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Estimating sporadic change point in the mean of polynomial profiles

Mona Ayoubi[1], *Reza Baradaran Kazemzadeh*[2]
Industrial Engineering Department
Tarbiat Modares University
Tehran, Iran
[1] mona.ayoubi@modares.ac.ir, [2] rkazem@modares.ac.ir

Abstract- *Identifying the actual time of a change brings a decrease in time range of searching for assignable causes leading to less cost. In this paper, the maximum likelihood approach is developed to estimate sporadic change point for the mean of a polynomial profile in Phase II which has not been performed yet in the literature. Estimation of the process parameters for the samples after the change point is carried out using filtering and smoothing estimation methods of dynamic linear models. The proposed procedures are applied after receiving an out-of-control signal from* T^2 *control chart. The performance of the proposed change point estimators is also compared to the step and drift estimators' performance under sporadic change in the process mean. Simulation results confirm the effectiveness of the proposed methods in estimating sporadic change point.*

Keywords—change point estimation; sporadic change; maximum likelihood estimator (MLE); dynamic linear model (DLM); filtering; smoothing; polynomial profiles; statistical process control.

I. INTRODUCTION

Control charts are very effective tools to monitor the process or product quality. But, when the control charts issue an out-of-control signal is not the actual time of the change and the process has shifted at an unknown time before the signal. Hence, change point estimation can help quality practitioners to search for assignable causes in a narrower time range with less cost. Some methods have been suggested for estimating the process change point which are EWMA and CUSUM built-in estimators, maximum likelihood estimator (MLE), clustering and artificial neural networks (ANN). Page [1] and Nishina [2] introduced CUSUM and EWMA built-in estimators, respectively. Samuel et al. [3, 4], Pignatiello and Samuel [5] and Noorossana et al. [6] concentrated on step change point estimation using MLE. Estimating drift change point using MLE was also considered by Perry and Pignatiello [7] and Perry et al. [8]. Perry et al. [9] and Noorossana and Shadman [10] developed MLE to detect monotonic change point. Amiri and Khosravi [11] and Niaki and Khedmati [12] concentrated on estimating monotonic change point estimation of high-yield processes. Ghazanfari et al. [13] and Alaeddini et al. [14] introduced a clustering approach for estimating step change point. Kazemi et al. [15] used fuzzy statistical clustering method to estimate drift change point. Finally, artificial neural network was applied by Ahmadzadeh [16] to estimate step change point in a multivariate process. Atashgar [17] also used ANN for estimating drift change point in a bivariate process. Amiri and Allahyari [18] comprehensively reviewed the literature of the change point estimation. Change-point estimation using MLE for multivariate processes was also focused by Nedumaran and Pignatiello [19], Movaffagh and Amiri [20], Nikai and Khedmati [21], and Niaki and Khedmati [22].

All researches in the change point estimation literature had been performed with the assumption of step, drift or monotonic changes in the process parameters. But, estimating the start point of chaotic changes (sporadic change) has been recently considered by Ayoubi et al. [23, 24] on the mean of a multivariate normal process and multivariate linear profiles, respectively. They used filtering and smoothing procedures which are two estimation methods of dynamic linear models (DLM) in order to estimate the process parameters after the sporadic change point. In this paper, we focus on estimating sporadic change point in the mean of a polynomial profile using MLE. For this purpose, we apply filtering and smoothing estimation procedures to estimate profile parameters after the change point.

A regression relationship between a response variable and one or more explanatory variables entitled profile can be also defined as quality of a process or product in many situations. In the literature of profiles, some methods for monitoring simple linear, multiple linear, polynomial, multivariate and non-linear profiles in both Phases I and II have been suggested.

For simple linear profile monitoring in Phase I, Kang and Albin [25] suggested two methods of T^2 and combined MEWA and R control charts. Mahmoud and Woodall [26] proposed the global F test to check the similarity of the regression lines of all samples. Mahmoud et al. [27] also suggested a control chart based on likelihood ratio test statistic. Among the methods of simple linear profile monitoring in Phase II, the most important methods were proposed by Kang and Albin [25] and kim et al. [28]. Kang

and Albin [25] suggested using the two methods of T^2 and EWMA-R for Phase II. kim et al. [28] recommended coding the *x*-values by subtracting the mean value leading to make the estimated intercept and slope independent. Hence, the parameters can be monitored, separately. They also proposed using three separate EWMA control charts to monitor the intercept, slope and standard deviation of the regression line.

The aforementioned methods suggested to monitor simple linear profiles are the basic methods which have been developed for monitoring multiple, polynomial and multivariate profiles.

The focus of this paper is on polynomial profiles. Some authors considered monitoring polynomial profiles in both Phases I and II. For example, Kazemzadeh et al. [29] developed three methods of LRT, global F test and T^2 control chart to monitor polynomial profiles in Phase I. Zou et al. [30] proposed using an MEWMA control chart for Phase II monitoring of general linear profiles consisting polynomial profiles. Kazemzadeh et al. [31] also developed four methods to monitor polynomial profiles in Phase II which are the T^2 control chart, MEWMA, MCUSUM-χ^2 and orthogonal polynomials. Finally, Amiri et al. [32] focused on monitoring polynomial profiles in an automotive industry.

Structure of the reminder of this paper is as follows. Section II discusses the underlying in-control model and monitoring method. Section III contains description of the DLM and adapting polynomial profiles model with DLM. The filtering and smoothing estimation methods are introduced in section IV. Section V reports the simulation results, and concluding remarks are presented in the final section.

