

Jobshop Scheduling for Skill-Dependent Make-to-Order System

Nasr Al-Hinai

Department of Mechanical and Industrial Engineering
Sultan Qaboos University
Muscat, Sultanate of Oman
nhinai@squ.edu.om

Sujan Piya

Department of Mechanical and Industrial Engineering
Sultan Qaboos University
Muscat, Sultanate of Oman
sujan@squ.edu.om

Abstract— In today's competitive environment many companies are shifting their production strategy from Make-to-Stock to Make-to-order (MTO) system. The reasons are to cater product as per the need of customer and to reduce inventory holding cost. In MTO system, production starts only when order arrives to the shop floor and gets confirmed. Production planning of such system includes scheduling of orders to the production periods and allocation of workers at different work centers. Complexity in such environment arises when the time to produce products is skilled dependent and from the shop floor condition. Under such situation, efficient scheduling of orders and allocation of workers at different work center play a major role to improve system performance. This research developed a mathematical model for workers allocation problem in a job shop production environment with an objective of minimizing makespan.

Keywords— Hierarchical workforce; Job Shop Scheduling; Makespan; Make-to-Order

I. INTRODUCTION

In a manufacturing system, production process involves the use of resources from personnel to materials so that finished products are ready to be delivered according to the specified schedule. The objective will be to maximize workplace efficiency by minimizing production time and cost. Schedule for production of product depends on the production strategy that the company implements and is basically concerned with the assignment of starting and completion times for different tasks to be performed on limited resources. The aim of scheduling is to notify the production facility when to make, by which worker, and on which machine or equipment. In a make-to-order (MTO) system, production starts only when order arrives to the shop floor and gets confirmed. Till then order remains as contingent on demand [1]. Once the order gets confirmed, it will then enter the production system based on the agreed due date and schedule. The production scheduling of such system includes assignment of confirmed orders to the production periods and to the workers at different work centers. Complexity in the system arises when the operation of an order to perform next and its processing time depends on the hierarchical nature of workforce where a higher qualified worker type can substitute a lower qualified one, but not vice-versa. Under such working environment, efficient scheduling of orders and allocation of workers at different work center

play a major role to improve the performance of a system. This research aims at developing a mathematical model for Job shop scheduling problem under hierarchical worker environment with an objective of minimizing the makespan.

The rest of the paper is structured as follows. Section II explores the past research carried out in this area. Section III discusses general assumptions and constraints incorporated in this research. Section IV presents and discusses proposed mathematical model. Section V discusses the Numerical example carried out to test the efficiency of proposed mathematical model. Finally, section VI gives the concluding remarks and future research directions.

II. LITERATURE REVIEW

Job shop scheduling and hierarchical workforce planning are indeed important aspects of production planning and control. In general, both aims at satisfying certain objectives and improves the overall productivity at the shop floor. The following sub-sections will review the literatures that have been carried out in these areas.

A. Job Shop Scheduling

Job shop scheduling problem (JSP) has been attracting attention of lots of researchers' over the past several decades. Different literatures have discussed it from different aspects. Some of them have focused on the objective function, while others on the constraints and some others have put their emphasis on solving techniques.

From the point of view of objective function, [2] has discussed job shop scheduling problem to minimize multiple objectives which includes makespan, total tardiness and total machine idle time. The problem was solved using a genetic algorithm.

Some researchers have addressed this problem by considering constraints on different issues. Constraints on setup time have been tackled from two different perspectives. Some researchers have assumed it to be sequence-independent and are included in the processing of operation [3]. On the other hand, others have considered set up time to be sequence-dependent [4]. Similarly, with respect to machine constraints, [5] has assumed that no breakdown of machine occurs throughout the production period. On the other hand, [6] has assumed random machine breakdowns.

Likewise, number of solution techniques has been developed over the past years to handle JSP. Reference [7] solved JSP by combining heuristic method and combinatorial branch and bound algorithm. References [8] and [9] both have used some heuristic dispatching rules to make schedule and then compared them with tabu search method. Similarly, [10] followed a similar heuristic approaches to handle machine assignment part of JSP and then used simulated annealing technique. Reference [11] has developed a new method for solving JSP using genetic algorithm and demonstrated its efficiency by comparing it with the standard benchmark. Reference [12] integrated more strategies in the genetic framework suggested by [13] and [14] to solve JSP that led to better results.

