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Two Calibrated Metaheuristic Algorithms for Solving a Multi-Plant Capacitated Lot-Sizing Problem in an Integrated Production-Distribution Network

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***Abstract*—In this paper a model is developed to investigate the value of coordination of production and distribution planning in a three-layer supply chain consisting of multiple suppliers, manufacturers, and distribution centers. The combinations of several factors such as purchasing, production, storage, backordering, and transportation are considered. The aim of the model is to find the optimal order, production, and shipment quantities, so that the cost of the whole supply chain is minimized, and demand is satisfied over a given planning horizon without violating the capacity restrictions of the plants and suppliers. Transfer decisions between plants are made when production at a plant cannot meet demand due to lack of adequate resources and it can be satisfied by other production plants. Since the proposed model is NP-hard, two metaheuristic algorithms namely simulated annealing and genetic algorithm are used to find the optimal or near-optimal solution within a moderate computation times. The results show that presented algorithms are computationally effective and beneficial for obtaining the optimal solution for the proposed lot-sizing problem.**

Keywords—capacitated lot-sizing; multi-plant; production and distribution planning; integrated supply chain; simulated annealing; genetic algorithm

I. INTRODUCTION

In order to remain competitive in today's ever-changing markets, companies have to examine alternative solutions for their logistics network. One of these solutions can be shifting from one plant manufacturing facility to multi-plant enterprise. Reference [1] distinguished supply chain coordination planning into two broad categories: coordination in terms of incorporating decisions of various functions including production planning, distribution, and marketing, and coordination of associating decisions within the same operation through several echelons of the corporation. The authors refer to the latter level of coordination as "multi-plant coordination". Each plant here refers to a manufacturing facility that is centered around related production processes. The multi-plant structure is a complex multi-stage manufacturing system, where each plant itself denotes a multi-stage system in which the flow of products may be serial, parallel, assembly or general [2]. In this case, lot-sizing problems become more complicated because of the interdependency between plants. In the multi-plant problem, there is no common resource between sites, and therefore, the production is controlled independently in each site. The multi-plant capacitated lot sizing problem (MPCLSP) with multiple products and time periods is comprised of multiple production centers that manufacture all the same products and allow inter-plant transfers [3].

Shifting from single plant to multi-plant organization offers several advantages such as saving on transportation cost and time, improving the customer service by locating the plant close to the customer, being close to low cost raw materials, flexibility in producing several products and specialization in activities, increasing the competitive advantage in the international economic arena, and so forth [4]. However, decision making in multi-plant systems has to attempt towards integration of several manufacturing plants' activities in such a way that they align their tasks in direction of improving overall performance of the enterprise. Each plant's internal function is as important as its relation with other plants since each plant is a part of the network [5]. The MPCLSP can be observed in several industries such as electric power generating industries [6], food and chemical process industries [7], automotive factories [8], steel corporations [9], production of thin film transistor-liquid crystal display [10], beverage industries [11], cultivation systems [12], and feed mill industries [13] where multiple plants producing the same products are located at different geographical locations in a country or scattered around the world.

Reference [14] addressed a capacitated master production planning and capacity allocation problem for a multi-plant manufacturing system with two serial stages in each plant. The resulting mixed binary linear programming model was solved by Lagrangian relaxation based heuristic algorithms. Reference [15] proposed a simulated annealing (SA) methodology to design a production and distribution system characterized by a central plant, multiple product families, multiple cross-docking sites, and multiple retailers. Reference [16] studied the problem of coordinating the short-term production and inter-facility transportation scheduling decisions between a plant that produces intermediate products and a finishing plant which processes the intermediate products into finished goods. Using real and simulated data from a process industry firm, the computational study showed that coordinated schedules yield significant cost savings resulting from the modest use of the expensive fast transport mode, coordinated product changeovers between plants and reduced intermediate product inventories. Reference [17] proposed an advanced process planning and scheduling model for the multi-plant chain, and developed an evolutionary algorithm to solve the model in order to minimize makespan and operation sequences with machine selections.

Reference [18] presented a mixed-integer linear programming model for the optimum planning of multi-plant, multi-supplier, and multi-grade petrochemical production. The model incorporated demand, capacity, raw-material availability, and sequencing constraints in order to maximize total profitability. Reference [19] considered a supply chain including multiple suppliers, multiple manufacturers and multiple customers, addressing a multi-site, multi-period, multi-product aggregate production planning problem under uncertainty. A robust multi-objective aggregate production planning model was presented, and was solved as a single-objective mixed integer programming model by applying the linear programming metrics method. Reference [20] addressed a multi-item, multi-plant lot-sizing problem with capacity restrictions, and inter-plant transfers, such that the total production, inventory, setup and transfer cost is minimized. A Lagrangian lower bound on the optimal cost value of the problem was proposed based on the decomposition of the problem into facility location and multi-commodity flow problems.