II. UNDERLYING MODEL AND MONITORNIG METHOD

Under in-control situations, the *k*th order polynomial model with fixed *x*-values is defined as bellow:

$$\begin{aligned} &y_{ij} = A_0 + A_1 x_i + A_2 x_i^2 + \ldots + A_k x_i^k + \varepsilon_{ij} \\ &i = 1,2,\ldots,n \qquad j = 1,2,\ldots \end{aligned} \tag{1}$$

where y_{ij} and x_i are the response and independent variables, respectively. Also, $\varepsilon_{ij} \sim N(0,\sigma^2)$ are error terms. The vector of model parameters is $\mathbf{u} = (A_0, A_1, \ldots, A_k)'$. The following equation is the matrix form of (1):

$$\mathbf{y}_j = \mathbf{X}.\mathbf{u} + \boldsymbol{\varepsilon}_j, \qquad for \quad j = 1,2,\ldots$$

$$or \quad \begin{bmatrix} y_{1j} \\ y_{2j} \\ \vdots \\ y_{nj} \end{bmatrix} = \begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^k \\ 1 & x_2 & x_2^2 & \cdots & x_2^k \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^k \end{bmatrix} \begin{bmatrix} A_0 \\ A_1 \\ \vdots \\ A_k \end{bmatrix} + \begin{bmatrix} \varepsilon_{1j} \\ \varepsilon_{2j} \\ \vdots \\ \varepsilon_{nj} \end{bmatrix} \tag{2}$$

In this paper, the T^2 control chart is used to monitor the process parameters. The T^2 statistic of *j*th sample for monitoring the above model parameters is as follows:

$$T_j^2 = (\hat{\mathbf{u}}_j - \mathbf{u})'\boldsymbol{\Sigma}^{-1}(\hat{\mathbf{u}}_j - \mathbf{u}), \tag{3}$$

where $\hat{\mathbf{u}}_j = (\hat{A}_{0j}, \hat{A}_{1j}, \ldots, \hat{A}_{kj})'$ is the vector of sample estimators for sample *j* with mean vector of $\mathbf{u} = (A_0, A_1, \ldots, A_k)'$ and covariance matrix of $\boldsymbol{\Sigma} = \sigma^2(\mathbf{X}'\mathbf{X})^{-1}$ in which $\mathbf{X}$ is the design matrix and σ^2 is the variance of error terms. Also, $\hat{\mathbf{u}}_j$ is calculated using the following formula:

$$\hat{\mathbf{u}}_j = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y}_j, \tag{4}$$

$\mathbf{y}_j$ is the observation vector of sample *j*. For the in-control *k*th order polynomial model, the statistic of T_j^2 follows a chi-square distribution with *k*+1degrees of freedom. Hence, the upper control limit (UCL) of the control chart is $\chi^2_{k+1,\alpha}$ where α is the specified type I error.

III. DYNAMIC LINEAR MODEL (DLM)

Special class of state space models is dynamic linear model in which the observations have normal distribution.

Dynamic regressions can be modeled by DLM which allows the regression parameters vary in each direction. This is the motivation of adapting the polynomial profiles with DLM. Because, chaotic changes (sporadic) in the mean of polynomial profiles can be also modeled by the time varying parameters. Petris et al. [33] stated that DLM consists of two equations, the first equation is the observation equation, and the second is the state equation. These equations are defined for $j \geq 1$. Hence, a prior guess about the mean vector and covariance matrix at time zero ($j = 0$) must be considered as a normal distribution of $\boldsymbol{\theta}_0 \sim MN_p(\mathbf{m}_0, \mathbf{C}_0)$. The DLM equations are as follows:

$$\begin{aligned} &\mathbf{y}_j = \mathbf{F}_j\boldsymbol{\theta}_j + \boldsymbol{\upsilon}_j, \quad \boldsymbol{\upsilon}_j \sim MN_m(\mathbf{0}, \mathbf{V}_j) \\ &\boldsymbol{\theta}_j = \mathbf{G}_j\boldsymbol{\theta}_{j-1} + \boldsymbol{\omega}_j, \quad \boldsymbol{\omega}_j \sim MN_p(\mathbf{0}, \mathbf{W}_j), \end{aligned} \tag{5}$$

In the above formula, $\mathbf{y}_j$ is a vector containing *m*-variate normal observations. $\boldsymbol{\theta}_j$ is a *p*-dimensional states vector. $\mathbf{F}_j$ is a known $m \times p$ matrix and $\mathbf{G}_j$ is a known $p \times p$ dimensional matrix. $\boldsymbol{\upsilon}_j$ and $\boldsymbol{\omega}_j$ are multivariate normal random vectors with zero mean and covariance matrices of $\mathbf{V}_j$ and $\mathbf{W}_j$, respectively.

In this paper, available data of $\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_j$ is denoted by D_j. State estimation can be carried out using filtering, smoothing or forecasting which are based on the conditional probability density of $p(\boldsymbol{\theta}_s \mid \mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_j)$. For $s = j$, filtering problem is defined. For $s < j$ smoothing problem is considered. Finally, forecasting problem is for $s > j$.