B. Hierarchical Workforce

Due to the impact of level of skill worker possess on the efficiency of production system, many researchers have devoted their attention towards scheduling of workers. More specifically, they have discussed different algorithms and approaches to find the best economic workers schedule. In this area of research, workers are classified into various hierarchies depending on their level of education, experience, performance and/or ability to do different job. The schedule aims at properly assigning the right person to the right job at the right time, which consequently will minimize the processing time and/or the associated production costs. Many types of systems have been considered in hierarchical workforce scheduling problem. Reference [15] considered a facility that works for 7 days a week with K types of workers having workload rules imposed on them while scheduling. The objective was to reduce the labor cost subjected to workers requirements and workload constraints. [16] developed a mathematical model for single shift workforce scheduling by classifying worker into three different categories namely full time worker, limited and unlimited part time workers. [17] also considered the problem for single shift system with 7 working days. He has developed a mathematical model that solves the problem by determining the most economical size of various hierarchies of workers, assigning them working and off days in each week. [18] further extended the problem to include multiple shifts scheduling of hierarchical employees on four days or three days workweeks scenario. Reference [19] shows that integer programming approach is well suited for solving the problem studied by [15]. Reference [20] introduced the concept of compressed workweeks in the model of [21]. Reference [22] has developed mixed-integer programming model for the problem under multi-shift scheduling based on which a specialized scheduling heuristic is subsequently developed. Recently, [23] has extended the model of [20] by developing mathematical models under the assumption that the work is divisible.

To the best of author's knowledge and from the above literature review it can be noticed that despite the importance of Job shop scheduling and workforce planning on production system, no previous research has been attempted to address these issues together. Therefore, this research attempts to address the gap by developing a mathematical model that integrates these two issues. For the purpose, we have classified worker hierarchy depending on the processing time that each level of worker needs to finish given task.

III. CONSTRAINTS AND ASSUMPTIONS

Job shop scheduling problem is a well-known research problem in the area of production planning and control. However, JSP combined with hierarchal workforce allocation problem is not yet being explored by the researcher. Constraints applicable to both JSP and hierarchical workforce individually are applicable to the problem that we are going to address. These constraints are as follow:

- Constraints and assumptions related to JSP:
 - There are (n) independent jobs indexed by (i) .
 - Each job (i) has (O_i) operations, and operations' sequence is given by (O_{ij}) .
 - There are (m) machines indexed by (k) .
 - The processing time of an operation (O_{ij}) on machine (k) is predefined and given by (t_{ijk}) .
 - No preemption is allowed.
 - Each machine can process at most one operation of an order at any time.
 - Precedence constraints in a job can be defined for any pair of operations. However, there is no precedence constraint among operations of different jobs.
 - Set up time of machines and movement of job from one machine to another is negligible and included in the processing time of the operations.
 - All jobs are ready to start at time zero.
 - Machines are available all the time i.e., no breakdown of machine occurs.
- Constraints and assumption related to hierarchical workforce:
 - There are S workers with different class based on the speed of their work:
 - Class A: can do the work faster than class B and C.
 - Class B: can do the work faster than class C.
 - Class C: do the work slower than A and B.
 - All classes of workers can work on all machines, but with different processing time.

Processing time of an operation by a given class of worker is deterministic and known.

IV. PROPOSED MODEL

The following mathematical model describes job shop scheduling integrated with hierarchal workforce allocation problem. The model consists of general constraints of job shop and hierarchal workforce. Table 1 shows the indices and notations that are used in the mathematical model.

TABLE I. NOTATIONS

| Indices | |
|-------------------|--|
| i | Number of job, $i=\{1,\dots,n\}$ |
| j | Number of operation, $j=\{1,\dots,h\}$ |
| k | Number of machine, $k=\{1,\dots,m\}$ |
| o | Worker type, $o=\{1,2,3\}$ |
| Input parameters | |
| P_{ijo} | Processing time that worker of type o takes to process operation j of job i on machine k |
| HW | Number of highly skill workers |
| MW | Number of semi- skill workers |
| LW | Number of low skill workers |
| M | Very big number |
| Output parameters | |
| t_{ijk} | Processing time of operation O_{ij} on machine k |
| St_{ij} | Starting time of operation O_{ij} |
| Ft_{ij} | Finishing time of operation O_{ij} |
| R_{ijk} | Starting time of machine k to process operation O_{ij} |
| Y_{ijo} | Binary variable that is equal to 1, if operation O_{ij} is processed by worker of type o ; otherwise 0. |
| X_{ijk} | Binary variable that is equal to 1 if operation O_{ij} starts before operation O_{ij} on the same machine k ; otherwise 0. |

