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A System with Multi-Failure Mode and Imperfect Repair

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Abstract – Systems' performance declines after successive usage, hence proper and adequate maintenance is required to prolong its operational duration. In this paper the performance of a repairable system subjected to several modes of failure is examined. A repairable system is defined as a system which, after failing to perform one or more of its functions satisfactorily, could be restored to a satisfactory performance by a method other than replacement of the entire system. Systems upon failure are perfectly, minimally, or imperfectly repaired. In Imperfect repair, the system performance is weakened after each repair and thus it fails more frequently. This article addresses the case where a repairable system is exposed to multiple failure modes with imperfect repair. The performance of the system is measured using availability, hence, a closed analytical form expression is derived for the long term probability the system being in the operational state under exponentially distributed times between failures and repair times.

Index Terms - steady state probability, exponential distribution; a repairable system; imperfect repair.

I. INTRODUCTION

Operating systems deteriorate over time and eventually failure to perform their intended function. Upon failure, inexpensive and critical systems are usually replaced, while non-critical and expensive ones are repaired several times before complete replacement. The type of repair conducted defines the performance of the system. Repair is either perfect where the system is restored to an as good as new state, minimal which bring the system to its status immediately before failure, or imperfect where the system becomes inferior after each repair.

In this paper, a repairable system with multiple failure modes and imperfect repair is examined. In recent years, increased attention has been paid in modeling repairable systems that undergo different modes of failure. Different models were presented with results for imperfect maintenance in [1]. Authors proposed a multi-component imperfect repair model that has dependent life lengths[2]. In [3] a system with several failure modes was studied. Genetic algorithm was used to find the optimal reliability of the system. The reliability of a system undergoing *f* failure modes were considered with and multiple repair facility [4]. While presenting a Markov model for the transient analysis of reliability of *k*-out-of-*N*: general repair systems with and without repair; *M* identical and independent components in the system were assumed.

The evaluated of the performance of a production line with unreliable machines with finite buffer was conducted [5]. Two machines were analyzed where each machine could fail in multiple failure modes. An algorithm for finding optimal series parallel topology for a system with two failure modes was presented in [6]. In this case, times to failure were assumed to be geometrically distributed. An investigation of the performance evaluation of production lines with deterministic processing times, multiple failure modes, and finite buffer capacity was conducted in [7]. In [8], a series system with multiple repairable components was considered in which only the failing component is repaired. A review on statistical methods for repairable systems was provided by [9]. Reliability measures for multi-state systems with two modes of failure for the case a flotation circuit of mineral processing plant was presented and discussed in [10]. A point process framework was depicted in [11] for modeling imperfect repair in a competing risks situation between failure and preventive maintenance

A reliable surrogate-based approach was developed in [12] for addressing the accuracy, efficiency and unimportant failure modes of a system simultaneously. The authors in [13] investigated the optimal replacement problem for a system with two types of failures; one repairable, and the other is un-repairable. The expressions for the limiting availability and the long-run average cost rate were derived for the optimal replacement policy *N*. Two maintenance polices were discussed in [14] for a single unit repairable systems undergoing degradation and shocks subject to competing and dependent failures. In [15] attempts were made to derive the static unavailability by using the micro-Markov models for the *k*-out-of-*n*: G systems subjected to multiple failure modes. The article [16] is concerned with the performance of a machine repair problem with mixed standbys, undergoing two modes of failure.

In this paper, Runge-Kutta method was used to obtain the transient state probabilities. The objective was to find machine availability, expected number of failed units, and the total expected cost per unit time. In this paper the behaviour of a system subjected to two types of failure modes with the objective of finding a proper maintenance policy. The two failure modes are divided into maintainable and non-maintainable. A mathematical model was constructed to identify the relationship and to exhibit the advantages of imperfect maintenance in maintenance decision making [17]. A system undergoing several dependent failure modes is addressed in [18]. Linear regression was utilized for establishing the relationship between ultimate state functions in different failure. The derived reliability is compared with the one obtained using Monte Carlo simulation. Authors in [19] presented general method to analyze the failure profile for a single repairable production system. Because of the complexity of the system with multiple failure modes, the intensity functions are analyzed using parametric models. Additionally two hypothesis testing procedures are developed to test whether dominating failure mode exists during a particular time interval and whether the failure profiles in two different time intervals are significantly different.

This paper is organized into several sections. Next section discusses the imperfect repair process and the main assumptions used to derive the various state probabilities and the availability of the system. Section III presents and explains the concept of multiple repair, while section IV provides a detailed approach for deriving the different parameters of the system. The last section deals with the conclusion and closing remarks.