A. Dynamic polynomial profile

Dynamic polynomial regression is a special case of DLM in which $\mathbf{G}_j = \mathbf{I}_{(k+1)}$. In this paper, DLM is used to estimate profiles parameters for each sample after the change point up

to the signal time of the T^2 control chart. T is represented the signal time of the T^2 control chart. Before unknown change point, τ, the parameters are known. But, after the change point, i.e. for $j = \tau+1, \tau+2, ..., T$, the parameters vary from their in-control values and must be estimated. Hence, the DLM model is appropriate for the parameters after the change point. The DLM in (5) is adapted to dynamic polynomial profiles as bellow:

$$\begin{aligned} &\mathbf{y}_j = \mathbf{X}.\mathbf{u}_j + \mathbf{v}_j, \quad \mathbf{v}_j \sim MN_n(\mathbf{0}, \mathbf{V}_j) \\ &\mathbf{u}_j = \mathbf{u}_{j-1} + \boldsymbol{\omega}_j, \quad \boldsymbol{\omega}_j \sim MN_{(k+1)}(\mathbf{0}, \mathbf{W}_j) \\ &for \quad j = \tau+1, \tau+2, ..., T \end{aligned} \tag{6}$$

$\mathbf{y}_j$ is the $n \times 1$ observation vector. Also, $\mathbf{F}_j = \mathbf{X}$ is considered for $j = \tau+1, \tau+2, ..., T$. The matrix of $\mathbf{V}_j = \sigma^2.\mathbf{I}_n$ is considered for the observation covariance matrix. $\mathbf{W}_j$ is the states covariance matrix with $(k+1)\times(k+1)$ dimension. The prior guess is also $\mathbf{u}_\tau \sim MN_{(k+1)}(\mathbf{m}_\tau, \mathbf{C}_\tau)$.

For smoothing estimations, first filtering problem, $p(\mathbf{u}_\tau \mid \mathbf{y}_{\tau+1}, ..., \mathbf{y}_j)$ for $j = \tau+1, \tau+2, ..., T$, must be considered. Then, smoothing problem, $p(\mathbf{u}_\tau \mid \mathbf{y}_{\tau+1}, ..., \mathbf{y}_T)$ for $j = T-1, T-2, ..., \tau+1$, can be focused. This is known as forward filtering backward smoothing procedure.

In dynamic linear models, Kalman-filter and Kalman-smoother were introduced to solve filtering and smoothing problems which are described due to the notations of Petris et al. [33] following of this section.

B. Kalman-filter

A prior guess for the posterior distribution of the states vector at time τ is necessary to start the filtering. It is assumed as bellow:

$$\mathbf{u}_\tau \mid D_\tau \sim MN_{(k+1)}(\mathbf{m}_\tau, \mathbf{C}_\tau) \tag{7}$$

Then for $j \geq \tau+1$, one-step-ahead predictive density of states, given D_{j-1} has multivariate normal distribution with mean of $\mathbf{a}_j$ and covariance matrix of $\mathbf{R}_j$ as below:

$$\begin{aligned} &\mathbf{a}_j = E(\mathbf{u}_j \mid D_{j-1}) = \mathbf{G}_j \mathbf{m}_{j-1} \\ &\mathbf{R}_j = \mathrm{var}(\mathbf{u}_j \mid D_{j-1}) = \mathbf{G}_j \mathbf{C}_{j-1} \mathbf{G}'_j + \mathbf{W}_j \end{aligned} \tag{8}$$

One-step-ahead predictive density of observations, given D_{j-1} has also multivariate normal distribution with mean $\mathbf{f}_j$ and covariance matrix of $\mathbf{Q}_j$ as follows:

$$\begin{aligned} &\mathbf{f}_j = E(\mathbf{y}_j \mid D_{j-1}) = \mathbf{F}_j \mathbf{a}_j \\ &\mathbf{Q}_j = \mathrm{var}(\mathbf{y}_j \mid D_{j-1}) = \mathbf{F}_j \mathbf{R}_j \mathbf{F}'_j + \mathbf{V}_j \end{aligned} \tag{9}$$

Finally, the posterior filtering density of the states vector given D_j is multivariate normal with mean $\mathbf{m}_j$ and covariance matrix of $\mathbf{C}_j$ as below:

$$\begin{aligned} &\mathbf{m}_j = E(\mathbf{u}_j \mid D_j) = \mathbf{a}_j + \mathbf{K}_G \mathbf{e}_j \\ &\mathbf{C}_j = \mathrm{var}(\mathbf{u}_j \mid D_j) = \mathbf{R}_j - \mathbf{K}_G \mathbf{F}_j \mathbf{R}_j, \end{aligned} \tag{10}$$

where $\mathbf{e}_j = \mathbf{y}_j - \mathbf{f}_j$ is the vector of forecast errors, and $\mathbf{K}_G = \mathbf{R}_j \mathbf{F}'_j \mathbf{Q}_j^{-1}$ is the gain matrix.

It must be noted that the covariance matrices of $\mathbf{R}_j$, $\mathbf{Q}_j$ and $\mathbf{C}_j$ in (8), (9) and (10) may not be positive definite or at least positive semi definite because of numerical insability or roundoff errors. To solve this problem, an algorithm based on singular value decomposition (SVD) proposed by Wang et al. [34] is used in this paper. This algorithm was also described in Ayoubi et al. [23, 24], completely.

C. Kalman-smoother

Conditional on all available data after the change point, D_T, the state vector has backward transition probabilities. Hence, for $j = T-1, T-2, ..., \tau+1$, if $\mathbf{u}_{j+1} \mid D_T \sim MN(\mathbf{s}_{j+1}, \mathbf{S}_{j+1})$ then $\mathbf{u}_j \mid D_T \sim MN(\mathbf{s}_j, \mathbf{S}_j)$ in which we have:

$$\begin{aligned} &\mathbf{s}_j = E(\mathbf{u}_j \mid D_T) = \mathbf{m}_j + \mathbf{E}_j(\mathbf{s}_{j+1} - \mathbf{a}_{j+1}) \\ &\mathbf{S}_j = \mathrm{var}(\mathbf{u}_j \mid D_T) = \mathbf{C}_j + \mathbf{E}_j(\mathbf{S}_{j+1} - \mathbf{R}_{j+1})\mathbf{E}'_j, \end{aligned} \tag{11}$$

where $\mathbf{E}_j = \mathbf{C}_j \mathbf{G}'_{j+1} \mathbf{R}_{j+1}^{-1}$.

