Parcel routing and scheduling combining fixed and variable timetables

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***Abstract*— Parcels sent from origin customers to destination customers are always routed through consolidation/sorting facilities referred to as hubs. The movement of shipments from one hub to another is typically carried out either by ground or by air. Practically, all parcel distribution companies have full control of their ground vehicles (trucks and vans), which enables them to schedule their departure times in such a way to reduce the shipments' journey. In contrast, only giant carriers can afford to have their own airplanes. Most small and middle size companies, which are subject of interest in this research, have to rely on commercial airlines for their air shipping and abide by the schedules decided by these companies. We propose a mixed integer formulation for the problem that optimizes the routing of the shipments (ground versus air) and the scheduling of their movements (departure times from customers and facilities). The model takes into account the variation of the flights (as offered by the airline carrier) from day to day and from season to season while ensuring a regular pickup time for each origin customer and a regular delivery time for each destination customer.**

Keywords—Shipment, hierarchical hub and spoke, airports, routing, scheduling, timetables, planning period.

I. INTRODUCTION

Parcel distribution companies pick up shipments from origin (sending) customers and deliver them to destination (receiving) customers. The performance of these companies highly depends on their success in optimally routing and scheduling their shipments from origins to destinations. As the number of origin-destination pairs of customers is usually very large and the shipment volume for a single origin-destination pair is usually very small, a direct transport from each origin to each destination would be too costly and impractical. Parcel distribution companies, being less than truckload carriers, usually consolidate the shipments of several neighbour customers at a consolidation facility called hub and use direct transports between hubs rather than between customers. Such a transportation network is referred to as Hub and Spoke.

In many cases the shipping cost can be further reduced by introducing an intermediate facility between hubs and customers; such a facility, which is usually smaller than a hub, is referred to as station. While the parcels of neighboring customers are consolidated at the station, those of neighboring stations are consolidated at the hub.

When routed through a two-level hub and spoke networks parcels undergo five movements on their way from origins to destinations: i) from origin customer to origin station (customer-station movement), ii) from origin station to origin hub (station-hub movement), iii) from origin hub to destination hub (hub-hub movement), iv) from destination hub to destination station (hub-station movement), and v) from destination station to destination customer (station-customer movement). Usually the customer-station movements involve short distances and low volumes transported on ground vehicles (such as vans). The station-hub movements usually involve moderate distances and medium volumes transported by ground vehicles (such as vans and trucks). The hub-hub movements usually involve long distances and high volumes typically transported by air (airplanes) or ground (vans, trucks trailers,...).

Most works related to hub location problems are limited to single level network design where a customer is directly connected to one or multiple hubs ([2], [7], [8], [11], [17]). There are, however, few works that deal with hierarchical hub location problem with two levels of facilities, namely stations and hubs. Reference [22] presents two sub-problems; the first one concerns the line haul movements and the second formulates the pickup-delivery movements. Reference [4] proposes a hierarchical network design model of three levels to locate two types of parcel ground distribution facilities (station and hub) where the objective is to minimize a non-linear function of cost. Reference [16] presents the generalized hub-and-spoke network for parcel distribution companies that integrate three types of networks, namely a hub-and-spoke network without stations, a hub-and-spoke network with stations necessarily connected through the hubs, and a hub-and-spoke network involving stations that can be directly connected.

Practically, all parcel distribution companies have full control of their ground vehicles (trucks and vans), which enables them to schedule the departure time of their ground vehicles in such a way to reduce the shipments' journey. In contrast, only giant carriers can afford to have their own airplanes. Most small and middle size parcel distribution companies, which are subject of interest in this research, have

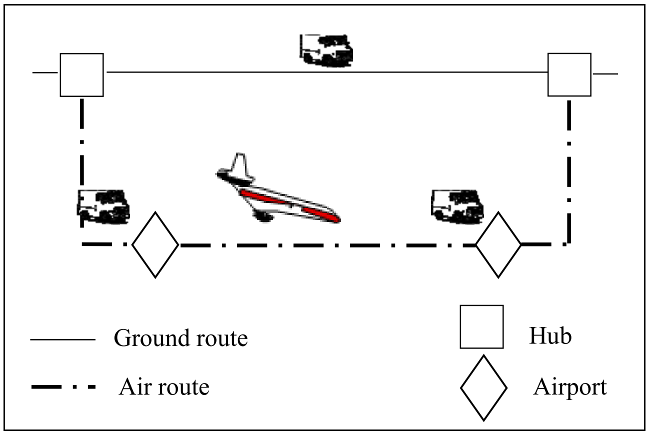

Fig. 1. The two possible hub-to-hub routes for each shipment

to rely on commercial airlines for their air shipping and abide by the schedules decided by the airline carriers. Fig. 1 shows how these companies transport their shipments either by a ground vehicle on a route called ground route, or by an airplane on a route called air route.

The selection of the appropriate transportation means between facilities has been the subject of several studies. Reference [3] considers the problem of designing airplane routes and schedules in order to pick up and deliver the shipments using a single hub. The problem involves a fleet of aircraft, a fleet of vehicles, and a set of shipments that require service. Reference [12] deals with the letter mail transportation network of the German postal service focusing on the next day delivery system. The two main transportation modes are air and ground. They determine airport and hub locations and network design service. Reference [25] develops a model and a solution algorithm to help carriers simultaneously solve for better fleet routes and appropriate timetables; the model is formulated as an integer multiple commodity network flow problem. Reference [20] studies a network composed of terminals (harbors, rail stations, airports or warehouses) and potential connections (sea routes, railways, airways, roads) and a planning horizon divided into a set of periods. Reference [21] presents a rail-truck intermodal transportation network that combines road networks with scale economies associated with railroad networks.

In this paper we are concerned with i) selecting between air and ground when routing the shipments from their origin hubs to their destination hubs and ii) determining the daily clock-time at which the shipment leaves each node of the route from its origin station to its destination station. We refer to this problem as the parcel routing and scheduling problem (PRSP).

The PRSP can be considered as service network design problem, which is concerned with operating the network generated by the network design problem; it involves mainly tactical decisions. References [9], and [10] present a state-of-the-art review of service network design modeling efforts and mathematical programming developments for network design. They focus on issues in freight transportation planning and operations. Railways and Less-Than-Truckload motor carriers are the basic examples.

