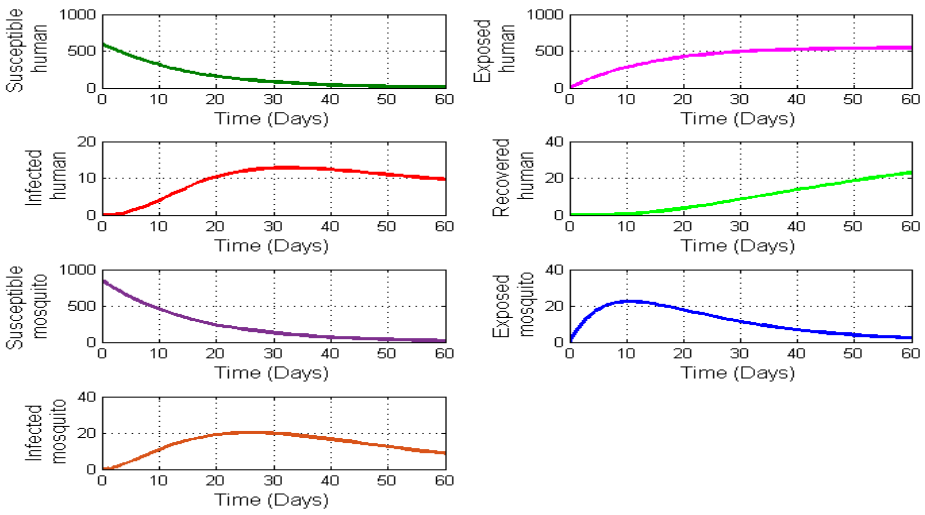

Figure 1: The graph of malaria transmission model (5.8)-(5.14) of the population with time (60 days) for $\alpha_4 = 0.0225$ and $\alpha_5 = 0.0246$. $R_0 < 1$

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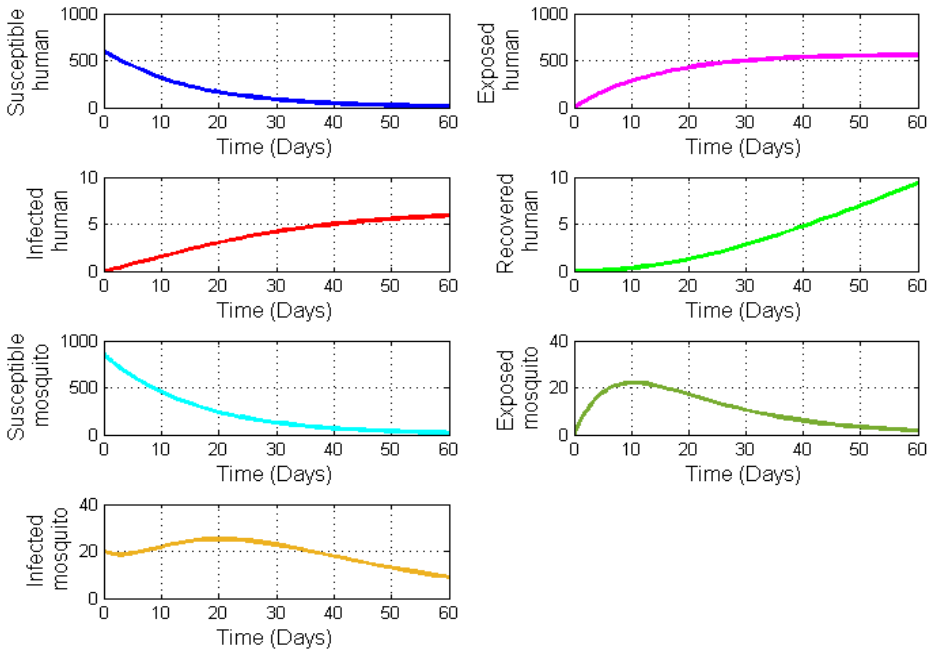

Figure 2: Numerical simulations for malaria transmission of the population with time (60 days) for $\alpha_4 = 0.0026$ and $\alpha_5 = 0.0035$.