It must be noted again that $\mathbf{S}_j$ in the above equation must be positive definite or at least positive semi definite. Hence, the SVD-based algorithm suggested by Zhang and Li [35] and explained completely by Ayoubi et al [23, 24] is applied in this paper.

IV. Derivation of the Proposed Maximum Likelihood Estimator

Both the profile parameters and the observational covariance matrix are known in Phase II. With the assumption of occurring shifts only in the mean of polynomial profiles, variance of each observation (σ^2) and the covariance matrix of the observations ($\mathbf{V}_j = \mathbf{V} = \sigma^2.\mathbf{I}_n$) is constant for all samples of $j = 1, 2, ..., \tau, \tau+1, ..., T$. But, the profile parameters are known for the samples before the change point and unknown for the samples after the change point. The unknown parameters must be estimated. Hence, $\mathbf{u}_j = \mathbf{u}$ for $j = 1, 2, ..., \tau$, and $\mathbf{u}_j = \hat{\mathbf{u}}_j$ for $j = \tau+1, \tau+2, ..., T$.

Likelihood function having a change point for polynomial profiles is as below:

$$L(\tau,\mathbf{u}_j \mid \mathbf{y}_j,\mathbf{X}) = \prod_{j=1}^{\tau} \frac{1}{(2\pi)^{\frac{n}{2}}.|\mathbf{V}|^{\frac{1}{2}}} e^{-\frac{1}{2}(\mathbf{y}_j-\mathbf{X}.\mathbf{u})'\mathbf{V}^{-1}(\mathbf{y}_j-\mathbf{X}.\mathbf{u})}$$
$$\times \prod_{j=\tau+1}^{T} \frac{1}{(2\pi)^{\frac{n}{2}}.|\mathbf{V}|^{\frac{1}{2}}} e^{-\frac{1}{2}(\mathbf{y}_j-\mathbf{X}.\mathbf{u}_j)'\mathbf{V}^{-1}(\mathbf{y}_j-\mathbf{X}.\mathbf{u}_j)} \tag{12}$$

Hence,

$$L(\tau,\mathbf{u}_j \mid \mathbf{y}_j,\mathbf{X}) = \frac{1}{(2\pi)^{\frac{\tau n}{2}}.|\mathbf{V}|^{\frac{\tau}{2}}} e^{-\frac{1}{2}\sum_{j=1}^{\tau}(\mathbf{y}_j-\mathbf{X}.\mathbf{u})'\mathbf{V}^{-1}(\mathbf{y}_j-\mathbf{X}.\mathbf{u})}$$
$$\times \frac{1}{(2\pi)^{\frac{(T-\tau)n}{2}}.|\mathbf{V}|^{\frac{(T-\tau)}{2}}} e^{-\frac{1}{2}\sum_{j=\tau+1}^{T}(\mathbf{y}_j-\mathbf{X}.\mathbf{u}_j)'\mathbf{V}^{-1}(\mathbf{y}_j-\mathbf{X}.\mathbf{u}_j)} \tag{14}$$

By taking the natural logarithm of (14), the following equation is obtained:

$$\ln(L(\tau,\mathbf{u}_j \mid \mathbf{y}_j,\mathbf{X})) = M - \frac{1}{2}\sum_{j=1}^{\tau}(\mathbf{y}_j-\mathbf{X}.\mathbf{u})'\mathbf{V}^{-1}(\mathbf{y}_j-\mathbf{X}.\mathbf{u})$$
$$-\frac{1}{2}\sum_{j=\tau+1}^{T}(\mathbf{y}_j-\mathbf{X}.\mathbf{u}_j)'\mathbf{V}^{-1}(\mathbf{y}_j-\mathbf{X}.\mathbf{u}_j) \tag{14}$$

where M is constant, and $\mathbf{u}_j$ is unknown and must be estimated. The proposed maximum likelihood change point estimator is as follows:

$$\hat{\tau} = \underset{0\le t\le T-1}{\arg\max}\left\{\begin{array}{l} -\frac{1}{2}\sum_{j=1}^{t}(\mathbf{y}_j-\mathbf{X}.\mathbf{u})'\mathbf{V}^{-1}(\mathbf{y}_j-\mathbf{X}.\mathbf{u}) \\ -\frac{1}{2}\sum_{j=t+1}^{T}(\mathbf{y}_j-\mathbf{X}.\hat{\mathbf{u}}_j)'\mathbf{V}^{-1}(\mathbf{y}_j-\mathbf{X}.\hat{\mathbf{u}}_j) \end{array}\right\} \tag{15}$$

In this paper, we estimate $\hat{\mathbf{u}}_j$ using Kalman-filtering, $\hat{\mathbf{u}}_j = E(\mathbf{u}_j \mid D_j) = \mathbf{m}_j$ for $j = \tau+1, \tau+2, ..., T$, and Kalman-smoothing, $\hat{\mathbf{u}}_j = E(\mathbf{u}_j \mid D_T) = \mathbf{s}_j$ for $j = T-1, T-2, ..., \tau+1$.