II. Imperfect Repairs

In this work, we examine the performance of a repairable systems exposed to failure. After each failure, it is imperfectly repaired resulting in a stepwise increasing failure rate of the process. Thus imperfect repairs can be taken in order to approximate a continuous degradation of the process combined with a stepwise improvement of the process after each repair. With imperfect repairs, the failure rate increases after each repair, while the repair rate decreases over time. However, after a specified number of repairs, the system is replaced by a new one. This is a renewal process and hence a new cycle will start. It is worth mentioning that in this analysis, all the derived models are based on the assumption that the time between failures and repair times are randomly and exponentially distributed with failure rates λ_i and repair rates μ_i, respectively. Moreover, it is assumed that the travel times between the operation and repair facility to be negligible; the existence of a single repair channel; and that after each replacement, the system becomes as good as new. The steady state availability is used as a performance measure.

III. Multiple Modes of Failure

This system in this analysis is assumed to undergo several modes of failure. In fact, many real world systems behave accordingly. Electrical systems failures, for example may be attributed to open circuit or short circuit failures, while, automobile failures may be due to malfunctions in the transmission system, the engine, or break system. Also you might have bearing, seal, gear and other failures within the transmission system. The possible ways through which a piping system could fail include pipe rupture, pipe clogging, and pipe leakage which is an example of several modes of failure for a system.

IV. Analysis of the System

This paper assumes that the system can fail in one of M different modes; a failure is due to mode m occurs with probability p_m, $m = 1, M$. The sum of the probabilities p_m equals one.

$$\sum_{m=1}^{M} p_m = 1 \tag{1}$$

The following terminologies are used:

λ_i: actual failure rate after exactly $i - 1$ failures without replacement.

$\mu_{i,m}$: repair rate of i^{th} repair and effective failure mode m with $i = 1, \ldots, n$ and $m = 1, \ldots, M$.

μ_{n+1}: replacement rate.

p_m: probability of failure mode m, $m = 1, \ldots, M$.

π_j: The long run probability of the system being in the operational state j, $j = 2v+1$,with $v = 0, 1, \ldots, n$

$\pi_{j,m}$: The long run probability of the system being in the failed state j due to failure mode m, $j = 2v$ with $v = 1, \ldots, n + 1$, $m = 1, \ldots, M$

k: number of times the system has been imperfectly repaired, where $k = 0$ means the system is new, $k = 1$ means the system has been repaired once, and so on.

n: number of repairs carried out on the system before a replacement by a new one.

The system states and transitions are illustrated in Figure 1. The operational states are represented by rectangles and have the odd numbers $2j + 1$, $j = 0, 1, n$. The ellipses represent the failed states, they are numbered by the even numbers and the mode of failure, i.e., $(2j + 2, m)$, $j = 0, 1, n$ and $m = 1, M$. For example, the failed state $(4, 3)$ means the system has failed the second time ($j = 2$) and the failure is due to failure mode $m = 3$. The total number of states in the system depends on both the number of repairs carried out and the number of failure modes considered. It is equal to $(n + 1)$ $(M + 1)$. The steady state Chapman-Kolomogorov equations for the system are as follows:

$$\begin{aligned}
&State1: \mu_{n+1}\sum_{m=1}^{M}\pi_{2n+2,m} = \lambda_1\pi_1 \\
&State(2i,m): \lambda_i p_m \pi_{2i-1} = \mu_{i,m}\pi_{2i,m} \quad ;m=1,\ldots,M \; and\; i=1,\ldots,n. \\
&States(2i+1): \lambda_{i+1}\pi_{2i+1} = \sum_{m=1}^{M}\mu_{i,m}\pi_{2i,m} \quad ;,i=1,\ldots,n. \\
&States(2n+2,m): \lambda_{n+1}p_m\pi_{2i+1} = \mu_{n+1}\pi_{2n+2,m} \quad m=1,\ldots,M.
\end{aligned} \tag{2}$$

From the above system of equations, closed general expressions for the steady state probabilities are derived, they are as follows:

$$\begin{aligned}
&\pi_{2j+1} = \frac{\sum_{m=1}^{M}\mu_{j,m}\pi_{2j,m}}{\lambda_{j+1}}\pi_{2j} \qquad j=1,\ldots,n \\
&\pi_{2j,m} = \frac{p_j\lambda_j}{\mu_{j,m}}\pi_{2j-1} \qquad j=1,\ldots,n \text{ and } m=1,\ldots,M \\
&\pi_{2n+2,m} = \frac{p_m\lambda_{n+1}}{\mu_{n+1}}\pi_{2n+1} \qquad m=1,\ldots,M
\end{aligned} \tag{3}$$

From the relationships in 3, the long run probabilities for the different states in terms of the initial state (1) are obtained as follows:

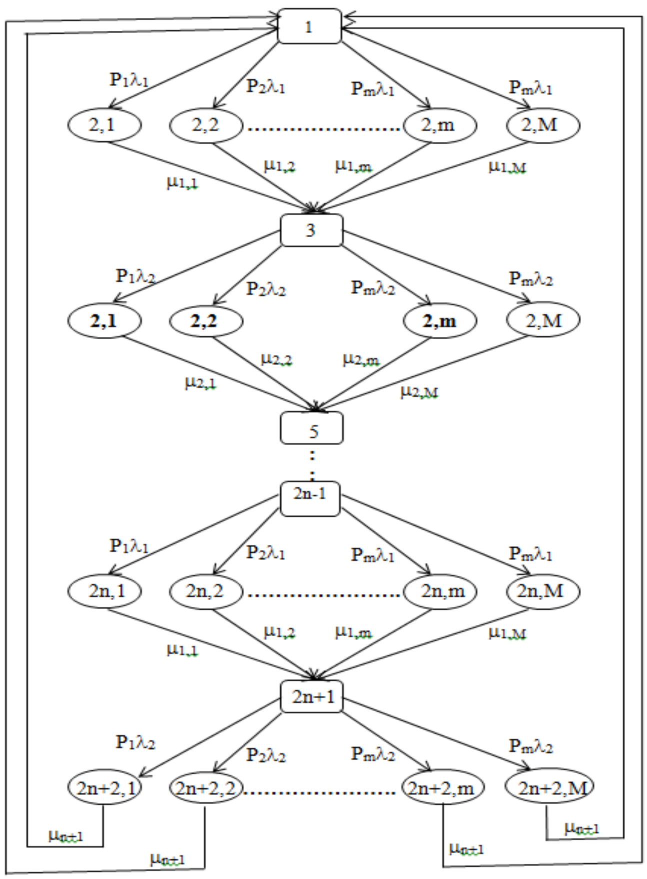

Fig. 1. A multiple component repairable system with multi- failure mode and imperfect repair.

$$\pi_{2j+1} = \frac{\lambda_1}{\lambda_{j+1}} \pi_1 \qquad j = 1, \ldots, n$$

$$\pi_{2j,m} = \frac{p_m \lambda_1}{\mu_{j,m}} \pi_1 \qquad j = 1, \ldots, n \text{ and } m = 1, \ldots, M \tag{4}$$

$$\pi_{2n+2,m} = \frac{p_m \lambda_1}{\mu_{n+1}} \pi_1 \qquad m = 1, \ldots, M$$

A closed from expression for π_1 is derived based on the above relationships as shown below

$$\pi_1 + \sum_{j=1}^{n} \sum_{m=1}^{M} \pi_{2j,m} + \sum_{j=1}^{n} \pi_{2j+1} + \sum_{m=1}^{M} \pi_{2n+2,m} = 1 \tag{5}$$

Solving the above equation for π_1 provides the following expression:

$$\pi_1 = \frac{1}{\lambda_1 \left[\sum_{m=1}^{M} p_m \sum_{j=1}^{n} \frac{1}{\mu_{j,m}} + \sum_{j=1}^{n+1} \frac{1}{\lambda_j} + \frac{1}{\mu_{n+1}} \right]} \tag{6}$$

The long run probability of being operational is given by the sum of the corresponding probabilities:

$$\sum_{j=0}^{n} \pi_{2j+1} = \left(\sum_{j=1}^{n+1} \frac{\lambda_1}{\lambda_j} \right) \pi_1 = \lambda_1 \left(\sum_{j=1}^{n+1} \frac{1}{\lambda_j} \right) \pi_1 \tag{7}$$

Substituting (7) into (6) yields the long-run operational probability for the system.

$$P_s = \frac{\left(\sum_{j=1}^{n+1} \frac{1}{\lambda_j} \right)}{\left[\sum_{m=1}^{M} \sum_{j=1}^{n} \frac{1}{\mu_{j,m}} + \sum_{j=1}^{n+1} \frac{1}{\lambda_j} + \frac{1}{\mu_{n+1}} \right]} \tag{8}$$

V. Conclusions and Future Research

Modern systems such as vehicles, aircraft, submerged vessels, electrical equipment; pipes among others exhibit multiple failure modes and hence acquire maintenance and repair. Most published literature in this area is based on perfect repair. This research work attempts to examine the real behavior of repairable systems through multiple failure and imperfect repair. Hence a general expression for the performance measure is derived which can be applied to derive expressions for systems exposed to all modes of failures.

For such systems, analysis becomes more tedious. Moreover, by increasing number of number of repairs and failure modes, the number of parameters describing the performance of the system rises, which in turn makes the system more complex and hard to analyze. Future research should address the behavior of such systems under different failure time and repair time distribution

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BIOGRAPHY

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