$$\text{Min } C_{max} = Ft_{nh} \quad (1)$$

Subject to:

$$\sum_{o=1}^r Y_{ijo} = 1 \quad i = 1, \dots, n \quad j = 1, \dots, h \quad (2)$$

$$t_{ijk} = \sum_{o=1}^r Y_{ijo} P_{ijo} \quad i = 1, \dots, n \quad j = 1, \dots, h \quad k = 1, \dots, m \quad (3)$$

$$Ft_{ij} = St_{ij} + t_{ijk} \quad i = 1, \dots, n \quad j = 1, \dots, h \quad k = 1, \dots, m \quad (4)$$

$$St_{i(j+1)} \geq St_{ij} + t_{ijk} \quad i = 1, \dots, n \quad j = 1, \dots, (h-1) \quad k = 1, \dots, m \quad (5)$$

$$S_{(i+1)jk} \geq F_{ijk} \quad i = 1, \dots, (n-1) \quad j = 1, \dots, h \quad k = 1, \dots, m \quad (6)$$

$$R_{ijk} + MX_{ijk} \geq t_{ijk} \quad (7)$$

$$R_{ijk} - R_{ijk} + M(1 - X_{ijk}) \geq t_{ijk} \quad (8)$$

$$Y_{ijo} \in \{0,1\} \quad (9)$$

$$X_{ijk} \in \{0,1\} \quad (10)$$

$$P_{ijo}, t_{ijk}, St_{ij}, R_{ijk}, R_{ijk} \geq 0 \quad (11)$$

The objective function (1) aims at minimize the makespan C_{max} of the process which is equals to the completion time of

last operation h of last job n in the schedule. Constraint (2) is to ensure that each operation j of job i will be processed by one worker of type o . There are three types of workers i.e., o equal to 1 represents highly skilled worker, o equal to 2 represents semi-skilled worker and o equal to 3 represents low skilled worker. Equation 2 is a binary variable. If worker of type o is assigned to operation j (O_{ij}) of job i its value will be 1; otherwise 0. Constraint (3) states that the processing time of operation j of job i on machine k will be equal to the processing time of assigned worker for that operation. Constraint (4) ensures that the finishing time of operation j of job i equals to the summation of starting time and processing time of operation j (O_{ij}) on machine k . Constraint (5) describes the precedence relationship between the operations of the same job. It ensures that the starting time of operation $(j+1)$ from job i is to be greater than or equal the finishing time of operation j from the same job. Constraint (6) ensures that among possible consequences of doing the job, the order of a specific consequence should be taken into account. Constraints (7) and (8) are resource constraints which ensure that each machine can process at most one operation at any time. X_{ijk} equals to 1 means that operation j (O_{ij}) is processed first on machine k before operation O_{ij} . In this case constraint (8) will be neglected. On the other hand, if X_{ijk} is equal to 0 then operation O_{ij} will be processed first on machine k before O_{ij} and constraints (7) will be neglected. Constraints (9) and (10) force the decision variables to be binary. While, constraint (11) ensures that all variables are non-negative.

The formulation (1)-(11) results into very large-scale programming model whose number of binary variables and constraints depends on the number of machines, jobs, operations, workers, and hierarchical workforce levels.

V. COMPUTATIONAL RESULTS

A typical job-shop scheduling problem is represented by a set of jobs $i = \{i_1, \dots, i_n\}$ and a set of resources or machines $k = \{k_1, \dots, k_m\}$. Each job i consists of a set of operations $j_i = \{j_{i1}, \dots, j_{im}\}$. In this work, the time required to perform each operation will vary depending on the skill of the worker who will be assigned to process the operation. We are considering three levels of workers' skills: highly skilled (HS), semi-skilled (SS) and poorly skilled (PS) and the processing time of operation will on average vary from short to long based on these skills, respectively.