V. SIMULATION STUDY

This section investigates the performance of the proposed estimators under sporadic change in the mean of polynomial profiles. Also, the step and drift estimators proposed by Kazemzadeh et al. [36] can be also used for polynomial profiles. They proposed the step estimator as bellow:

$$\hat{\tau} = \underset{0\le t\le T-1}{\arg\max}\left\{\begin{array}{l} -\frac{1}{2}\sum_{j=1}^{t}(\mathbf{y}_j-\mathbf{X}.\mathbf{u})'\mathbf{V}^{-1}(\mathbf{y}_j-\mathbf{X}.\mathbf{u}) \\ -\frac{1}{2}\sum_{j=t+1}^{T}(\mathbf{y}_j-\mathbf{X}.\hat{\mathbf{u}}_1)'\mathbf{V}^{-1}(\mathbf{y}_j-\mathbf{X}.\hat{\mathbf{u}}_1) \end{array}\right\} \tag{16}$$

In the above equation, $\hat{\mathbf{u}}_1 = \frac{(\mathbf{X}'\mathbf{X})^{-1}\left[\sum_{j=t+1}^{T}\mathbf{X}'\mathbf{y}_j\right]}{(T-t)}$. Also the drift estimator is:

$$\hat{\tau} = \underset{0\le t\le T-1}{\arg\max}\left\{\begin{array}{l} -\frac{1}{2}\sum_{j=1}^{t}(\mathbf{y}_j-\mathbf{X}.\mathbf{u})'\mathbf{V}^{-1}(\mathbf{y}_j-\mathbf{X}.\mathbf{u}) \\ -\frac{1}{2}\sum_{j=t+1}^{T}(\mathbf{y}_j-\mathbf{X}.\mathbf{u}-(j-t)\mathbf{X}\hat{\mathbf{k}})'\mathbf{V}^{-1}(\mathbf{y}_j-\mathbf{X}.\mathbf{u}-(j-t)\mathbf{X}\hat{\mathbf{k}}) \end{array}\right\}, \tag{17}$$

where $\hat{\mathbf{K}} = \frac{(\mathbf{X}'\mathbf{X})^{-1}\left[\sum_{j=t+1}^{T}(j-t)\mathbf{X}'(\mathbf{y}_j-\mathbf{X}.\mathbf{u})\right]}{\sum_{j=t+1}^{T}(j-t)^2}$.

Hence, this section also concentrates on the comparisons of the proposed sporadic estimators to the step and drift estimators' performance under sporadic changes in the mean of polynomial profiles. Performance assessments are carried out using 5000 simulation runs. In each run, the change point estimators are applied after receiving an out-of-control signal from the T^2 control chart. The overall in-control ARL of 200 ($\alpha = 0.005$) is considered for the monitoring method which is consistent with the profile literature. We consider $\tau = 25$ for the simulations. To deal with false alarms before $\tau = 25$, the out-of-control sample is omitted and substituted with an in-control sample. This procedure can be used for the Shewhart-type control charts and is repeated in this paper to obtain 25 in-control samples.

For the dynamic polynomial profiles the matrix of $\mathbf{G}_j = \mathbf{I}_{(k+1)}$ and $\mathbf{W}_j = 10^{-6}.\mathbf{I}_{(k+1)}$ are assumed.

The underlying in-control model with the fixed x-values of $x = 1, 2, ..., 10$ is considered as (18). To reduce the effect of multicolinearity in polynomial profiles, the average of x-values is subtracted from each x. Hence, the fixed x-values are -4.5, -3.5, -2.5, -1.5, -0.5, 0.5, 1.5, 2.5, 3.5 and 4.5. Therefore, the matrix of $\mathbf{X}$ is equal to $\mathbf{X} = \begin{bmatrix} 1 & -4.5 & 20.25 \\ 1 & -3.5 & 12.25 \\ \vdots & \vdots & \vdots \\ 1 & 3.5 & 12.25 \\ 1 & 4.5 & 20.25 \end{bmatrix}$.

$$y_{ij} = 3 + 2x_i + x_i^2 + \varepsilon_{ij}$$
$$i = 1, 2, ..., 10 \qquad j = 1, 2, ... \tag{18}$$

The UCL of the T^2 control chart is $\chi^2_{3,0.005} = 12.8382$ to have an in-control ARL of 200. For the prior guess of the state, the known mean vector and covariance matrix of the profile parameters in Phase II are considered. Thus, $\mathbf{m}_\tau = \mathbf{u} = (3, 2, 1)'$ and $\mathbf{C}_\tau = \sigma^2.(\mathbf{X}'\mathbf{X})^{-1}$. Also the value of $\sigma^2 = 1$ is assumed. To impose the sporadic shifts to the parameters the following chaotic changes are considered:

$$\mathbf{u}_j = \mathbf{u} - \mathbf{k}_1, \quad for \quad j = \tau_1+1, ..., \tau_1+10$$
$$\mathbf{u}_j = \mathbf{u}_{(\tau_1+10)} + (j-\tau_2).\mathbf{k}_2, \quad for \quad j = \tau_2+1, ..., \tau_2+10 \tag{19}$$
$$\mathbf{u}_j = \mathbf{u}_{(\tau_2+10)} - \mathbf{k}_3, \quad for \quad j = \tau_3+1, ..., T.$$

The above changes consist of a decreasing step, an increasing drift and a decreasing step changes. $\tau_1 = 10$, $\tau_2 = 20$ and $\tau_3 = 30$ is considered. Also, $\mathbf{k}_1$ and $\mathbf{k}_3$ are vectors containing the magnitudes of step change in each parameter. $\mathbf{k}_2$ is a vector consisting of magnitudes of the linear drift slopes in the parameters. Dimensions of $\mathbf{k}_1$, $\mathbf{k}_2$ and $\mathbf{k}_3$ are the same as $\mathbf{u}$ dimension. Another sporadic change in the parameters considered in this paper is as bellow:

$$\begin{aligned} \mathbf{u}_j &= \mathbf{u} + \mathbf{k}_1, \quad for \quad j = \tau_1 + 1, \ldots, \tau_1 + 10 \\ \mathbf{u}_j &= \mathbf{u} + \mathbf{k}_2, \quad for \quad j = \tau_2 + 1, \ldots, T. \end{aligned} \tag{20}$$

Each element of $\mathbf{k}_1$ and $\mathbf{k}_2$ in (20) is a normal random variable. Simulations results are reported in Tables I and II.