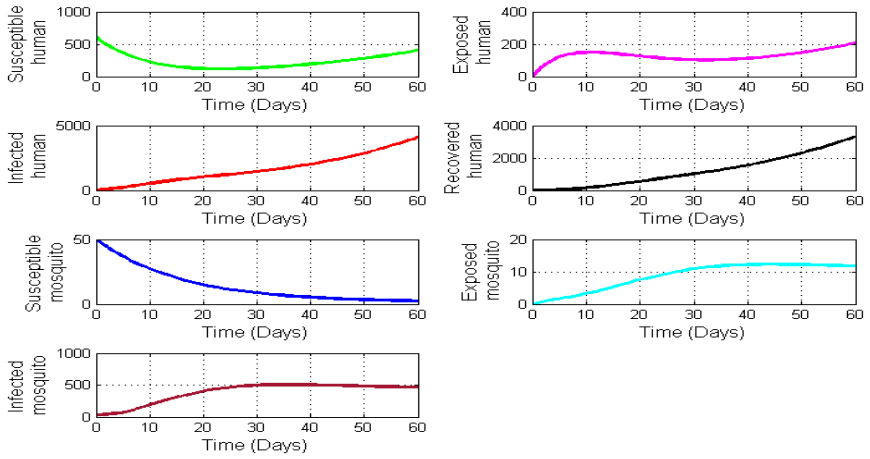

Figure 3: Numerical simulations for malaria transmission of the population with time (60 days) for $\alpha_4 = 0.26$ and $\alpha_5 = 0.35$, $R_0 > 1$

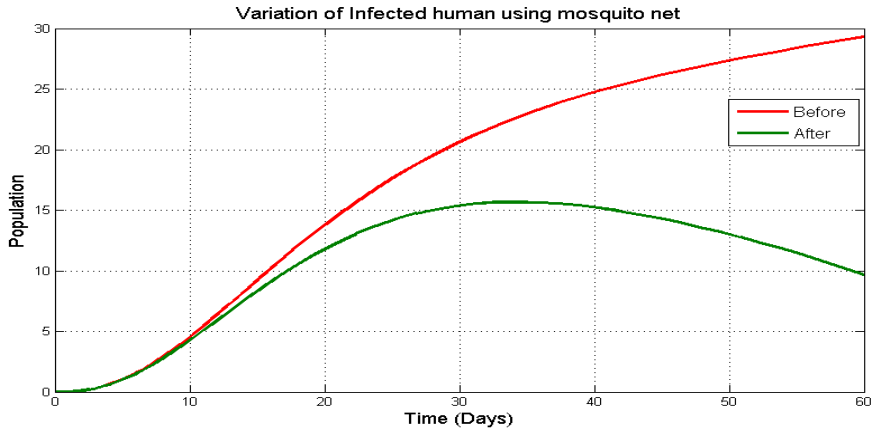

Figure 4: The infected human population is significantly decreasing after using mosquito net.

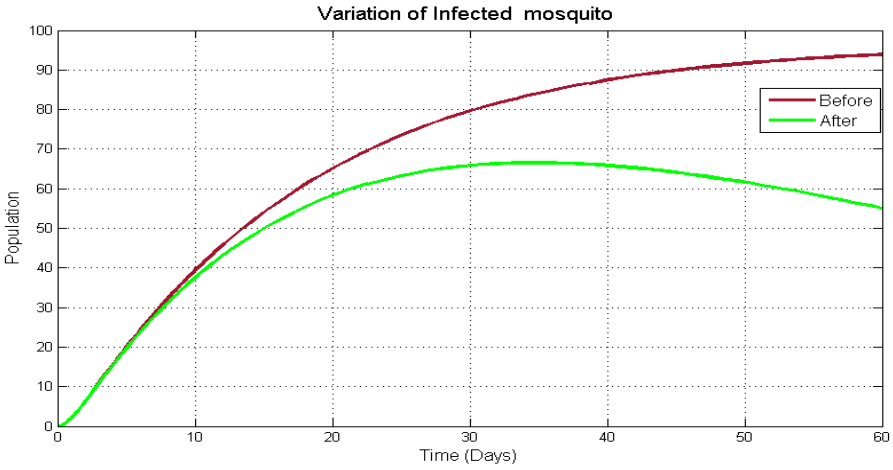

Figure 5: The infected mosquito population is significantly decreasing after using specific electromagnetic frequency.

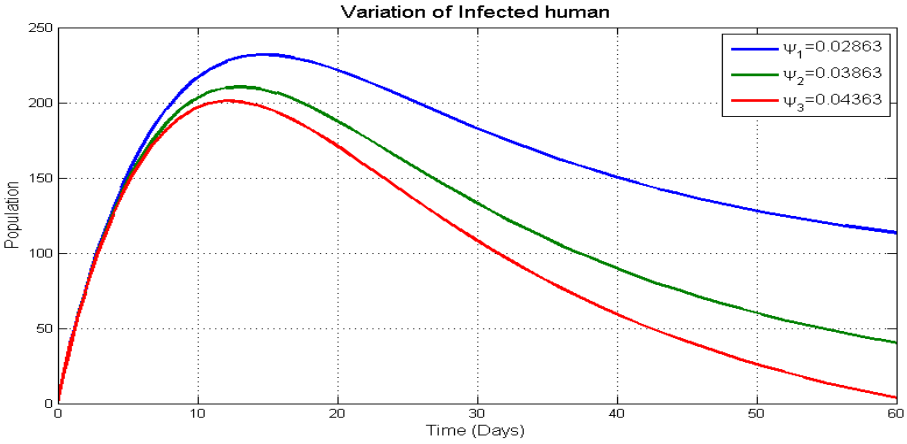

Figure 6: Variation of infected human by using mosquito net for $\psi_1 = 0.02863$, $\psi_2 = 0.03863$ and $\psi_3 = 0.04863$.