Based on the existing literature, little attention has been paid to the multi-plant capacitated lot-sizing problem in integrated production-distribution systems. Therefore, this study attempts to address these shortcomings by developing a comprehensive mathematical model and solution approaches for such problem. Moreover, from the metaheuristic viewpoint, the contribution of the study is to find out how metaheuristic algorithms perform for the MPCLSP, as previously they have been applied mostly to other production-related problems, in particular scheduling, but not to this exact lot-sizing problem.

II. Model Description And Mathematical Formulations

This study is concerned with the multi-item multi-period lot-sizing problem in a three-echelon supply chain environment consisting of multiple suppliers, manufacturers, and distribution centers, considering deterministic dynamic demand and finite time horizon. The objective of three-layer supply chain problem is to determine the purchasing, production, transportation, storage, and shortage schedule for the products that are optimal from a system's perspective, in addition to minimizing the cost of the whole supply chain.

It is assumed that production takes place in a multi-plant manufacturing company, where the plants are geographically spread in different locations of a country. Each product is made of raw materials which are provided by the suppliers. It is designated that for each raw material type there is only one particular supplier. If the quality of raw materials is not acceptable, they can be rejected, but plants do not pay for the rejected raw materials, and their associated cost is paid by the respective supplier. Each plant is characterized by its own inventory and production capacities. It is possible to store excess production at the plant storage which has capacity limit, but no storage is possible for end products at distribution centers. Any of the products produced in each plant can be transported to any of the distribution centers that are located in different areas. Obviously, demand in a distribution center is served by the closest plant. Transfer decisions between plants are made when demand observed at a plant can be satisfied by other production plants to cope with under-capacity of that particular plant. It should be noted that the customer would pay only for the transportation from the nearest plant. The transportation cost from other plants to the plant where demand has been placed, has to be borne by the company. Since all factories, suppliers, and distribution centers are spread out geographically, the transportation cost can vary. Transportation cost is composed of a fixed cost plus a variable cost that depends on the amount transported and the distance between entities. Homogenous vehicles of a given capacity are stationed at each supplier and plant to deliver products from suppliers to plants, between the production plants, and from plants to distribution centers. In addition, backordering is allowed when demand at a distribution center cannot be entirely satisfied.

A. Definition of Symbols

The following notations are used to formulate the model.

***Indices*:**

Notation	Definition
i	Product, $i \in \{1,2,...,N\}$
k	Raw material, $k \in \{1,2,...,K\}$
v	Resource, $v \in \{1,2,...,V\}$
m	Supplier, $m \in \{1,2,...,M\}$
j, l, l'	Plant, $j,l,l' \in \{1,2,...,J\}$
w	Distribution center, $w \in \{1,2,...,W\}$
t	Period, $t \in \{1,2,...,T\}$

***Parameters*:**

Notation	Definition
d_{iwt}	Demand of product i at distribution center w in period t
A_{ijt}	Setup time of product i at plant j in period t
P_{ijt}	Production time of product i at plant j in period t
F_{jt}	Total available production time at plant j in period t
x_{ki}	Amount of raw material k required to produce a unit of product i
E_{kmt}	Number of raw material k that can be provided by supplier m in period t
λ_{kmjt}	Percentage of rejected raw material k delivered by supplier m to plant j in period t
R_{vij}	Amount of resource v required to produce a unit of product i at plant j
N_{vjt}	Total amount of resource v available at plant j in period t
π_{kmt}	Ordering cost of raw material k at supplier m in period t
τ_{kmt}	Purchasing cost of raw material k at supplier m in period t
S_{ijt}	Setup cost for product i at plant j in period t
O_{ijt}	Production cost of product i at plant j in period t
H_{kjt}	Holding cost of raw material k at plant j in period t
H'_{ijt}	Holding cost of product i at plant j in period t
B_{iwt}	Backordering cost of product i at distribution center w in period t
ϖ_{mj}	Distance between supplier m and plant j
μ_{jl}	Distance between plant j and plant l
ζ_{jw}	Distance between plant j and distribution center w
σ_{kj}	Storage capacity for raw material k at plant j
σ'_{ij}	Storage capacity for item i at plant j
ς_k	Vehicle available capacity respect to raw material k
ς'_i	Vehicle available capacity respect to product i
η^F	Fixed transportation cost of vehicle
η^V	Variable transportation cost of vehicle per trip
ρ_k	Safety stock coefficient respect to raw material k
ε_j	Performance percentage of available time at plant j
ξ_{vj}	Productivity percentage of resource v at plant j
δ	A very large number
δ'	A very large number