Some traffic distribution problems, especially those concerning public transport, use pre-defined timetables. Reference [14] focuses on planning the multiple-leg journeys using public transport services. The repertoire of transport services may include walking, fixed-route public transport, and demand-responsive modes such as taxis. A journey-planning problem is defined by a request to travel at minimal generalized cost from a given origin to a given destination. Reference [18] proposes a model and algorithm for solving the equilibrium assignment problem in a congested, dynamic and schedule-based transit network. Reference [23] is interested in the scheduled-based public transit network, where each node of the network has a list of scheduled departure times. The authors developed an algorithm solution to find the K shortest paths in the schedule-based transit network with multiple stations. It is a route-enumeration-based algorithm.

Reference [13] presents the model called parcel distribution timetabling problem, which is the most related to ours. The authors present a model generating an effective timetable for time-definite parcel distribution companies operating a hierarchical hub-and-spoke network. More recently [5] used the modulo function to formulate clock times. The resulting nonlinear PDTP looks for the optimal departure times in order to minimize the waiting time at hubs and stations.

Until 2011 there were no works in the literature related to the parcel distribution timetabling problem, except those of [15], [24], and [19] that explicitly include the decisions concerning the arrival and departure times at each node, in addition to minimizing the time rather than the cost. However, their model has eventually some limits when applied to parcel distribution because it does not use clock-times. Reference [24] indicates that the transient times, which arise from non-simultaneous arrivals at hubs (typically spent for unloading, loading, and sorting operations), can constitute a significant portion of the total delivery time. Therefore, they studied the latest arrival hub location problem, which determines the locations of hubs, the allocation of non-hubs to hubs, and the associated routing structure between non-hubs and hubs with multiple stopovers so as to minimize the arrival time of the latest arriving cargo at destination. The model explicitly includes the decisions concerning the arrival time at intermediate stops. The objective of the model is to minimize the arrival time of the latest arriving vehicle at destinations (minmax).

This paper is organized in 4 sections with section 1 being the introduction. Section 2 defines the problem and presents its mathematical formulation. Section 3 illustrates a real world application of the model. Finally, Section 4 concludes the paper with some recommendations.

II. MATHEMATICAL FORMULATION

PRSP model aims at determining for each day of the planning period (often a season, or a year): i) the route of each shipment from its origin hub to its destination hub, ii) the clock-time at which the shipment leaves each node of the route from its origin station to its destination station. The leaving clock-times from the stations and hubs are controlled by the distribution company; the vehicle schedule in this case is referred to as *variable schedule*. However, the leaving clock-times from airports are controlled by the airline carriers; the flight schedule in this case is referred to as *fixed schedule*.

A. Sets and parameters

When modelling the PRSP, the nodes of the network represent whole cities; what we call station i actually refers to the station located at city i; what we call hub j is the hub located at city j; and what we call airport k is the airport located at city k. Each city hosting a hub must host a station. However, a city hosting a station does not necessarily host a hub. Every station services the city where it is located (we refer to this city as the main customer) in addition to several other small cities (villages) in the neighbourhood of that cities (we refer to these small cities as remote customers). The revenue and the volume generated by the remote customers is usually too small to justify the complexity resulting from their inclusion in the formulation. For this reason, we limit the customers to the main ones and omit the remote ones.

The sets I, J, and I_j are all provided by the network design, which precedes the routing and scheduling phases. Each station is assigned to exactly one hub. The set K is provided by the timetables of the airline carrier. Each airport $k_1 \in K$ has at least one regular flight. Usually, the airports of the set K are big airports located at or close to the big cities.

The time spent on each of the movements of the parcels is called *travel time*. We denote by $\delta_{n_1n_2}$ the travel time between two nodes n_1 and n_2. This applies to $\delta_{i_1j_1}, \delta_{j_1k_1}, \delta_{k_1k_2}, \delta_{k_2j_2}, \delta_{j_1j_2}, \delta_{j_2i_2}$. The travel time between the origin and the destination airports is the time elapsing from the take-off clock-time at origin airport to the landing clock-time at destination airport.

At each node, a shipment undergoes a variety of processing activities that include sorting/consolidation, scanning, weighing, manifesting, X-raying, pickup/delivery, and/or loading/unloading. We denote by : γ_n the processing time of shipment at node n. This applies to: γ_{i_1}, γ_{j_1}, γ_{k_1}, γ_{k_2}, γ_{j_2}, γ_{i_2}.

Even if we assume that all the shipments are equally important, the number of shipments varies from one pair of stations to another. The model must give higher priority to the pairs having higher volume. To reflect the differences between origin-destination pair of stations we assign to each pair (i_1, i_2) a relative economic weight denoted by $\lambda_{i_1i_2}$. These weights usually combine such dimensions as the revenue generated by the parcels between each pair of customers, the operating cost of the trip between the two customers, and the commitments of the competitors for that pair of customers. They are ideally provided by the decision maker to reflect the relative importance of each dimension to the company.

The parameter $\varphi_{k_1k_2fl}$ represents the leaving clock-time of the f^{th} trip from airport k_1 to airport k_2 on day l. This information is available in the timetables provided by the airline carrier. Moreover, pickup or delivery cannot be performed at any time; in particular it may be very inconvenient (or even unacceptable) for customers to be visited between 10:00pm and 5:00am. For delivery to start and for pickup to end at convenient times to customers, the shipments to be delivered should leave a station i no earlier than a time α_i and those collected should be in the station no later than a time β_i. We refer to $[\alpha_i, \beta_i]$ as the time-window interval for the station i.

B. Decision variables

The PRSP model determines at each day the optimal movements between each origin-destination pair of hubs. We denote by $y_{n_1n_2}^{j_1j_2l}$ the binary variable which takes 1 if the shipment originating from hub j_1 and destined to hub j_2 at day l goes on the link from node n_1 to node n_2 and 0 otherwise. $y_{n_1n_2}^{j_1j_2l}$ is one of the following: $y_{j_1j_2}^{j_1j_2l}, y_{j_1k_1}^{j_1j_2l}, y_{k_1k_2}^{j_1j_2l}, y_{k_2j_2}^{j_1j_2l}$. Between the origin-destination pair of airports (k_1, k_2) there are f flights. To select the optimal flight we denote by $y_{k_1k_2f}^{j_1j_2l}$ the binary variables which take 1 if shipment originating from hub j_1 and destined to hub j_2 at day l goes on the f^{th} trip from airport k_1 to airport k_2 and 0 otherwise.

