An Optimal Preventive Maintenance Plan According to Power Generation for a Wind Turbine

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Abstract—This paper deals a production and maintenance optimization problem for a wind turbine system in order to satisfy a random demand under given service level. A jointly optimization is made by combining between energy production and maintenance plan in order to establish an economical energy production planning and an optimal maintenance strategy according to the wind turbine degradation. The key of this study is to consider the influence of the energy production rate on the degradation degree of wind turbine. We prove simultaneously, with stochastic constraints of production and maintenance problem, an economical energy production plan and a maintenance planning which minimizes the average total cost of holding, production and maintenance. A numerical example is studied in order to apply the developed approach.

Keywords—maintenance; production; wind turbine; optimization; service level

I. INTRODUCTION

The renewable energies have become increasingly complex in response to economic growth and continuously increasing power demand. The wind and solar energies are considered among the renewable energies that have become the most efficient to achieve sustainable development. The renewable energies technologies and equipment’s are generally operated under more or less stationary conditions and influenced by several factors. [1] and [3] showed the variations of wind speed from season to another and from day to day. In the last years, the renewable energy has become one of the efficient ways to reach sustainable development but it is influenced by several factors in terms of reliability deterioration. Among these factors, we can consider that the variability of wind has a significant impact on the wind turbine equipment availability and the random variation of the climate is considered as constraint for the application of maintenance actions (preventive and corrective maintenances).

Concerning maintenance strategies in the energy generation, the operation and maintenance is considered as a sizeable share of a wind turbine generating. The most common maintenance practice is to perform scheduled maintenance on a regular basis. [4] determines the availability of the wind turbine according to the distance to shore, average (offshore) wind speed and of course amount of money to be spend for maintenance. [2] discuss recent studies on wind energy facility reliability and maintenance. [6] and [7] studies the wind power operations and the different critical factors that have significant impact on the turbine’s reliability and maintenance. [8] and [9] proposed several analytical models for predicting the lifetime of a system and developed an operation and maintenance strategy by quantifying risks and uncertain-ties based on the lifetime prediction. [10] and [11] treated the same problem of the last work by using the Markov process.

The previous works treated the production and maintenance problem of the power generating system. In this context, this paper proposes an optimal preventive maintenance based on an optimal production plan for a wind turbine power generation.

Our approach consists to minimize the total production and maintenance cost under several constraints in order to obtain the best combination of energy production and maintenance actions satisfying the random demand of electricity during a finite horizon. Inspiring from works of [12], our mathematical model shows the influence the influence of the production rate on the degradation degree of the wind turbine, and consequently on the maintenance strategy.
The remainder of the paper is organized as follows: Section II is presented the mathematical model of the problem which consists to minimize simultaneously the total average production and maintenance cost of the wind turbine system under operational and service level constraints. An analytical study is developed to determine the analytical expressions of the components of total cost and constraints. A numerical example is discussed in section III. Finally, section IV presents the conclusions of this work.

II. MODELING PRODUCTION/MAINTENANCE PROBLEM

Our model is a finite-horizon, we are concerned with the problem of jointly optimal energy production and maintenance planning problem formulation of a manufacturing system composed of a wind turbine that considered as energy recovery device provided by the Kinetic energy of wind and which provisions the load when there is a lack of electricity, and stores the surplus in battery system for energy storage when the power generated exceeds the load. In practice, a wind turbine is used to recover energy from the wind, against part, this one, it deflects the wind before that reaches the surface covered by the rotor. According to Betz law [13], a wind turbine can therefore never recover the total energy supplied by the wind.

The wind turbine is subject to a random failure. The probability degradation law of wind turbine is described by the probability density function of time to failure \( f(t) \) and for which the failure rate \( \lambda(t) \) increases with time and according to the production rate. That’s why; a preventive maintenance action is planned according to the production rate in order to reduce the machine failure and to improve the overall reliability and availability of the wind turbine.

Our objective is to establish simultaneously an economical production plan and an optimal preventive maintenance strategy of a wind turbine satisfying the randomly power demand with a given service level over a finite time horizon. The use of the production plan in maintenance strategy is justified by the fact of taking into account the natural influence of the production rate on the evolution of the failure rate of the wind turbine.