Table I represents the performance of the change point estimators under sporadic shifts defined in (19), and Table II reports the performance of the estimators under sporadic shifts defined in (20).

TABLE I. ACCURACY AND PRECISION PERFORMANCES OF THE STEP, DRIFT AND PROPOSED MLES UNDER SPORADIC CHANGES IN THE PARAMETERS. $\tau_1 = 10$, $\tau_2 = 20$ AND $\tau_3 = 30$, $N = 5000$ REPLICATIONS.

$\mathbf{k}_1 = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}, \mathbf{k}_2 = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}$, $\mathbf{k}_3 = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}$			$\begin{bmatrix} 0.03 \\ 0.03 \\ 0.03 \end{bmatrix}, \begin{bmatrix} 0.001 \\ 0.001 \\ 0.001 \end{bmatrix}, \begin{bmatrix} 0.01 \\ 0.01 \\ 0.01 \end{bmatrix}$	$\begin{bmatrix} 0.05 \\ 0.05 \\ 0.05 \end{bmatrix}, \begin{bmatrix} 0.002 \\ 0.002 \\ 0.002 \end{bmatrix}, \begin{bmatrix} 0.01 \\ 0.01 \\ 0.01 \end{bmatrix}$	$\begin{bmatrix} 0.05 \\ 0.05 \\ 0.05 \end{bmatrix}, \begin{bmatrix} 0.002 \\ 0.002 \\ 0.002 \end{bmatrix}, \begin{bmatrix} 0.025 \\ 0.025 \\ 0.025 \end{bmatrix}$	$\begin{bmatrix} 0.075 \\ 0.075 \\ 0.075 \end{bmatrix}, \begin{bmatrix} 0.003 \\ 0.003 \\ 0.003 \end{bmatrix}, \begin{bmatrix} 0.03 \\ 0.03 \\ 0.03 \end{bmatrix}$	$\begin{bmatrix} 0.09 \\ 0.09 \\ 0.09 \end{bmatrix}, \begin{bmatrix} 0.005 \\ 0.005 \\ 0.005 \end{bmatrix}, \begin{bmatrix} 0.03 \\ 0.03 \\ 0.03 \end{bmatrix}$
Accuracy Performances	$\hat{ARL} = E(T) - \tau_1$		59.3032	39.1698	27.2582	22.902	23.534
	$\bar{\hat{\tau}}_{Step}$		10.392 (22.8588)	9.5896 (3.0604)	9.8206 (2.615)	9.8256 (0.658)	9.8158 (0.525)
	$\bar{\hat{\tau}}_{Drift}$		1.329 (98.6618)	0.396 (93.552)	1.0164 (83.8648)	0.759 (87.0634)	0.3864 (93.2492)
	$\bar{\hat{\tau}}_{proposed\ filtering}$		9.426 (14.8876)	9.4336 (4.2104)	9.5462 (3.8974)	9.748 (1.1228)	9.7276 (0.8832)
	$\bar{\hat{\tau}}_{proposed\ smoothing}$		9.7614 (16.665)	9.5478 (3.261)	9.7254 (2.841)	9.804 (0.7104)	9.8082 (0.5458)
Precision Performances	$\hat{P}_0$	Step	0.3194	0.5584	0.5848	0.7812	0.8408
		Drift	0.002	0	0	0	0
		Proposed (Filtering)	0.2894	0.54	0.5564	0.7626	0.8024
		Proposed (Smoothing)	0.3196	0.5538	0.5814	0.7802	0.8382
	$\hat{P}_1$	Step	0.5588	0.8006	0.8166	0.9396	0.9556
		Drift	0.0072	0	0.001	0	0
		Proposed (Filtering)	0.5208	0.7748	0.7778	0.92	0.9326
		Proposed (Smoothing)	0.5602	0.7966	0.812	0.9366	0.9542
	$\hat{P}_3$	Step	0.767	0.9406	0.948	0.992	0.9926
		Drift	0.0288	0.0036	0.013	0	0
		Proposed (Filtering)	0.7404	0.9142	0.92	0.9786	0.9836
		Proposed (Smoothing)	0.7678	0.9366	0.943	0.9904	0.9924
	$\hat{P}_5$	Step	0.8594	0.977	0.9848	0.9978	0.9982
		Drift	0.0614	0.0202	0.065	0.019	0.005
		Proposed (Filtering)	0.853	0.9622	0.9668	0.993	0.9948
		Proposed (Smoothing)	0.8616	0.9744	0.9804	0.9972	0.998
	$\hat{P}_7$	Step	0.9152	0.991	0.9942	0.999	0.9994
		Drift	0.1066	0.0702	0.1794	0.126	0.0514
		Proposed (Filtering)	0.9166	0.9826	0.984	0.9978	0.999
		Proposed (Smoothing)	0.9214	0.9896	0.991	0.9988	0.9994
	$\hat{P}_{10}$	Step	0.9642	1	0.9994	1	1
		Drift	0.9798	1	0.9998	1	1
		Proposed (Filtering)	0.9906	1	1	1	1
		Proposed (Smoothing)	0.9806	1	0.9998	1	1

TABLE II. ACCURACY AND PRECISION PERFORMANCES OF THE STEP, DRIFT AND PROPOSED MLES UNDER SPORADIC CHANGES IN THE PARAMETERS. $\tau_1 = 10$ AND $\tau_2 = 20$, $N = 5000$ REPLICATIONS.