***Decision Variables*:**

Notation	Definition
Q_{ijt}	Quantity of product *i* produced at plant *j* in period *t*
α_{kmjt}	Purchase amount of raw material *k* shipped from supplier *m* to plant *j* in period *t*
I_{kjt}	Inventory level of raw material *k* stored at plant *j* at the end of period *t*
I'_{ijt}	Inventory level of product *i* stored at plant *j* at the end of period *t*
C_{ijwt}	Quantity of product *i* that is available to be shipped from plant *j* to distribution center *w* in period *t*
Z_{ijlt}	Quantity of product *i* transferred from plant *j* to plant *l* in period *t*
Y^{j}_{iwt}	Total number of product *i* shipped from plant *j* to distribution center *w* in period *t*
b_{iwt}	Shortage amount of product *i* at distribution center *w* in period *t*
ϕ_{mjt}	Number of vehicles required to ship products from supplier *m* to plant *j* in period *t*
ν_{jlt}	Number of vehicles required to transfer products from plant *j* to plant *l* in period *t*
Ω_{jwt}	Number of vehicles required to ship products from plant *j* to distribution center *w* in period *t*
χ_{ijt}	$\begin{cases}1 & \text{If there is a setup for product } i \text{ at plant } j \text{ in period } t\\ 0 & \text{Otherwise}\end{cases}$
φ_{kmjt}	$\begin{cases}1 & \text{If an order for raw material } k \text{ is allocated to supplier } m \text{ by plant } j \text{ in period } t\\ 0 & \text{Otherwise}\end{cases}$

B. The Mathematcal Model

***Cost Function*:**

$$\begin{aligned}\text{Min } f = &\sum_k\sum_m\sum_j\sum_t \varphi_{kmjt}\pi_{kmt} + \alpha_{kmjt}\tau_{kmt} + \sum_i\sum_j\sum_t \chi_{ijt}S_{ijt} + Q_{ijt}O_{ijt}\\ &+ \sum_k\sum_j\sum_t I_{kjt}H_{kjt} + \sum_i\sum_j\sum_t I'_{ijt}H'_{ijt}\\ &+ \eta^F\sum_m\sum_j\sum_t \phi_{mjt} + \eta^F\sum_j\sum_{l\neq j}\sum_t \nu_{jlt} + \eta^F\sum_j\sum_w\sum_t \Omega_{jwt}\\ &+ \eta^V\sum_m\sum_j\sum_t \varpi_{mj}\phi_{mjt} + \eta^V\sum_j\sum_{l\neq j}\sum_t \mu_{jl}\nu_{jlt} + \eta^V\sum_j\sum_w\sum_t \zeta_{jw}\Omega_{jwt}\\ &+ \sum_i\sum_w\sum_t b_{iwt}B_{iwt}\end{aligned} \tag{1}$$

Equation (1) is the objective function of the proposed model, where the sum of procurement, production, inventory, transportation, and shortage costs over the planning horizon should be minimized.

***Constraints*:**

$$\alpha_{kmjt} = \max\left\{0, \sum_i \rho_k(x_{ki}Q_{ijt}) - I_{kj(t-1)}\right\} \qquad \forall k, m, j, t \tag{2}$$

Equation (2) shows the required amount of raw material *k* that plant *j* must purchase from supplier *m* in period *t*. If total amount of raw material *k* used in production of all items multiplied by a safety stock coefficient (ρ_k) is less than the existing inventory of raw material *k*, then the factory does not need to order any raw material in period *t*.

$$I_{kj(t-1)} = 0 \qquad \forall k, j, t = 1 \tag{3}$$

Initial inventory level of raw materials is considered to be zero as shown in (3).

$$I_{kjt} = I_{kj(t-1)} + \alpha_{kmjt} - \lambda_{kmjt}\alpha_{kmjt} - \sum_i x_{ki} Q_{ijt} \qquad \forall k, m, j, t \tag{4}$$

Equation (4) represents the balance equation for the inventory of raw materials at plants at the end of period *t*.

$$\alpha_{kmjt} \le \delta \varphi_{kmjt} \qquad \forall k, m, j, t \tag{5}$$

Equation (5) describes that a plant cannot place a procurement order without charging an ordering cost. φ_{kmjt} is a binary variable with value of 1 if an order is allocated to supplier *m* at time *t*, otherwise, it is 0.

$$\sum_j \alpha_{kmjt} \le E_{kmt} \qquad \forall k, m, t \tag{6}$$

Equation (6) ensures that the order size of raw materials released for each supplier is limited by its capacity.

$$I'_{ij(t-1)} = 0 \qquad \forall i, j, t = 1 \tag{7}$$

Equation (7) shows the initial inventory level of products at the beginning of planning horizon.

$$I'_{ijt} = I'_{ij(t-1)} + Q_{ijt} + \sum_{l' \ne j} Z_{il'jt} - Y^j_{iwt} - \sum_{l \ne j} Z_{ijlt} \quad \forall i, j, w, t \tag{8}$$

Equation (8) is the inventory balance equation for finished items at plants.