A. Notation

\begin{align*}
N & : \text{number of periods over the considered time horizon} \\
\Delta t & : \text{length of a production period} \\
N \Delta t & : \text{horizon of energy production} \\
B(k) & : \text{energy storage level during period } k (k = 1, \ldots H) \\
w_v(k) & : \text{energy power output during period } k (k = 1, \ldots H) \\
p_d(k) & : \text{average power demand during period } k (k = 1, \ldots H) \\
V_d & : \text{variance of demand} \\
C_p & : \text{production unit cost} \\
C_s & : \text{storage unit cost} \\
M_p & : \text{preventive maintenance cost} \\
M_c & : \text{corrective maintenance cost} \\
C_M & : \text{total maintenance cost} \\
f(t) & : \text{probability density function associated with the wind turbine time to failure} \\
\lambda(t) & : \text{Wind turbine failure rate} \\
V_1 & : \text{wind speed upstream of the turbine} \\
V_2 & : \text{wind speed downwind of the turbine} \\
V & : \text{wind speed passing through the turbine rotor} \\
\theta & : \text{probabilistic index}
\end{align*}

B. Problem Formulation

The aim of the production/maintenance plan is to determine the best combination of production, storage levels and failure rate of the wind turbine power generation system that minimizes the total costs over a finite horizon; taking into consideration the requirement of satisfying the fluctuating demand and the constraints on major variables. The problem can be stated as follows:
\[ \text{Min } f(W, B) = \text{Min } \sum_{k=1}^{N} f_k(W, B) \]
\[ = \text{Min } C_s \cdot E[B(H)] + \sum_{k=1}^{N-1} C_p \cdot E[W_v(k)] + C_s \cdot E[B(k)] + \varphi_j(j, W_v(j)) \]
\[ B(k) = B(k-1) + w_v(k) - P_d(k) \]  
\[ \text{Prob}(B(k) \geq 0) \geq \theta, \forall k \in \{1..H\} \]
\[ 0 \leq C_p \leq C_{\text{max}} \]

Where \( \{f_k, k = 1, \ldots, N\} \) denotes functions that represent the output power and battery storage costs, \( \{\varphi_j, j = 1, \ldots, N\} \) denotes functions for maintenance costs, the analytical expressions of these functions. The paper set defines the amount of stored energy for each time period \( k \) (2). The constraint (3) imposes the service level requirement for each period and denotes the lower physical limit of storage variables. Therefore, the chance constraint is used to guarantee feasible production plan. This constraint can also help the manager to analyses different and alternative scenarios of manufacturing and, therefore, help him to get insights about the future use of the wind turbine. The constraint (4) defines the maximum or the optimal value of the performance coefficient \( C_p \). Note that for a simple wind turbine; physically it’s impossible to recuperate more that 59% of the kinetic energy of wind [13].

C. Production Policy

The idea is to develop the expected output power and energy storage over a finite time horizon \([0, H]\). It’s assumed that the horizon \( H \) is portioned equally into \( N \) periods. The demand is satisfied at the end of each period.

Thus, this kind of problem can be formulated as a linear-stochastic optimal control problem under threshold stock level constraint. Actually the uncertainty about fluctuation of the demand brings randomness to the balance (2). Consequently, the inventory and production variables are stochastic and their statistics depend on the probabilistic distribution function of the demand.

- Average energy storage

The system model is described by an amount of stored energy at period \( k \) can be obtained using the following equation:
\[ E[B(k)] = E[B(k-1)] + E[w_v(k)] - E[P_d(k)] \]  

Where:
\[ B(0) = B_0 \]
\[ P_d(k) \] is the electric power demand at period \( k \).
The average amount of energy stored during period $k$ is given by:

$$E[B(k)] = \int_{(k-1)\Delta t}^{k\Delta t} tB(t) \, dt \quad (6)$$

- **Average output power**

Respecting the wind turbine efficiency constraint, the performance of every period $k$ cannot exceed a given maximal performance factor $C_p,max$.

Considering the wind speed passing through the turbine is uniform as $V$, with its value as $V_1$ upwind, and as $V_2$ downwind at a distance from the rotor. Extraction of mechanical energy by the rotor occurs by reducing the kinetic energy of the air stream from upwind to downwind.

The extractable power $P$ from the wind turbine can be expressed as:

$$P_o = P_k \cdot C_p \quad (7)$$

With:

$P_k$: the kinetic power content of the undisturbed upstream wind stream with $V=V_1$ and over a cross sectional area $S$:

$$P_k = \frac{1}{2} \rho \cdot S \cdot V_1^3 \quad (8)$$

And

$C_p$: the performance coefficient is a dimensionless measure of the efficiency of a wind turbine in extracting the energy content of a wind stream:

$$C_p = \frac{1}{2} \cdot (1-b^2) \cdot (1+b) \quad b = \frac{V_2}{V_1} \quad (9)$$

The average output power by time unit of the wind turbine can be calculated using the following equation:

$$P(V) = \int_{LB}^{LH} \frac{df(V)}{dV} P_o(V) \, dV \quad (10)$$

With:

$df(V) / dV$: The Weibull probability density function associated with the wind speed random variable $V_u$ is given by:

$$\frac{df(V)}{dV} = \frac{h}{A} \left( \frac{V}{A} \right)^{h-1} \exp \left( -\left( \frac{V}{A} \right)^b \right) \quad (11)$$

- $LB$: minimal velocity of the wind turbine started the production
- $LH$: maximal velocity