The PRSP model determines at each day the optimal leaving clock-time of each movement. We denote by $x_n^{j_1j_2l}$ the leaving clock-time from node n of the shipment originating from hub j_1 and destined to hub j_2 on day l. $x_n^{j_1j_2l}$ is one of the following: $x_{j_1}^{j_1j_2l}, x_{k_1}^{j_1j_2l}, x_{k_2}^{j_1j_2l}$. When the leaving clock-time from node n is independent from j_1 and j_2 it is denoted by x_n^l, which applies to $x_{j_2}^l$. When the leaving clock-time from node n is independent from j_1, j_2, and l it is denoted by x_n. x_n refers to the origin station as x_{i_1} and to the destination station as x_{i_2}.

Clock-time is a value between 00:00 and 24:00, excluded; whenever it reaches 24:00, it is reset to 00:00 and the number of elapsing days is incremented by one [13]. To keep track of the number of elapsing days we denote by $z_{n_1n_2}^{j_1j_2l}$ the integer number of calendar days between the leaving clock-times from nodes on the link $n_1 n_2$; this number is related to the shipment originating from hub j_1 and destined to hub j_2 on day l. $z_{n_1n_2}^{j_1j_2l}$ is one of the following: $z_{i_1j_1}^{j_1j_2l}$, $z_{j_1k_1}^{j_1j_2l}$, $z_{k_1k_2}^{j_1j_2l}$, $z_{k_2j_2}^{j_1j_2l}$, $z_{j_1j_2}^{j_1j_2l}$. Similarly to the variables $x_n^{j_1j_2l}$, when the number of calendar days between n_1 and n_2 is independent from j_1 and j_2 the variable is denoted by $z_{n_1n_2}^l$, which applies to $z_{j_2i_2}^l$.

The optimal routes and leaving clock-times minimize the total shipping time of all shipments between all origin-destination pairs of stations. We denote by $w_{i_1i_2}$ the shipping time between the leaving clock-time from origin station i_1 and the leaving clock-time from destination station i_2 during the planning period.

C. Constraints

The constraints of the PRSP model are composed of three types: Routing, scheduling and bound constraints.

1) Routing constraints

The routing constraints ensure that each shipment, at each, day moves from its origin hub to its destination hub either directly on a ground route or indirectly on an air route via an origin-destination pair of airports. The routing constraints include the following 4 groups:

i) When the shipment moves on an air route, it goes from the origin hub j_1 to exactly one airport, which means that $\sum_{k\in K} y_{j_1k}^{j_1j_2l}=1$. However, when the shipment moves on a ground route it goes from the origin hub j_1 to the destination hub j_2, which means that the shipment wouldn't go from the origin hub j_1 to any airport, i.e. $\sum_{k\in K} y_{j_1k}^{j_1j_2l}=0$. By adding the variable $y_{j_1j_2}^{j_1j_2l}$, the two cases can be formulated as:

$$y_{j_1j_2}^{j_1j_2l}+\sum_{k\in K} y_{j_1k}^{j_1j_2l}=1 \qquad j_1,j_2\in J \quad ,l\in L \tag{1}$$

ii) When the shipment enters an origin airport k_1 it must leave it to another airport. This means if $y_{j_1k_1}^{j_1j_2l}=1$ then $\sum_{k\in K} y_{k_1k}^{j_1j_2l}=1$. However, when the shipment moves on a ground route, it doesn't enter any origin airport k_1 and it doesn't leave it to any other airport. This means, if $y_{j_1k_1}^{j_1j_2l}=0$ then $\sum_{k\in K} y_{k_1k}^{j_1j_2l}=0$. The two cases can be formulated as:

$$\sum_{k\in K} y_{k_1k}^{j_1j_2l}=y_{j_1k_1}^{j_1j_2l} \qquad j_1,j_2\in J \quad ,k_1\in K\ ,l\in L \tag{2}$$

ii) When the shipment moves from an origin airport k_1 to a destination airport k_2 it would move on one flight scheduled by the airline carrier:

$$\sum_{f\in F_{k_1k_2l}} y_{k_1k_2f}^{j_1j_2l}=y_{k_1k_2}^{j_1j_2l} \qquad j_1,j_2\in J \quad ,k_1,k_2\in K\ ,l\in L \tag{3}$$

iv) When the shipment enters a destination airport k_2 it must leave it the destination hub j_2. This means if $\sum_{k\in K} y_{kk_2}^{j_1j_2l}=1$ then $y_{k_2j_2}^{j_1j_2l}=1$. However, when the shipment moves on a ground route, it doesn't enter any destination airport k_2 and it doesn't leave it to the destination hub j_2. This means, if $\sum_{k\in K} y_{kk_2}^{j_1j_2l}=0$ then $y_{k_2j_2}^{j_1j_2l}=0$. The two cases are formulated as:

$$\sum_{k\in K} y_{kk_2}^{j_1j_2l}=y_{k_2j_2}^{j_1j_2l} \qquad j_1,j_2\in J \quad ,k_2\in K\ ,l\in L \tag{4}$$

2) Scheduling constraints

The scheduling constraints consist of: i) time precedence constraints, ii) airline schedule constraints, and iii) regularity constraints.

i) Time precedence constraints

These constraints, which represent the first component of the scheduling constraints are intended to ensure that a shipment can leave a node only after it has arrived to and has completed its processing at that node [13]. Since the leaving clock-time from any node depends on that of its predecessor these constraints must be applied to all the movements of the shipment. Their formulation from the last movement to the first (going backward) is as follows:

- The shipment can leave the destination station i_2 only after it has already left the destination hub j_2, has performed the trip from the destination hub j_2 to the destination station i_2, and has completed the processing operations at the destination station i_2:

$$x_{i_2}+24\times z_{j_2i_2}^{l}\ge x_{j_2}^{l}+\delta_{j_2i_2}+\gamma_{i_2} \qquad i_2\in I_{j_2},\ j_1,j_2\in J \quad ,l\in K \tag{5}$$