It is supposed that if during period *t* there is a transfer into plant *j*, there cannot be any transfer out from plant *j* to other plants during that period. Hence:

$$Z_{ijlt} \times Z_{il'jt} = 0 \qquad \forall i, j, l \,\&\, l' \ne j, t \tag{9}$$

$$Q_{ijt} \le \delta' \chi_{ijt} \qquad \forall i, j, t \tag{10}$$

Equation (10) forces χ_{ijt} to be nonzero if Q_{ijt} is nonzero.

$$\sum_i (P_{ijt} Q_{ijt} + A_{ijt} \chi_{ijt}) \le F_{jt} \varepsilon_j \qquad \forall j, t \tag{11}$$

Equation (11) limits the production time available at a plant during period *t*. The overall time consumptions for production and setup in each plant for all products must be lower than or equal to the available time capacity. It also considers the available time's performance.

$$\sum_i R_{vij} Q_{ijt} \le N_{vjt} \xi_{vj} \qquad \forall v, j, t \tag{12}$$

Equation (12) ensures that a manufacturer does not plan beyond the available resources (machine or human) of each plant in each period. It also considers resources' productivity.

$$C_{ijwt} + \sum_{l \ne j} Z_{ijlt} \le I_{ijt-1} + Q_{ijt} + \sum_{l' \ne j} Z_{il'jt} \qquad \forall i, j, w, t \tag{13}$$

Equation (13) shows that the number of products available to be transferred from plant *j* to distribution center *w* and other plants in period *t* should not exceed the previous period inventory and production quantity in plant *j* as well as transferred products to plant *j* in period *t*.

$$\sum_{l' \ne j} Z_{il'jt} \le \max\left\{(b_{iw(t-1)} + d_{iwt}) - C_{ijwt}, 0\right\} \qquad \forall i, j, w, t \tag{14}$$

Equation (14) restricts the transfer quantity from other plants to plant *j* during period *t*. It implies that if total amount of item *i* available at plant *j* to be transferred to distribution center *w* in period *t* is greater than backorder amount from previous period and demand at distribution *w* in period *t*, then plant *j* does not need outsourcing. In this condition C_{ijwt} will be equal to Y_{iwt}^{j}. Otherwise, plant *j* needs to request the shortage amount of item *i* from other plants.

$$\sum_{k} \frac{\alpha_{kmjt}}{\varsigma_k} \le \phi_{mjt} \qquad \forall m, j, t \tag{15}$$

Equation (15) calculates the number of vehicles used for transportation of raw materials from suppliers to plants.

$$\sum_{i} \frac{Z_{ijlt}}{\varsigma_i'} \le \nu_{jlt} \qquad \forall j, l \ne j, t \tag{16}$$

$$\sum_{i} \frac{Y_{iwt}^{j}}{\varsigma_i'} \le \Omega_{jwt} \qquad \forall j, w, t \tag{17}$$

Equations (16) and (17) determine the number of vehicles required for delivery of products from a plant to other plants and distribution centers respectively.

$$b_{iwt} = \max\left\{(b_{iw(t-1)} + d_{iwt}) - Y_{iwt}^{j}, 0\right\} \qquad \forall i, j, w, t \tag{18}$$

Equation (18) limits the backorder quantity in period *t* by the current demand plus the backorder amount from the previous period. The shortage in period *t* will be zero if the amount of demand of item *i* at distribution center *w* in period *t* plus its previous backorder is equal to or smaller than total quantity of item *i* transferred to distribution center *w*.

$$I_{kjt} \le \sigma_{kj} \qquad \forall k, j, t \tag{19}$$

$$I_{ijt}' \le \sigma_{ij}' \qquad \forall i, j, t \tag{20}$$

Equations (19) and (20) determine the upper limit of inventory level for each type of raw material and product in plants respectively.

$$\begin{aligned} &Q_{ijt}, \alpha_{kmjt}, I_{kjt}, I_{ijt}', b_{iwt}, Z_{ijlt}, C_{ijwt}, Y_{iwt}^{j} \ge 0 && \forall k, i, m, j, l \ne j, w, t \\ &\phi_{mjt}, \nu_{jlt}, \Omega_{jwt} \ge 0, \text{integer} && \forall m, j, l \ne j, w, t \\ &\chi_{ijt}, \varphi_{kmjt} \in \{0,1\} && \forall k, i, m, j, t \end{aligned} \tag{21}$$

Equation (21) enforces the restrictions of non-negativity and binary nature on the decision variables.