	$\mathbf{k}_1 = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}$,		$\begin{bmatrix} N(0.15,10^{-4}) \\ N(0.02,10^{-4}) \\ N(0.06,10^{-4}) \end{bmatrix}$,	$\begin{bmatrix} N(0.15,10^{-4}) \\ N(0.02,10^{-4}) \\ N(0.06,10^{-4}) \end{bmatrix}$,	$\begin{bmatrix} N(0.15,10^{-4}) \\ N(0.02,10^{-4}) \\ N(0.06,10^{-4}) \end{bmatrix} \times 1.2$,	$\begin{bmatrix} N(0.15,10^{-4}) \\ N(0.02,10^{-4}) \\ N(0.06,10^{-4}) \end{bmatrix} \times 1.4$,	$\begin{bmatrix} N(0.15,10^{-4}) \\ N(0.02,10^{-4}) \\ N(0.06,10^{-4}) \end{bmatrix} \times 1.5$,
	$\mathbf{k}_2 = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}$		$\begin{bmatrix} N(0.15,10^{-2}) \\ N(0.02,10^{-4}) \\ N(0.06,10^{-2}) \end{bmatrix}$	$\begin{bmatrix} N(0.15,10^{-2}) \\ N(0.02,10^{-4}) \\ N(0.06,10^{-2}) \end{bmatrix} \times 1.4$	$\begin{bmatrix} N(0.15,10^{-2}) \\ N(0.02,10^{-4}) \\ N(0.06,10^{-2}) \end{bmatrix} \times 1.6$	$\begin{bmatrix} N(0.15,10^{-2}) \\ N(0.02,10^{-4}) \\ N(0.06,10^{-2}) \end{bmatrix} \times 2$	$\begin{bmatrix} N(0.15,10^{-2}) \\ N(0.02,10^{-4}) \\ N(0.06,10^{-2}) \end{bmatrix} \times 2.3$
Accuracy Performances	$A\hat{R}L = E(T) - \tau_1$		15.1524	13.2666	12.2222	11	11.0892
	$\bar{\hat{\tau}}_{Step}$		9.985 (1.9794)	17.6504 (82.8684)	15.5548 (62.7384)	12.892 (28.742)	9.903 (0.2942)
	$\bar{\hat{\tau}}_{Drift}$		4.3166 (40.3866)	18.0728 (93.522)	15.8552 (92.544)	9.255 (16.6418)	5.6588 (22.556)
	$\bar{\hat{\tau}}_{\substack{proposed \\ filtering}}$		16.526 (79.138)	13.9172 (58.4372)	11.9154 (37.2862)	10.0932 (1.1072)	9.8272 (0.5124)
	$\bar{\hat{\tau}}_{\substack{proposed \\ smoothing}}$		9.9116 (2.1832)	16.5108 (73.1136)	14.1966 (51.3334)	10.2308 (2.0144)	9.8866 (0.3194)
Precision Performances	$\hat{P}_0$	Step	0.6206	0.0948	0.2204	0.5794	0.8658
		Drift	0.0098	0.0002	0	0.0668	0.0004
		Proposed (Filtering)	0.101	0.1736	0.3268	0.8006	0.8438
		Proposed (Smoothing)	0.6194	0.1348	0.2778	0.7878	0.8616
	$\hat{P}_1$	Step	0.8488	0.1444	0.3106	0.6854	0.9716
		Drift	0.0514	0.001	0.0006	0.3388	0.0236
		Proposed (Filtering)	0.161	0.2728	0.462	0.9448	0.9586
		Proposed (Smoothing)	0.8464	0.204	0.392	0.9358	0.9688
	$\hat{P}_3$	Step	0.9624	0.1808	0.3668	0.7124	0.9982
		Drift	0.2654	0.005	0.009	0.7468	0.3852
		Proposed (Filtering)	0.2096	0.3588	0.5694	0.9888	0.994
		Proposed (Smoothing)	0.9596	0.2556	0.4638	0.978	0.9978
	$\hat{P}_5$	Step	0.9892	0.1968	0.3854	0.7152	0.9988
		Drift	0.5094	0.0164	0.0352	0.8596	0.7626
		Proposed (Filtering)	0.2384	0.4046	0.6164	0.9926	0.9972
		Proposed (Smoothing)	0.986	0.279	0.4878	0.9834	0.9986
	$\hat{P}_7$	Step	0.9964	0.2066	0.396	0.7158	1
		Drift	0.7048	0.043	0.078	0.882	0.9266
		Proposed (Filtering)	0.2604	0.4442	0.6544	0.9938	0.9992
		Proposed (Smoothing)	0.9942	0.294	0.5054	0.9852	1
	$\hat{P}_{10}$	Step	0.9998	0.8998	0.9424	1	1
		Drift	1	0.9732	0.985	1	1
		Proposed (Filtering)	0.8712	0.9694	0.9892	1	1
		Proposed (Smoothing)	1	0.9432	0.9768	1	1

Each table is divided into two segments of accuracy and precision of the change point estimators. In the accuracy section, the mean square errors of the estimators are appeared in the parentheses. In the precision section, $\hat{P}_0 = \hat{p}(|\hat{\tau} - \tau| = 0)$, $\hat{P}_1 = \hat{p}(|\hat{\tau} - \tau| \le 1)$, $\hat{P}_3 = \hat{p}(|\hat{\tau} - \tau| \le 3)$, $\hat{P}_5 = \hat{p}(|\hat{\tau} - \tau| \le 5)$, $\hat{P}_7 = \hat{p}(|\hat{\tau} - \tau| \le 7)$ and $\hat{P}_{10} = \hat{p}(|\hat{\tau} - \tau| \le 10)$ show precisions of the three change point estimators. $E(T)$ is also the average time at which the T^2 control chart issues an out-of-control signal. Hence, the average run length is calculated using the formula of $A\hat{R}L = E(T) - \tau_1$.