- If the shipment moves on an air route, then it can leave the destination hub j_2 only after it has already left the destination airport k_2, has arrived from the destination airport k_2 to the destination hub j_2, and has completed the processing operations at the destination hub j_2. If the shipment moves on a ground route then, it can leave the destination hub j_2 only after it has already left the origin hub j_1, has performed the trip from the origin hub j_1 to the destination hub j_2, and has completed the processing operations at the destination hub j_2. The leaving clock time from the destination hub j_2 must be determined from either the origin hub j_1 or the destination airport k_2 (not from both). We formulate this by constraints (6) and (7):

$$x_{j_2}^{j_1j_2l}+24\times z_{k_2j_2}^{j_1j_2l}\ge x_{k_2}^{j_1j_2l}+\delta_{k_2j_2}+\gamma_{j_2}+M\left(y_{k_2j_2}^{j_1j_2l}-1\right) \qquad j_1,j_2\in J\ ,\ k_2\in K\ ,l\in L \tag{6}$$

$$x_{j_2}^{j_1j_2l}+24\times z_{j_1j_2}^{j_1j_2l}\ge x_{j_1}^{j_1j_2l}+\delta_{j_1j_2}+\gamma_{j_2}+M(y_{j_1j_2}^{j_1j_2l}-1) \qquad j_1,j_2\in J\ ,\ k_1\in K\ ,l\in L \tag{7}$$

where M is a large number. When $y_{j_1j_2}^{j_1j_2l}=1$, $M\left(y_{j_1j_2}^{j_1j_2l}-1\right)=0$, which means that the constraints (7) aplly and the constraints (6) become redundant because $y_{k_2j_2}^{j_1j_2l}=0$ and $M\left(y_{k_2j_2}^{j_1j_2l}-1\right)=(-M)$; The relation between $y_{j_1j_2}^{j_1j_2l}=0$ and $y_{k_2j_2}^{j_1j_2l}$ is defined in constraints (1), (2), and (4). However, when $y_{j_1j_2}^{j_1j_2l}=0$, the constraints (7) become redundant and the constraints (6) apply because $y_{k_2j_2}^{j_1j_2l}=1$.

- The shipment can leave the destination airport k_2 only when it has already left the origin airport k_1, has performed the trip from the origin to the destination airports, and has completed the processing operations at the destination airport k_2:

$$x_{k_2}^{j_1 j_2 l} + 24 \times z_{k_1 k_2}^{j_1 j_2 l} \geq x_{k_1}^{j_1 j_2 l} + \delta_{k_1 k_2} + \gamma_{k_2} + M(y_{k_1 k_2}^{j_1 j_2 l} - 1) \qquad j_1, j_2 \in J \;, k_1, k_2 \in K, l \in L \qquad (8)$$

$y_{k_1 k_2}^{j_1 j_2 l} = 0$ when $y_{j_1 j_2}^{j_1 j_2 l} = 1$ which makes constraints (8) redundant. However, $y_{k_1 k_2}^{j_1 j_2 l} = 1$ when $y_{j_1 j_2}^{j_1 j_2 l}$ which makes constraints (8) apply. The relation between $y_{j_1 j_2}^{j_1 j_2 l}$ and $y_{k_1 k_2}^{j_1 j_2 l}$ is defined in constraints (1) and (2).

- The shipment can leave the origin airport k_1 only when it has already left the origin hub j_1, has performed the trip from the origin hub j_1 to the origin airport k_1, and has completed the processing operations at the origin airport k_1:

$$x_{k_1}^{j_1 j_2 l} + 24 \times z_{j_1 k_1}^{j_1 j_2 l} \geq x_{j_1}^{j_1 j_2 l} + \delta_{j_1 k_1} + \gamma_{k_1} + M(y_{j_1 k_1}^{j_1 j_2 l} - 1) \qquad j_1, j_2 \in J \;, k_1 \in K, l \in L \qquad (9)$$

$y_{j_1 k_1}^{j_1 j_2 l} = 0$ when $y_{j_1 j_2}^{j_1 j_2 l} = 1$; in this case constraints (9) are redundant. However, $y_{j_1 k_1}^{j_1 j_2 l} = 1$ when $y_{j_1 j_2}^{j_1 j_2 l} = 0$ and constraints (8) apply. The relation between $y_{j_1 j_2}^{j_1 j_2 l}$ and $y_{j_1 k_1}^{j_1 j_2 l}$ is defined in constraints (1).

- The shipment can leave the origin hub j_1 only when it has already left the origin station i_1, has performed the trip from the origin station i_1 to the origin hub j_1, and has completed the processing operations at the origin hub j_1:

$$x_{j_1}^{j_1 j_2 l} + 24 \times z_{i_1 j_1}^{j_1 j_2 l} \geq x_{i_1} + \delta_{i_1 j_1} + \gamma_{j_1} \qquad j_1, j_2 \in J \;, i_1 \in I_{j_1}, l \in L \qquad (10)$$

ii) Airline schedule constraints

These contraints, which represent the second component of the scheduling constraints, are intended to embed the departure clock-time from an origin airport with the flight time fixed by the airline carrier. The leaving clock-time from the origin airport k_1 should abide by the schedule decided by the airline carrier

$$x_{k_1}^{j_1 j_2 l} = \sum_{k \in K} \sum_{f \in F_{k_1 k l}} \varphi_{k_1 k f l} \, y_{k_1 k f}^{j_1 j_2 l} \qquad j_1 \in J \;, j_2 \in J \;, k_1 \in K, l \in L \qquad (11)$$

The constraints (11) apply only when the shipment moves on an air route. When the shipment is transported by ground route, constraints (11) become redundant. Indeed, when $y_{j_1 j_2}^{j_1 j_2 l} = 0$, $\sum_{k \in K} y_{j_1 k}^{j_1 j_2 l} = 1$ (see constraints (1)), $\sum_{k \in K} y_{k_1 k}^{j_1 j_2 l} = 1$ (see constraints (2)), and $\sum_{f \in F_{k_1 k l}} y_{k_1 k_2 f}^{j_1 j_2 l} = 1$ (see constraints (3)).