JSP was proven to be NP-hard problem. In our proposed model, the JSP is even harder to solve due to the addition of hierarchical workforce. Nevertheless, to test the efficiency of the proposed model a small scale test case problem with three jobs with three operations each and three machines was randomly generated. Table 2 shows the details of the problem. The first and second column in Table 2 shows the job and its operation number, respectively. The processing times of each operation by each worker type are shown in columns three to five. The last column in the table refers to the assignment of the operation to the machine. The number of workers from each skill level is set as follows: one HS worker, two SS workers and three PS workers.

ERROR! REFERENCE SOURCE NOT FOUND.PROCESSING TIMES OF TEST CASE

| Job# | Operation# | Processing Times | | | Machine# |
|------|------------|------------------|----|----|----------|
| | | HW | SW | PW | |
| 1 | 1 | 12 | 12 | 14 | 1 |
| | 2 | 8 | 8 | 12 | 2 |
| | 3 | 4 | 9 | 14 | 3 |
| 2 | 1 | 9 | 10 | 11 | 2 |
| | 2 | 4 | 9 | 13 | 3 |
| | 3 | 12 | 6 | 11 | 1 |
| 3 | 1 | 6 | 9 | 10 | 3 |
| | 2 | 5 | 7 | 13 | 1 |
| | 3 | 10 | 8 | 10 | 2 |

The test case problem was solved by implementing the mathematical model using LINGO. The makespan using the mathematical model was found to be equal to 24. To test the capability of the mathematical model to hold a larger size problem, another test case of 8 jobs 8 operations test case was attempted. However, LINGO was unable to handle this size. This finding emphasis on the importance of developing heuristic or meta-heuristic approaches to solve such NP-hard problem.

VI. CONCLUSION AND FUTUREWORK

This paper developed a mathematical model to allocate hierarchical nature of workforce to different orders at various work centers in a job shop environment. This allocation problem is integrated with order scheduling problem. The developed model helps a company to identify required number of workers with different level of hierarchies, order processing sequence, and allocation of different workers to the available work centers that minimizes makespan.

JSP is a well-known NP-hard problem. The complexity is exacerbated by integrating the issue of hierarchical workforce allocation in JSP. The developed mathematical model can be used to solve only small size problem in terms of number of jobs and level of workers hierarchy. It is necessary to develop some intelligent method such as heuristic or meta-heurist to deal with medium to large size industrial problem. Therefore, in the near future, some intelligent methods will be developed and extensive numerical analysis will be conducted to compare them with mathematical model developed in this research. Also, the present research can be extended as follows:

- In MTO system, confirmed order will have due date attached to it based on the outcome of negotiation between the customer and company. However, in the present model, neither the constraint on completion time of order, with respect to the due date, has been considered nor the effect of tardiness or earliness.
- The next avenue for the research expansion may be to consider the constraint on working hours of workers, e.g. each worker works only 8hrs per day and max 5 days a week with 2 day rest period.

ACKNOWLEDGMENT

Authors would like to extend their gratitude and appreciations to Sultan Qaboos University for providing the necessary funding and assistance to conduct this work.