III. Solution Algorithms

Since the MPCLSP is considered as NP-hard problem, using the exact methods may encounter difficulties for solving medium to large size instances. Furthermore, both deterministic and heuristic optimization methods may not be able to solve such problem efficiently. Therefore, the SA algorithm is employed to solve the proposed mathematical problem. Furthermore, since no benchmark for the MPCLSP can be found in the literature, the genetic algorithm (GA) is applied as well to solve the problem and to verify the solution. These methods are explained in the following subsections.

A. SA Algorithm

The SA algorithm is an effective stochastic search method for solving combinatorial and global optimization problems proposed by Kirkpatrick [21]. The basic idea is inspired from the physical process of cooling molten material to solid form. Based on this procedure, the SA explores different areas of the solution space of a problem by annealing from a high to a low temperature. During the search process both good solutions as well as low quality solutions are accepted with a nonzero probability related to the temperature in the cooling schedule at that time. This feature can prevent getting trapped in local minima. In the beginning, this probability is large, and it will be reduced during the execution with a positive parameter such as temperature [21]. The main steps of the SA algorithm are described below.

1) Initialization of Parameters

In this step, the input parameters of the SA algorithm are initialized. The parameters are:

i. Initial temperature (D_0): it is the starting point of temperature computation in every iteration. D_0 should be adequately high to escape a premature convergence. Basically, the SA algorithm starts with an initial temperature where almost all worsening moves are accepted regardless of the objective function value.

ii. Population size (N_{pop}): it is the number of sustaining solutions in every iteration. In this study, a population-based SA is utilized. Unlike the conventional SA, population-based SA iterates a population of solutions rather than a single solution. During each iteration, it explores the candidate solutions around several promising samples, and prevents the candidate solution from being stagnated in one local optimum. This not only enhances the search speed, but also yields a solution near the global optimum [22, 23].

iii. Iteration: it shows the number of iteration in each temperature.

iv. Final temperature (D_U): the temperature will remain fixed once it reaches the lowest temperature limit.

2) Solution Representation

To design the SA, a suitable representation scheme that shows the solution characteristics is required. In this study, the solution structure is constructed of the quantities of the produced items (lot size Q) with $N \times J \times T$ dimensions where N shows the total number of products, J indicates total number of plants, and T denotes total number of periods. The general structure of the solution representation is shown in Fig. 1.

$$\begin{array}{c} \\ \text{Product} \end{array} \begin{array}{c} \text{Plant} \\ \begin{bmatrix} Q_{11t} & Q_{12t} & \cdots & Q_{1Jt} \\ Q_{21t} & Q_{22t} & \cdots & Q_{2Jt} \\ \vdots & \vdots & \cdots & \vdots \\ Q_{N1t} & Q_{N2t} & \cdots & Q_{NJT} \end{bmatrix} \end{array}$$

Fig. 1. Solution representation

This presentation technique encodes only the variable for the lot size but derives other decision variables by making use of the problem-specific knowledge.

3) Creating the Initial Solution

The SA generates a randomly initial population of g solutions within the boundary of the component. Let Q_g represent the gth solution in the population. Then, each solution is generated by:

$$Q_g = \text{round}\left[\text{lb}\,Q(i, j, t), \text{ub}\,Q(i, j, t)\right] \tag{22}$$

Where g denotes the size of population (g = 1, 2, …, N_{pop}), and lb$Q$ and ub$Q$ are the lower and upper bounds for variable Q respectively. Therefore, (22) produces integer random numbers for variable Q within the predetermined limits.

4) Cooling Schedule

System temperature determines the degree of randomness towards solution, and it is reduced with a known plan in accordance with the progress of solution procedure. In reality, system temperature is a solution subspace of the problem accepted in each iteration. As the algorithm progresses and the temperature decreases, inappropriate solutions have smaller chance of being accepted. Cooling schedule determines the functional form of the change in temperature required in the SA. A geometric temperature reduction rule, which is the most commonly utilized decrement rule, is applied for this study. If the temperature at uth iteration is D_u, then the temperature at (u+1)*th* iteration is given by [24]:

$$D_{u+1} = z \times D_u \tag{23}$$

Where z denotes the cooling factor and it is obtained as shown in (24):

$$z = (D_U \,/\, D_0)^{(1/\text{max iteration})} \tag{24}$$

5) Neighborhood Representation

The neighborhood search structure is a procedure which generates a new solution that slightly changes the current solution and prevents the fast convergence of the SA procedure. The following process is used to delineate the neighborhood configuration.