iii) Regularity constraints

These constraints, which are the third component of the scheduling constraints, are intended to ensure that each shipment leaves its origin and destination stations at the same time during all the planning period; i.e. the leaving time cannot vary from one day to another. This is because the distribution company must provide its customers with regular commitments for all days of the planning period. However, since the airline carrier's timetable varies from day to day and from season to season, the use of such schedule will lead to irregular delivery times. In order to enhance regular commitment the distribution company has to commit the longest time to its customers; e.g. if the shipping time between an origin-destination pair of stations is 25 hours for 300 days of the year and 30 hours for the remaining days, then the company has to make a commitment of 30 hours. The time elapsing from the departure from an origin station i_1 to the departure from a destination station i_2 on a given day l is:

$$x_{i_2} + 24 \times \left(z_{i_1 j_1}^{j_1 j_2 l} + z_{j_1 k_1}^{j_1 j_2 l} + z_{k_1 k_2}^{j_1 j_2 l} \right) + 24 \times \left(z_{k_2 j_2}^{j_1 j_2 l} + z_{j_1 j_2}^{j_1 j_2 l} + z_{j_2 i_2}^{l} \right) - x_{i_1} \qquad i_1 \in I_{j_1}, i_2 \in I_{j_2}, l \in L \;\; j_1, j_2 \in J \;, k_1, k_2 \in K$$

The longest shipping time will be:

$$w_{i_1 i_2} \geq x_{i_2} + 24 \times \left(z_{i_1 j_1}^{j_1 j_2 l} + z_{j_1 k_1}^{j_1 j_2 l} \right) + 24 \times \left(z_{k_1 k_2}^{j_1 j_2 l} + z_{k_2 j_2}^{j_1 j_2 l} + z_{j_1 j_2}^{j_1 j_2 l} + z_{j_2 i_2}^{l} \right) - x_{i_1} \qquad i_1 \in I_{j_1}, i_2 \in I_{j_2}, l \in L \;\; j_1, j_2 \in J \;, k_1, k_2 \in K \qquad (12)$$

3) Bound constraints

The x variables, which determine the leaving clock-times, have to belong to the interval $[0, 24)$. Moreover, the variables x_i should be within their time-window interval $[\alpha_i, \beta_i]$, which is necessarily included in $[0, 24)$. The binary variables y belong to the set $\{0, 1\}$. The z variables, which determine the number of calendar days, are integer variables.

$$x_{i_1} \in \left[\alpha_{i_1}, \beta_{i_1} \right] \qquad i_1 \in I \qquad (13a)$$

$$x_{i_2} \in \left[\alpha_{i_2}, \beta_{i_2} \right] \qquad i_2 \in I \qquad (13b)$$

$$x_{j_1}^{j_1 j_2 l}, x_{k_1}^{j_1 j_2 l}, x_{k_2}^{j_1 j_2 l}, x_{j_2}^{l} \in [0, 24) \qquad j_1, j_2 \in J \;, k, k_1 \in K, l \in L \qquad (13c)$$

$$y_{j_1 k_1}^{j_1 j_2 l}, y_{k_1 k_2}^{j_1 j_2 l}, y_{k_2 j_2}^{j_1 j_2 l}, y_{j_1 j_2}^{j_1 j_2 l}, y_{k_1 k_2 f}^{j_1 j_2 l} \in \{0, 1\} \qquad j_1, j_2 \in J \;, k_1, k_2 \in K, \; f \in F_{k_1 k_2 l}, l \in L \qquad (13d)$$

$$z_{i_1 j_1}^{j_1 j_2 l}, z_{j_1 k_1}^{j_1 j_2 l}, z_{k_1 k_2}^{j_1 j_2 l}, z_{k_2 j_2}^{j_1 j_2 l}, z_{j_1 j_2}^{j_1 j_2 l}, z_{j_2 i_2}^{l} \in \square \qquad i_1 \in I_{j_1}, i_2 \in I_{j_2}, j_1, j_2 \in J \;\; k, k_1 \in K, l \in L \qquad (13e)$$

D. *The objective function*

Customers are usually sensitive to cost and time, the most successful parcel distribution companies are the ones that incur the lowest shipping cost and the shortest shipping time. We define the shipping time of a shipment as the time spent by the shipment in its journey from its origin station to its destination station. In our case there is no cost minimization because the resources are allocated by the network design problem. The

PRSP rather focuses on minimizing the weighted average shipping time per origin destination pair, where $\lambda_{i_1i_2}$ is the weight of the pair of customers (i_1,i_2). The obtained objective function is: min $\sum_{i_1\in I}\sum_{i_2\in I}\lambda_{i_1i_2}w_{i_1i_2}$, where $w_{i_1i_2}$ is the shipping time between the leaving clock-time from origin station i_1 and the leaving clock-time from destination station i_2 during the planning period.

III. APPLICATION TO A REAL WORLD CASE

We apply the proposed PRSP model to a real-world case study. All the information about the studied company and data needed to solve the model are presented in the first sub-section. We present in the second sub-section the modifications made on the PRSP model. The third sub-section presents the generated solution.

A. *Data*

The following application is a real-world case related to one of the licensees of Federal Express Corporation (FedEx). This company whose identity is concealed as desired by its managers will be referred to as GSP or Global Service Participant (the name used by FedEx for its licensees). GSP is a one-thousand-employee Company and is basically a ground carrier, where a great part of its shipment is routed by ground. However, the remaining part is routed using the domestic airline carrier's fleet.

The actual network design of GSP is the one shown in Fig. 2. The network is composed of 24 stations located at the most important cities. They are denoted by $\{1,\ldots,24\}$. These 24 cities represent around 97% of the total economic weight of all cities. We assume that the number of customers is 24 and each one of them is assigned to the station hosted by the same city. The set of hubs is denoted by the first five nodes: $\{1,\ldots,5\}$. The hubs are directly connected except for the pairs (3,4), (3,5), and (4,5). Since, our model requires that all hubs to be directly connected, in Fig. 2, the missed hub-hub links are artificially added.

The shortest travel time between cities depends on such factors as the distance, the type of road, and the type of vehicle used. GSP couriers spend on average 3.5 hours a day on pickup and 2.5 hours on delivery. The operation managers of the company estimate in-station processing times to be: i) 30 minutes for outgoing shipments, which are consolidated before they are shipped, ii) one hour for incoming shipments, which are split up before they are delivered. The in-hub processing time is estimated at 30 minutes. The processing time at each airport is estimated at 2 hours.