REFERENCES

- [1] S. Piya, K. Takahashi, and K. Morikawa, "Quotation and capacity planning with contingent orders in make-to-order system," Proceeding of 20th International Conference on Production Research, 2009 (in CD ROM)
- [2] D. Y. Sha, and H. H. Lin, "A multi-objective PSO for job-shop scheduling problems," Expert Systems with Applications, vol. 37 (2), pp. 1065-1070, 2010.
- [3] S. Piya, K. Takahashi, and K. Morikawa, "Order acceptance and rejection decision for MTS and MTO production systems," Asia Pacific Journal of Industrial Management, vol. 1(1), pp. 24-36, 2008.
- [4] A. Manikas, and Y. L. Chang, "Multi-criteria sequence-dependent job shop scheduling using genetic algorithms," Computers and Industrial Engineering, vol. 56(1), pp. 179-185, 2009.
- [5] K. Ploydanai, and A. Mungwattana, "Algorithm for solving job shop scheduling problem based on machine availability constraint," International Journal on Computer Science and Engineering, vol. 2, pp. 1919-1925, 2010.
- [6] N. Al-Hinai, and T.Y. ElMekkawy, "Robust and stable flexible job shop scheduling with random machine breakdowns using a hybrid genetic algorithm," International Journal of Production Economics, vol. 132, pp. 279-291, 2011.
- [7] D. Applegate, and W. Cook, "A computational study of the job shop scheduling problem," ORSA Journal On Computing, vol. 02, pp. 149-157, 1991.
- [8] P. Brandimarte, "Routing and scheduling in a flexible job shop by tabu search," Annals of Operations Research, vol. 41, pp. 157-183, 1993.
- [9] J. Paulli, "A hierarchical approach for the FMS scheduling problem," European Journal of Operational Research, vol. 86(1), pp. 32-42, 1995.
- [10] B. Bona, P. Brandimarte, C. Greco, and G. Menga, "Hybrid hierarchical scheduling and control systems in manufacturing," IEEE Transactions on Robotics and Automation, vol. 6(6), pp. 673-686, 1990.
- [11] Y. Gen, Tsujimura and E. Kubota, "Solving job-shop scheduling problem using genetic algorithm," Proceedings of 16th International Conference on Computers & Industrial Engineering, pp. 576 -579, 1994.
- [12] F. Pezzella, G. Morganti, and G. Ciaschetti, "A genetic algorithm for the flexible job-shop scheduling problem," Computers & Operations Research, vol. 35, pp. 3202-3212, 2008.
- [13] I. Kacem, S. Hammadi, and P. Borne, "Approach by localization and multi-objective evolutionary optimization for flexible job-shop scheduling problems," IEEE Transactions on Systems, Man and Cybernetics, vol. 32 (1), pp. 1-13, 2002.
- [14] I. Kacem, S. Hammadi, and P. Borne, "Pareto-optimality approach for flexible job-shop scheduling problems: Hybridization of evolutionary algorithms and fuzzy logic," Mathematics and Computers in Simulation, vol. 60(3), pp. 245-276, 2002.
- [15] R. K. Y. Hung, "Hierarchical workforce scheduling," PhD thesis, Case Western Reserve University, 1990.
- [16] F. Du-Sean, "Hierarchical workforce scheduling with limited substitution as well as full-time and part-time workforce mix," PhD thesis, Case Western Reserve University, 1991.
- [17] R. Narasimhan, "An algorithm for single shift scheduling of hierarchical workforce," European Journal of Operational Research, vol. 96, pp. 113-121, 1996.
- [18] R. Narasimhan, "An algorithm for multiple shift scheduling of hierarchical workforce on four-day or three-day workweeks," INFOR, vol. 38, pp. 14-32, 2000.
- [19] A. Billonnet, "Integer programming to schedule a hierarchical workforce with variable demands," European Journal of Operational Research, vol. 114, pp. 105-114, 1999.

- [20] S. U. Seckiner, H. Gokcen and M. Kurt, "An integer programming model for hierarchical workforce scheduling problem," *European Journal of Operational Research*, vol. 183, pp. 694-699, 2007.
- [21] A. Billonnet, "Integer programming to schedule a hierarchical workforce with variable demands," *European Journal of Operational Research*, vol. 114, pp. 105-114, 1999.
- [22] S.M. Al-Yaqoob and H.D. Sherali, "Multiple shift scheduling of hierarchical workforce with multiple work centers," *Journal Informatica*, vol. 18(3), 325-342, 2007.
- [23] C. Ozguven and B. Sungur, "Integer programming models for hierarchical workforce scheduling problems including excess off-days and idle labor times," *Applied Mathematical Modelling*, vol. 37, pp. 9117-9131, 2013.

BIOGRAPHY

Nasr Al-Hinai is an Assistant Professor, in Mechanical and Industrial Engineering in Sultan Qaboos University (SQU). He earned his B.S. in Mechanical Engineering from SQU, Masters of Science in Advanced Manufacturing Technology and Systems Management from UMIST-UK, and PhD in Production Planning from University of Manitoba-Canada. His research interests include Production Planning and Control, Optimization, Meta-heuristics, Six Sigma, and Product development.

Sujan Piya is an Assistant Professor at the Department of Mechanical and Industrial Engineering, Sultan Qaboos University, Sultanat of Oman. He received his Master and Doctoral degree in Industrial and systems engineering from Hiroshima University, Japan. His research interests include production planning and control, operations and supply chain management, quotation and negotiation in MTO system.