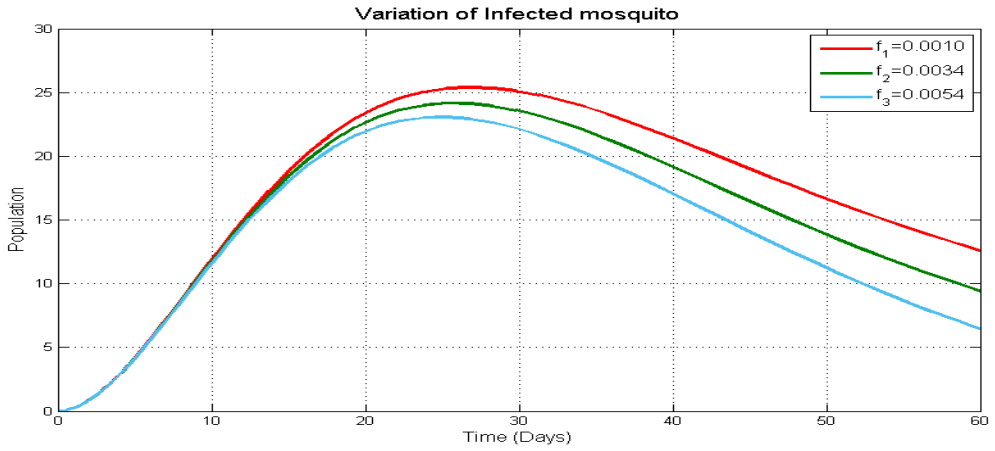

Figure 7: Variation of the infected mosquito by using specific electromagnetic frequency for $f_1 = 0.0010$, $f_2 = 0.0034$ and $f_3 = 0.0054$.

6. Contributions:

We conclude that the people all over the world are still under threat due to the epidemic disease malaria. The main objective of this study is to propose a model for the transmission of malaria between mosquito to human and human to mosquito and also establish a possible way of reducing the virus transmission. In this paper, a malaria transmission model is presented and analyzed to that effect. Figure 2 shows the behavior of the solution for the selected parameter values when $R_0 < 1$. Figure 3 shows the behavior of the solution for the selected parameter values when $R_0 > 1$. These show that there is decline in both the susceptible population of humans and mosquitoes with time and both the recovered human and mosquitoes population increase with time. The contact rate has a great impact on transmission disease.

So, if we can work on reducing mosquitoes through source reduction (removal and clearance of breeding sites), then the entire mosquito population can be eliminated, which will lead to total elimination of the malaria disease. The basic reproductive number has been computed to determine the stability of the disease because theoretical determination of threshold conditions for R_0 is of important public heath interest. It was realized that whenever $R_0 < 1$, the disease free equilibrium point is locally asymptotically stable and unstable whenever $R_0 > 1$.

In Figures 4 and 6, we investigate that the infected human population decrease as the increase of the using of mosquito net and the number of infected human population increase as the decrease of the using of mosquito net. In Figures 5 and 7, we observe that, the infected human mosquito decrease as the increase of the value of specific electromagnetic frequency and the number of infected mosquito population increase as the decrease of the value of specific electromagnetic frequency.

7 Future Recommendations:

Malaria is one of the life-threatening disease in the world. It's typically transmitted through the bite of an infected *Anopheles* mosquito. Infected mosquitoes carry the *Plasmodium* parasite. Many of the people all over the world are mostly affected by malaria infection. So, it is time to take prevent step against malaria infection. If the symptoms of malaria appear then it may be defended. In this case, public awareness and the use of producing antibodies against malaria infection is most important to control malaria infection. So that, it is necessary to carry out comprehensive researches in order to find out more relevant strategies to incorporate in eradicating malaria infection. We propose that our future work will be focused on:

- How to reduce the infection rate from mosquito to human and human to mosquito by using natural treatment.

- Applying the control strategies of malaria transmission.
- Giving emphasize on mass education, public awareness to clean out our surroundings and media campaigns against the transmission dynamics of malaria infection.

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