i. An integer random number in range [1, N_{pop}] is generated in order to select a solution (Q_g). The total numbers of elements selected for change are $\mathrm{n}Q = N \times J \times T$.

ii. Two integer random numbers r_1 and r_2 are produced in order to select the elements of the solution for alteration. The considered range for r_1 is [1, nQ-1] and r_2 is [r_1+1, nQ].

iii. The value of selected elements of solution Q_g is changed using (25):

$$Q'_g\left(r_1 : r_2\right) = Q_g\left(r_1 : r_2\right) + 0.1 \times r \times \left(\mathrm{ub}Q_g - \mathrm{lb}Q_g\right) \tag{25}$$

Where r is a random number generated from the continuous uniform distribution within the range of [-1, 1]. The size of generated random numbers is equal to the upper bound Q_g. The result obtained by (25) is rounded to attain an integer value. However, there is a possibility that the calculated value by (25) exceeds the upper and lower bounds of Q_g. Therefore:

$$\begin{cases} Q'_g = \min(Q'_g, \mathrm{ub}Q_g) \\ Q'_g = \max(Q'_g, \mathrm{lb}Q_g) \end{cases} \tag{26}$$

6) Main Loop of the SA

The SA begins with a high temperature and selects initial solutions (s_0) randomly. Next, a new solution (s_n) within the neighborhood of the current solution (s) is computed in each iteration. In the minimization problem, if the value of the objective function, $f(s_n)$, is smaller than the previous value, $f(s)$, the new solution is accepted. Otherwise, the SA algorithm uses a stochastic function given in (27) for accepting the new solution in order to prevent the local optimum trap.

$$a = \exp(-\Delta f \,/\, D) \tag{27}$$

Where $\Delta f = f(s_n) - f(s)$ and D is the current state temperature. This procedure is repeated until the termination condition is reached.

B. GA Algorithm

The GA is considered as an evolutionary algorithm and a population-based method that attempts to finds the optimal or near-optimal solutions through conducting a random search. Fundamental of the GA was primarily instated by Holland [25]. This algorithm is based upon "survival of the fittest" principles by Darwin Theory of Evolution and simulates the process of natural evolution. The GA method has been effectively used for solving continuous and discrete combinatorial problems [26]. Simplicity and capability of finding quick reasonable solutions for intricate searching and optimization problems have brought about a growing interest over the GA. The general steps of the GA can be summarized as follow [27]:

i. Encoding solutions of problem into chromosomes.

ii. Creating initial population of solutions randomly.

iii. Evaluating chromosomes in terms of their fitness in order to select parents.

iv. Applying genetic operators (crossover and mutation) in order to reproduce new chromosomes (offspring).

v. Evaluating the new population.

vi. Maintain the best chromosomes among parents and offspring.

vii. If stopping criteria is met, then stop. Otherwise, go to step iii.

1) Initilization of Parameters

The initial information required to begin a GA includes the number of chromosomes kept in each generation called population size (N_{pop}), the probability of operating crossover (P_c), the probability of operating mutation (P_m), and maximum number of generations.

2) Chromosome Representation

The algorithm starts with encoding the variables of the problem as finite-length strings or chromosomes as shown in (22). After generation of the chromosomes, each chromosome is evaluated using the objective function given in (1).

3) Selection

The selection process of the chromosomes in the mating pool is based on the roulette wheel selection. The selection probability, a_g, for individual g (g denotes the size of population and g = 1, 2, …, N_{pop}), with objective function value f_g, is calculated by (28):

$$a_g = \frac{f_g}{\sum_{g=1}^{Npop} f_g} \tag{28}$$

Although all individuals in the population have a chance of being selected to reproduce the next generation, those with higher fitness value are more likely to be selected for the mating pool.

4) Crossover and Mutation

In this study, the arithmetic crossover operator that linearly combines the parent chromosome vector is used to produce offspring based on (29) and (30).

$$offspring_{(1)} = y \times parent_{(1)} + (1-y) \times parent_{(2)} \tag{29}$$

$$offspring_{(2)} = y \times parent_{(2)} + (1-y) \times parent_{(1)} \tag{30}$$

Where y is a random vector in range [0, 1], and has a dimension equal to the size of the selected part (say the first part) of the chosen parent. Because variable Q is integer, the amounts of produced offsprings are rounded.

The solution spaces that are not discovered by the crossover operator are found using the mutation operator. The neighborhood structure of SA is used for the mutation operator of GA.

5) Stopping Criteria

The process of generating new chromosomes and searching for better solutions are continued until reaching the maximum number of generations without any significant improvement in the solution.