Although there are three airline carriers in GSP country, GSP relies only on the largest one to ship by air. The other two companies, established recently, are too small to ensure any regular flights. There are 26 airports in the country; but only six of them offer regular flights. One of these airports is located at a city not hosting stations, and the remaining five are located at cities hosting stations; they are denoted by $\{1,2,5,6,10,25\}$. Between each origin-destination pair of airports, we assume the maximum number of regular flights is

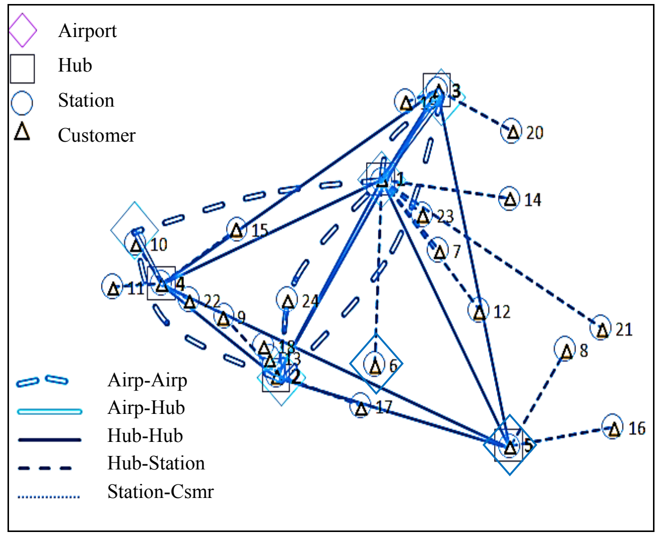

Fig. 2. Actual GSP's distribution network

2. If, for any pair of airports, there are more than 2 regular flights, then we select the two closest to midnight.The timetable provided by the airline carrier during one year is divided into 6 periods; each period encompasses regular flights. Every two periods differ by at least one flight. Since the flights are the same within each period, we assume that the number of days is equal to the number of periods, i.e. 6.

B. *Modifications on the model*

The PRSP model applied to GSP involves 34,235 variables and 113155 constraints. In this sub-section we introduce some modifications on the PRSP model in order to reduce the size of the problem and take into consideration the specifications of the real world case.

1) *Calculation of the time-window interval*

While the number of stations is equal to the number of customers we can define the time-window interval of each station according to the respective time-window interval of each customer. Let be $\left[\alpha_i^{'},\beta_i^{'}\right]$ the time-window interval of customer i and $\gamma_i^{'}$ be the processing time of customer i (pick-up time for origin customer and delivery time for destination customer). Since the customer and the station are located in the same city the travel time between them is assumed to be zero.

If i is an origin station, then:

The earliest clock-time α_i at which the shipment can leave the origin station i will be the earliest clock-time $\alpha_i^{'}$ at which it can visit the origin customer i plus the processing-time $\gamma_i^{'}$ at that customer plus the processing-time γ_i at the origin station i : $\alpha_i = \alpha_i^{'} + \gamma_i^{'} + \gamma_i$

The latest clock-time β_i at which the shipment can leave the origin station i will be the latest clock-time $\beta_i^{'}$ at which it can leave the origin customer i plus the processing-time γ_i at the origin station i : $\beta_i = \beta_i^{'} + \gamma_i$

If i is a destination station, then:

The earliest clock-time α_i at which the shipment can leave destination station i will be the earliest clock-time $\alpha_i^{'}$ at which it can visit destination customer i : $\alpha_i = \alpha_i^{'}$

The latest clock-time β_i at which the shipment can leave the destination station i will be the latest clock-time $\beta_i^{'}$ at which it can leave the origin customer i minus the processing-time $\gamma_i^{'}$ at that customer: $\beta_i = \beta_i^{'} - \gamma_i^{'}$

The time-window interval of each origin station i is $\left[\alpha_i^{'} + \gamma_i^{'} + \gamma_i , \beta_i^{'} + \gamma_i\right]$, and the time-window interval of each destination station i is $\left[\alpha_i^{'}, \beta_i^{'} - \gamma_i^{'}\right]$. Since the time-window interval of each customer is $[8,20]$, the time-window interval of each origin station is $[12,20.5]$ while the time-window interval of each destination station is $[8,17.5]$.

2) Calculation of the lower bound of the shipping time variables

The ideal route is when the shipment leaves its origin station at the end of the time-window interval (20.5 o'clock) and leaves its destination station at the beginning of its time-window interval (8 o'clock). This means that the corresponding shipping time will be the time elapsing from 21 o'clock to 8 o'clock the next day (11.5 hours). Consequently, the lower bound for any shipping time is 11.5 hours.

3) Elimination of bad air routes

The travel time of the air route is the sum of the travel time spent on its links $\delta_{j_1k_1} + \sum_{f \in F_{k_1k_2l}} \delta_{k_1k_2} y_{k_1k_2f}^{j_1j_2l} + \delta_{k_2j_2}$ plus the processing time at its nodes $\gamma_{k_1} + \sum_{f \in F_{k_1k_2l}} \gamma_{k_2} y_{k_1k_2f}^{j_1j_2l}$. For each origin-destination pair of hubs, if the travel time of the air route is greater than the travel time $\delta_{j_1j_2}$ of the ground route, then this air route is eliminated ($y_{j_1k_1}^{j_1j_2l} + y_{k_1k_2}^{j_1j_2l} + y_{k_2j_2}^{j_1j_2l} < 3$). This can be formulated as follows:

$$\begin{aligned}&\delta_{j_1k_1} + \gamma_{k_1} + \sum_{f \in F_{k_1k_2l}} \left(\delta_{k_1k_2} + \gamma_{k_2}\right) y_{k_1k_2f}^{j_1j_2l} + \\ &\delta_{k_2j_2} - \delta_{j_1j_2} \le M\left(3 - y_{j_1k_1}^{j_1j_2l} - y_{k_1k_2}^{j_1j_2l} - y_{k_2j_2}^{j_1j_2l}\right)\end{aligned} \qquad j_1, j_2 \in J, \; k_1, k_2 \in K, l \in L \tag{14}$$

4) Requirements of the GSP company

The shipments transported on ground routes between hubs 3 and 4 and between hubs 3 and 5 are routed via hub 1, and those between hubs 4 and 5 are routed via hub 2. To meet this requirement, we assume that the travel time between hub-hub link $\delta_{j_1j_2}$ is equal to the travel time $\delta_{j_1j_0}$ between origin hub j_1 and transit hub j_0, plus the processing time γ_{j_0} at transit hub j_0, plus the travel time $\delta_{j_0j_2}$ between transit hub j_0 and destination hub j_2; i.e. $\delta_{j_1j_2} = \delta_{j_1j_0} + \gamma_{j_0} + \delta_{j_0j_2}$.