Results of Tables I and II show that the proposed change point estimators have superior performance compared to the step and drift estimators in detecting sporadic change point in the mean of polynomial profiles in all range of shifts.

TABLE III. COMPARISON OF THE PROPOSED CHANGE POINT ESTIMATORS PERFORMANCE WITH THE PERFORMNACE OF THE DRIFT ESTIMATOR UNDER TWO TYPES OF SPORADIC CHANGES.

					$\hat{P}_0$			$\hat{P}_1$			$\hat{P}_3$			$\hat{P}_5$			$\hat{P}_7$			$\hat{P}_{10}$		
	$A\hat{R}L$	$\bar{\hat{\tau}}_{Drift}$	$\bar{\hat{\tau}}_{Filtering}$	$\bar{\hat{\tau}}_{Smoothing}$	Drift	Filtering	Smoothing	Drift	Filtering	Smoothing	Drift	Filtering	Smoothing	Drift	Filtering	Smoothing	Drift	Filtering	Smoothing	Drift	Filtering	Smoothing
$\mathbf{k}_1 = \begin{bmatrix} 0.01 \\ 0.01 \\ 0.01 \end{bmatrix}, \mathbf{k}_2 = \begin{bmatrix} 0.00125 \\ 0.00125 \\ 0.00125 \end{bmatrix}, \mathbf{k}_3 = \begin{bmatrix} 0.0375 \\ 0.0125 \\ 0.0375 \end{bmatrix}$	48.692	16.0616 (207.6352)	22.988 (276.7746)	26.9582 (355.0342)	0.0194	0.0252	0.0136	0.0512	0.0654	0.0398	0.1156	0.1366	0.08	0.1752	0.1926	0.1084	0.237	0.2456	0.131	0.5478	0.3298	0.1662
$\mathbf{k}_1 = \begin{bmatrix} 0.05 \\ 0.05 \\ 0.05 \end{bmatrix}, \mathbf{k}_2 = \begin{bmatrix} 0.002 \\ 0.002 \\ 0.002 \end{bmatrix}, \mathbf{k}_3 = \begin{bmatrix} 0.01 \\ 0.01 \\ 0.01 \end{bmatrix}$	39.2604	0.3822 (93.9362)	9.401 (4.151)	9.5246 (3.3286)	0	0.5402	0.5694	0.0002	0.7764	0.808	0.0032	0.9172	0.936	0.0212	0.9602	0.9712	0.0632	0.984	0.988	0.9998	1	1

Because, $\bar{\hat{\tau}}$ of the proposed estimators are closer to the actual change point of $\tau = 10$, and the precision performances are desirable. Ayoubi et al. [16, 17] demonstrated that the performance of the filtering estimator is deteriorated when the number of profile parameters increases. But, we considered only a second-order polynomial profile with three mean parameters in this paper. Also, in some situations when the sporadic shifts get close to one of the existing change types (step, drift or monotonic), the corresponding estimator of that change type may perform better than the proposed estimators. But, in some other sporadic changes the proposed estimators may perform better. Table III shows the comparison of the proposed estimators and the drift estimator in which the first row shows the results based on sporadic changes in (19) in which the drift estimator has better performance than the proposed estimators. The second row reports the results based on the increasing step, decreasing drift and increasing step changes in which the drift estimator does not perform well compared to the proposed estimators.

Form table III, it can be concluded that when there is no knowledge about the type of sporadic changes and the magnitudes of shifts, we do not know which of the existing estimators can perform better than the proposed estimators. Hence, the best choice is applying the proposed estimators to estimate sporadic change point.

VI. CONCLUSIONS

In this paper, the polynomial profile model was adapted to dynamic linear model to consider time-varying parameters. The model with time-varying parameters can include changes in each direction. The main contribution of this paper is that we focused on estimating sporadic change point in the mean of polynomial profiles which had not been performed previously. Hence, we applied filtering and smoothing estimation methods of dynamic linear models to estimate profile parameters after the change point. Simulation results showed the effectiveness of the proposed estimators to estimate sporadic changes compared to the existing step and drift estimators in terms of both accuracy and precision of the estimators. Because, $\bar{\hat{\tau}}$ of the proposed estimators are closer to the actual change point of $\tau = 10$. The precision performance is better because of less values of mean square error.

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Reza Baradaran Kazemzadeh is an Associate Professor in Tarbiat Modares University. He received a BS degeree in Operations Research and an MS degeree in Industrial Engineering. He holds a PhD degree in Technology Management from the University of Bradford. His research interests are quality engineering and management, profile monitoring, applied multivariate statistics, and operations management. His publications have appeared in Quality & Quantity, European Journal of Operational Research, Communications in Statistics, and the International Journal of Advanced Manufacturing Technology.

Mona Ayoubi is currently a PhD candidate of Industrial Engineering at Tarbiat Modares University. She holds BS and MS degrees in Industrial Engineering. Her research interests include statistical quality control, profile monitoring and change point estimation. Her Publications have appeared in Communications in Statistics, the International Journal of Advanced Manufacturing Technology, and Quality & Reliability Engineering International.