IV. Results And Discussions

The following parameter sizes for the proposed MPCLSP are considered: N = 3, K = 3, V = 1, M = 3, J = 3, W = 2, and T = 6. The demands and time independent costs are uniformly distributed in the specified intervals: $d_{iwt} \in$ [500, 700] and integer, $A_{ijt} \in$ [4, 8], $P_{ijt} \in$ [2, 5], $\pi_{kmt} \in$ [0.1, 1], $\tau_{kmt} \in$ [1, 4], $S_{ijt} \in$ [1, 4], $O_{ijt} \in$ [3, 7], $H_{kjt} \in$ [2, 4], $H'_{ijt} \in$ [2, 6], and $B_{iwt} \in$ [26, 29]. Applied optimizers are written and coded in MATLAB software version R2012a and are run on a laptop with 2.5-GHz AMD and 4GB RAM. To find the values of the algorithms' parameters, each algorithm is employed through pilot runs, each time changing the parameters in their corresponding ranges and obtaining the response values. Several combinations of N_{pop} in the range [10, 50], number of neighbors in the range [5, 15], D_0 in the range [10, 50], and D_U in the range [1, 0.001] for the SA, and N_{pop} in the range [50, 200], P_c in the range [0.1, 1], and P_m in the range [0.1, 1] for the GA are implemented. The parameter settings of both algorithms are given in Table I.

TABLE I. THE PARAMETERS' VALUES OF THE ALGORITHMS

Algorithm	Parameter	Parameter Value
SA	N_{pop}	40
	Number of neighbors	15
	D_0	30
	D_U	0.001
GA	N_{pop}	200
	P_c	0.9
	P_m	0.5

In order to compare the performances of the two algorithms, 10 different optimization runs are carried out. The results are reported in Table II. Table III provides an insight into each component of the total cost of the supply chain. The production quantities for the defined planning horizon obtained by SA and GA approaches are reported in Table IV. In Table V, the interaction between plants and the amounts of transportation are presented.

TABLE II. OBJECTIVE FUNCTION VALUES

Run No.	SA	GA
	Fitness ($)	Fitness ($)
1	1,981,209.86	1,999,064.25
2	1,992,357.31	1,999,436.01
3	1,994,099.90	2,000,269.90
4	1,995,588.08	2,001,715.64
5	1,999,781.36	2,003,745.26
6	2,000,547.48	2,004,399.16
7	2,003,531.15	2,005,925.49
8	2,009,853.73	2,008,387.88
9	2,012,092.36	2,009,817.74
10	2,014,126.68	2,012,281.89

TABLE III. OBJECTIVE FUNCTION COMPONENTS FOR THE BEST RUN OF THE APPLIED ALGORITHMS

Cost	Total Cost	Procurement	Production	Inventory	Transportation	Shortage
SA	1,981,209.86	313,576.77	102,812.09	167,941.00	1,396,880.00	0
GA	1,999,064.25	316,373.76	102,751.49	161,819.00	1,418,120.00	0

TABLE IV. THE PRODUCTION QUANTITIES OBTAINED BY THE BEST RUN OF SA AND GA

Plant *j*	Product *i*	Period *t*											
		1		2		3		4		5		6	
		SA	GA	SA	GA	SA	GA	SA	GA	SA	GA	SA	GA
1	1	745	656	765	361	148	662	787	312	476	673	188	195
	2	289	537	544	256	489	588	749	664	691	570	244	464
	3	755	423	394	519	490	713	564	473	392	265	546	445
2	1	561	601	787	740	380	631	658	566	441	571	616	472
	2	768	750	701	685	800	581	496	497	409	431	467	453
	3	685	689	800	732	751	682	800	393	374	533	129	449
3	1	85	126	86	93	110	125	53	95	158	84	37	107
	2	200	129	152	131	30	102	14	59	44	92	0	81
	3	163	173	37	152	63	136	58	89	63	129	32	75

TABLE V. INTERACTIONS BETWEEN PLANTS AND TRANSPORTATION QUANTITIES OBTAINED BY THE BEST RUN OF SA AND GA

Product *i*	Plant *j*	Plant *l*	Period *t*											
			1		2		3		4		5		6	
			SA	GA	SA	GA	SA	GA	SA	GA	SA	GA	SA	GA
1	1	2	65[1]	0	0	0	0	0	0	0	0	0	0	0
2			0	0	0	0	0	0	0	144	47	0	0	0
3			0	0	0	0	0	0	0	0	0	0	0	0
1	2	1	0	0	0	73	0	0	0	0	0	0	61	97
2			61	0	0	27	0	0	0	0	0	0	0	0
3			0	0	0	0	0	0	0	0	0	0	13	0
1	3	1	0	24	0	166	237	0	0	96	0	0	288	315
2			200	13	0	247	147	62	0	0	0	10	13	86
3			0	17	51	181	110	0	0	0	184	289	71	185
1	3	2	4	29	0	0	0	0	0	0	0	0	0	0
2			0	0	0	0	0	0	0	0	0	99	63	77
3			0	0	0	0	0	0	0	0	0	0	0	22

[1] It means that 65 units of product type 1 are transported from plant 1 to plant 2 during period 1.