In addition, to ensure that the solution provided by the PRSP model is feasible for the company, which hub network is not fully connected, we impose the following two restrictions:

- The leaving clock-time from an origin hub j_1 to a destination hub j_2 is the same as the leaving clock-time from an origin hub j_1 to its intermediate hub j_0:

$$x_{j_1j_2}^{j_1j_2l} = x_{j_1j_0}^{j_1j_2l} \qquad \begin{aligned}&\left(j_1, j_2\right) \in \{(3,4),(3,5),(4,5)\} \\ &\left(j_1, j_0\right) \in \{(3,1),(3,1),(4,2)\}\end{aligned} \tag{15}$$

- The arrival clock-time at destination hub j_2 from origin hub j_1 is the same as the arrival clock-time from transit hub j_0:

$$\begin{aligned}&x_{j_1j_2}^{j_1j_2l} + \delta_{j_1j_2} - 24 \times z_{j_1j_2}^{j_1j_2l} = &&\left(j_1, j_2\right) \in \{(3,4),(3,5),(4,5)\} \\ &x_{j_0j_2}^{j_1j_2l} + \delta_{j_0j_2} - 24 \times z_{j_0j_2}^{j_1j_2l} &&\left(j_1, j_0\right) \in \{(3,1),(3,1),(4,2)\}\end{aligned} \tag{16}$$

C. Generated solution

By solving the PRSP model on the standard software CPLEX 12.5 run on Intel(R) Core(TM) i3 CPU M380 @ 2.53GHz with 4.00GB of RAM we reach in 4 hours a feasible solution with a weighted average shipping time equal to 18.30 hours (18 hours and 18 minutes). The solution remained the same with no improvement after 6 additional hours. In this solution, there are ground and air routes. The shipments are transported between the pair of hubs, which are not directly connected, on air routes except from origin hub 5 to destination hub 4, where there are no regular flights from airport 5 to airport 4. The current distribution process of the company, which is performed manually, uses one air route from station 3 to station 2 and generates a weighted average shipping time equal to 22.61 hours (22 hours and 37 minutes). In this real world case, the PRSP model improves the current solution by around 19.6%.

IV. CONCLUSION

This paper defined and formulated the PRSP (parcel routing and scheduling problem). The study is concerned especially with small and medium parcel distribution companies that cannot afford to have their own airplanes and have to rely on commercial airlines for their air shipping. The PRSP model determines, for each shipment and for each day of the planning period, the route to follow (air or ground) and the time at which the shipment leaves each node on the route (the schedules of air routes should abide by the schedules fixed by the airline carrier). The optimal routes and schedules should minimize the weighted average shipping time per origin-destination pair of stations.

The proposed model was successfully applied to a real-world case; it generates a feasible solution better than the current one in an acceptable running time (4 hours). The improvement of the solution is very important (19.6%). The improvement can be even more significant as the number of regular flights increases.

Despite its merits, the PRSP model cannot reach the optimal solution. To obtain a near optimal solution better than that generated by the PRSP model, a heuristic method is recommended. This heuristic can be one of the well known methods in the literature such as *the tabu search method*, or a new heuristic based on the PRSP model with valid inequalities.

REFERENCES

[1] S. Alumur, B.Y. Kara, “Network hub location problems: The state of the art,” European Journal of Operational Research, vol. 127, 1, pp. 1–21, 2008.

[2] T. Aykin, “The hub location and routing problem,” European Journal of Operational Research, vol. 83, pp. 200–219, 1995.

[3] C. Barnhart, R.R. Schneur, “Air Network Design for Express Shipment Service,” Operations Research, vol. 44, pp. 852–863, 1996.

[4] O. Ben–Ayed, “Hierarchical parcel distribution facility ground network design problem,” International Journal of Logistics Economics and Globalisation, vol. 2, pp. 250–269, 2010.

[5] O. Ben–Ayed, “Timetabling hub–and–spoke parcel distribution inter–facility network,” International Journal of Advanced Operations Management, vol. 5, pp. 159–180, 2013a.

[6] O. Ben–Ayed, “Parcel distribution network design problem,” Operational Research, vol. 13, pp. 211–232, 2013b.

[7] J.F. Campbell, “Location-allocation for distribution systems with transshipments and transportation economies of scale,” Annals of Operations Research, vol. 40, pp. 77–99, 1992.

[8] J.F. Campbell, “Integer programming formulations of discrete hub location problems,” European Journal of Operational Research, vol. 72, pp. 387–405, 1994.

[9] T.G. Crainic, “Service network design in freight transportation,” European Journal of Operational Research, vol. 122, pp. 272–288, 2000.

[10] T.G. Crainic, G. Laporte, “Planning models for freight transportation,” European Journal of Operational Research, vol. 97, pp. 409–438, 1997.

[11] A.T. Ernst, M. Krishnamoorthy, “Efficient algorithms for the uncapacitated single allocation p-hub median problem,” Location Science, vol. 4, pp. 139–154, 1996.

[12] T. Grünert, H.-J. Sebastian, M. Thärigen, “The Design of a Letter-Mail Transportation Network by Intelligent Techniques,” PROCEEDINGS OF THE HAWAI’I INTERNATIONAL CONFERENCE ON SYSTEM SCIENCES, 1999.

[13] S. Hamzaoui, O. Ben-Ayed, “Parcel distribution timetabling problem,” Operations Management Research, vol. 4, pp. 138–149, 2011.

[14] M.E.T. Horn, “An extended model and procedural framework for planning multi-modal passenger journeys,” Transportation Research Part B: Methodological, vol. 37, pp. 641–660, 2003.

[15] B.Y. Kara, B.C. Tansel, « The Latest Arrival Hub Location Problem,” Management Science, vol. 47, pp. 1408–1420, 2001.