To compare the performance of the applied algorithms statistically, the one-way analysis of variance (ANOVA) is utilized based on the objective function values of 10 experiments. This process is performed using Minitab software. Table VI shows the ANOVA results.

TABLE VI. THE ANOVA RESULTS FOR OBJECTIVE FUNCTION VALUES

Source	Degree of freedom (DF)	SS	MS	F-test	*p*-value
Optimization Engines	1	87593349	87593349	1.42	0.248
Error	18	1107842021	61546779		
Total	19	1195435370			

The hypothesis is that there is no difference in the average costs of the algorithms. The null hypothesis is rejected when the *p*-value turns out to be less than a predetermined significance level. The *p*-value obtained from the results is 0.248, which indicates the null hypothesis cannot be rejected at 95% confidence level, meaning that there is no significant difference between mean values of objective function obtained by two algorithms. Furthermore, the convergence paths of SA and GA for the best run are plotted in Fig. 2.

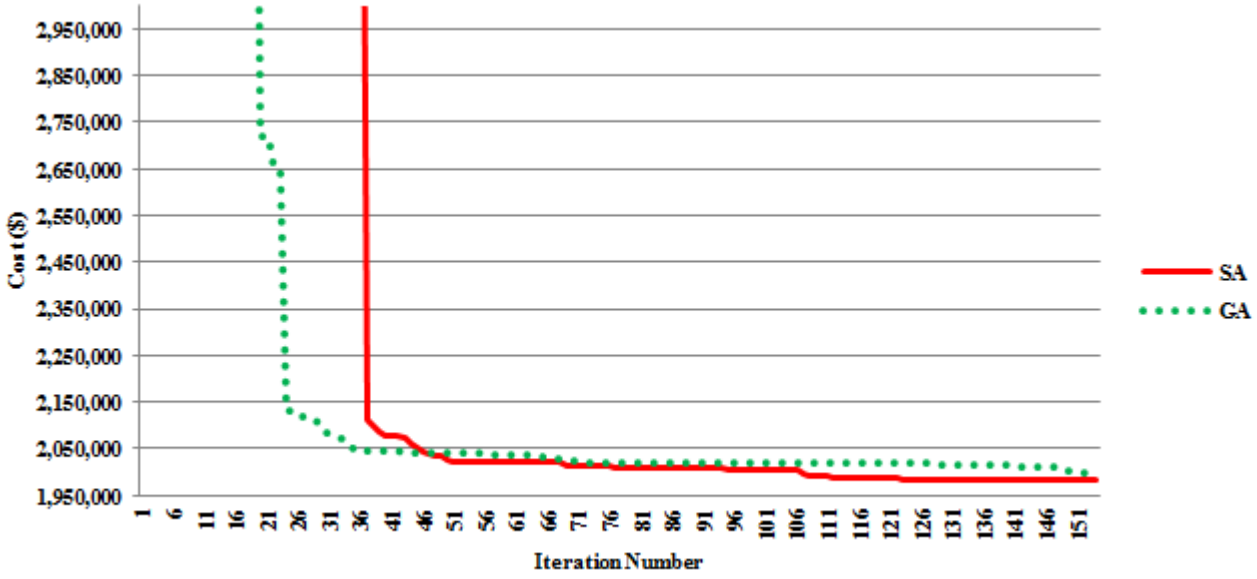

Fig. 2. The convergence path of the best result obtained by SA and GA methodologies

As it can be seen in Fig. 2, the SA offered better optimal solution in terms of function value compared to GA, while GA converged to the near optimal solution faster than SA in the early iterations of the algorithm.

V. CONCLUSIONS

In this paper, the multi-plant coordination problem is analyzed by considering different functions such as supply planning, production and inventory planning, and distribution planning. The objective of this problem is to coordinate the production plans of several manufacturing plants located at multiple locations, so that the overall performance of the firm is improved. A mathematical programming model is presented to distribute the raw materials to the suitable plants in order to meet demand of distribution centers and to achieve the objective of minimizing the multi-plant manufacturing costs composed of procurement, production, inventory, transportation, and shortage costs. Since the model is NP-hard, metaheuristic algorithms as solution methods are adopted to find cost effective and quality solutions for the proposed problem. Metaheuristic approaches namely SA and GA are applied to solve the model. The statistical results show that both presented algorithms can efficiently solve the proposed model in terms of solution quality as well as computational effort.

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