[16] C.-C. Lin, S.-H. Chen, “An integral constrained generalized hub-and-spoke network design problem,” Transportation Research Part E: Logistics and Transportation Review, vol. 44, pp. 986–1003, 2008.

[17] A. Marín, L. Cánovas, M. Landete, “New formulations for the uncapacitated multiple allocation hub location problem,” European Journal of Operational Research, vol. 172, pp. 274–292, 2006.

[18] M.H. Poon, S.C. Wong, C.O. Tong, “A dynamic schedule-based model for congested transit networks,” Transportation Research Part B: Methodological, vol. 38, pp. 343–368, 2004.

[19] P.Z. Tan, B.Y. Kara, “A hub covering model for cargo delivery systems,” Networks, vol. 49, pp. 28–39, 2007.

[20] N. Teypaz, S. Schrenk, V.-D. Cung, “A decomposition scheme for large-scale Service Network Design with asset management,” Transportation Research Part E: Logistics and Transportation Review, vol. 46, pp. 156–170, 2010.

[21] M. Verma, V. Verter, “A lead-time based approach for planning rail-truck intermodal transportation of dangerous goods,” European Journal of Operational Research, vol. 202, pp. 696–706, 2010.

[22] M. Wasner, G. Zäpfel, “An integrated multi-depot hub-location vehicle routing model for network planning of parcel service,” International Journal of Production Economics, vol. 90, pp. 403–419, 2004.

[23] W. Xu, S. He, R. Song, S.S. Chaudhry, “Finding the K shortest paths in a schedule-based transit network,” Computers & Operations Research, vol. 39, pp. 1812–1826, 2012.

[24] H. Yaman, B.Y. Kara, B.Ç. Tansel, « The latest arrival hub location problem for cargo delivery systems with stopovers,” Transportation Research Part B: Methodological, vol. 41, pp. 906–919, 2007.

[25] S. Yan, C.-H. Tseng, “A Passenger Demand Model for Airline Flight Scheduling and Fleet Routing,” Comput. Oper. Res., vol. 29, pp.1559–1581, 2002.

APPENDIX

TABLE I. NOTATIONS

Names	Definition	Names	Definition
I	The set of stations i	K	The set of airports k
J	The set of hubs j	L	The set of working days in the planning period. Each element l in L is a 24-hour day starting at midnight.
I_j	The set of stations assigned to hub j	$F_{k_1k_2l}$	Set of trips f from airport k_1 to airport k_2 at day l.
α_i	The earliest clock-time at which shipments can leave station i $\alpha_i \in [0,24)$	β_i	The latest clock-time at which shipments can leave station i. $\beta_i \in [0,24)$
$\alpha_i^{'}$	The starting time of the time-window interval for customer i. It is the earliest clock-time at which we begin servicing the customer i.	$\beta_i^{'}$	The ending time of the time-window interval for customer i. It is the latest clock-time at which we finish servicing the customer i.
$\lambda_{i_1i_2}$	The economic percentage weight of shipment from origin station i_1 to destination station i_2.	$\delta_{n_1n_2}$	The travel time between nodes n_1 and n_2. $\delta_{n_1n_2}$ is one of the following: $\delta_{i_1j_1}, \delta_{j_1k_1}, \delta_{k_1k_2}, \delta_{k_2j_2}, \delta_{j_1j_2}, \delta_{j_2i_2}$
γ_n	The processing time of shipments at node n. γ_n is one of the following: γ_{i_1}, γ_{j_1}, γ_{k_1}, γ_{k_2}, γ_{j_2}, γ_{i_2}	$\varphi_{k_1k_2fl}$	The leaving clock-time of trip f from airport k_1 to airport k_2 on day l.
$y_{n_1n_2}^{j_1j_2l}$	1 if shipment originating from hub j_1 and destined to hub j_2 at day l goes on the link from node n_1 to node n_2 0 otherwise $y_{n_1n_2}^{j_1j_2l}$ is one of the following: $y_{j_1j_2}^{j_1j_2l}, y_{j_1k_1}^{j_1j_2l}, y_{k_1k_2}^{j_1j_2l}, y_{k_2j_2}^{j_1j_2l}$	$y_{k_1k_2f}^{j_1j_2l}$	1 if shipment originating from hub j_1 and destined to hub j_2 at day l goes on the f^{th} trip from airport k_1 to airport k_2 0 otherwise
$x_{n_1}^{j_1j_2l}$	The leaving clock-time from node n_1 of the shipment originating from hub j_1 and destined to hub j_2 on day l $x_{n_1}^{j_1j_2l}$ is one of the following: $x_{j_1}^{j_1j_2l}$, $x_{k_1}^{j_1j_2l}$, $x_{k_2}^{j_1j_2l}$	x_n	The leaving clock-time from node n. x_n refers to the origin station as x_{i_1} and to the destination station as x_{i_2}
$x_{j_2}^l$	The leaving clock-time of the shipment from node j_2 at day l	$w_{i_1i_2}$	The shipping time between the leaving clock-time from origin station i_1 and the leaving clock-time from destination station i_2 during the planning period
$z_{n_1n_2}^{j_1j_2l}$	The integer number of calendar days between the leaving clock-times from nodes on the link n_1 n_2; this number is related to the shipment originating from hub j_1 and destined to hub j_2 on day l $z_{n_1n_2}^{j_1j_2l}$ is one of the following: $z_{i_1j_1}^{j_1j_2l}$, $z_{j_1k_1}^{j_1j_2l}$, $z_{k_1k_2}^{j_1j_2l}$, $z_{k_2j_2}^{j_1j_2l}$, $z_{j_1j_2}^{j_1j_2l}$	$z_{j_1j_2}^l$	The integer number of calendar days between the leaving clock-times from nodes on the link $j_1 j_2$ at day l

BIOGRAPHY

Mondher Hachicha is a PhD candidate in Quantitative Methods at the University of Sfax in Tunisia. He has already submitted his dissertation for defence to the thesis committee. He obtained his Master's degree in Management Science from HEC Montreal in Canada. He is the winner of a scholarship award from the Canadian International Development Agency (CIDA) for outstanding performance. He taught Operations Research at HEC Montreal and the University of Sfax. He worked as the distribution operations manager in one of the largest dairies in Tunisia for about four years.

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