Consequently, the average output power during period $k$ is expressed as follows:
Service level

In order to solve our optimization problem, we transform the service level constraint into a deterministic equivalent constraint by specifying certain minimum cumulative output power quantities that depend on the service level requirements.

\[
\text{Prob}(B(k) \geq 0) \geq \theta, \forall k \in \{1..N\}
\]

For \( k=0,1,..,N \), we have:

\[
W_r(k) \geq P_{\theta}(B(k),\theta)
\]

\[
P_{\theta}(B(k),\theta) = \varphi^{-1}_d(\theta) \cdot V_d - B(k-1) + E[P_d(k)] \quad \text{with} \quad k = 1,..,N
\]

Where

\( P_{\theta}(\cdot) \): represents a minimum cumulative production quantity
\( V_d \): variance of demand at period \( k \)
\( \varphi_d \): cumulative Gaussian distribution function with mean \( E[P_d(k)] \) and finite variance \( V_d \geq 0 \)
\( \varphi^{-1}_d \): inverse distribution function.

D. Maintenance Policy

The maintenance strategy under consideration is the well-known preventive maintenance with minimal repair [8]. The wind turbine will work during a finite horizon \( N \Delta t \). The replacement or the preventive maintenance actions are practiced at periodic time’s \( jT \) following which the unit is considered as good as new. When a unit fails for the wind turbine between preventive maintenance actions, only minimal repair is made. It is assumed that the repair and replacement times are negligible. If we assume that \( \lambda(t) \) represents the wind turbine failure rate function, the average total cost by time unit of the maintenance action is expressed as following:

\[
\Phi_T = \frac{M_p + M_c \times \int_0^T \lambda(t)dt}{T}
\]

It has proven the existence of an optimal preventive maintenance period \( T^* \) in the case of increased failure rate [14].

- Maintenance policy optimization
The objective of maintenance strategy optimization is to determine the optimal period \( j^* \) for which the preventive maintenance is performed according to the production plan established during \( N \) periods. We noted that \( j^* \) can exceed \( N \), it means that no preventive maintenance to do. It’s clear that the maintenance strategy is correlated to the system degradation. That’s why we approved the influence of production rate on the degradation degree of wind turbine and consequently on the failure rate \( \lambda(t) \). Since that, this last is cumulated at the beginning of each production interval and progresses in each interval according to production rate.

Formally the failure rate in the interval \( j \) is expressed as following:

\[
\lambda_j(t) = \lambda_{j-1}(\Delta t_{j-1}) + \frac{W_{v}(j)}{U_{\text{max}}} \cdot \lambda_{n}(t) \quad \forall \ t \in \left[0, \Delta t \right]
\]  

(15)

Since that we can express \( \lambda_i(t) \) as following:

\[
\lambda_i(t) = \lambda_0(t_0) + B_i + \frac{W_{v}(i)}{U_{\text{max}}} \cdot \lambda_{n}(t)
\]  

(16)

With:

\[
B_i = \sum_{l=1}^{i-1} \frac{W_{v}(l)}{W_{v_{\text{max}}}} \cdot \lambda_{n}(\Delta t_l)
\]  

With:

\[
\lambda_{n}(k) \text{ is the nominal failure rate which corresponds to the maximal production capacity } W_{v_{\text{max}}}. 
\]

Since that we can expressed the average total cost per time unit of maintenance action:

\[
\phi(j) = \frac{M_p + M_c \times A_j}{j \cdot \Delta t} \quad \text{With } \quad j = 1, \ldots, N
\]  

(18)

With \( A_j \) represent the average number of failure is given by:

\[
A_j = \sum_{i=1}^{j} \int_{0}^{\Delta t} \lambda_i(t) dt = \sum_{i=1}^{j} \int_{0}^{\Delta t} \left[ \lambda_0(t_0) + B_i + \frac{W_{v}(i)}{W_{v_{\text{max}}}} \cdot \lambda_{n}(t) \right] dt
\]

\[
\Rightarrow A_j = j \cdot \int_{0}^{\Delta t} \lambda_0(t_0) dt + \sum_{i=1}^{j} \int_{0}^{\Delta t} B_i dt + \sum_{i=1}^{j} \int_{0}^{\Delta t} \frac{W_{v}(i)}{W_{v_{\text{max}}}} \cdot \lambda_{n}(t) dt
\]
By minimizing the average total maintenance cost, we obtain the optimal period \( j^* \Delta t \), if it exists, for which we practiced the preventive maintenance action.

The existence of the optimal period \( j^* \), determines by the following relation:

\[
\exists j^* \text{ if } \chi_{j-1} \leq \frac{M_p}{M_c} \leq \chi_j \quad \text{With} \quad \chi_j = j \cdot A_{j+1} - (j+1) \cdot A_j
\]  

(20)

III. NUMERICAL EXAMPLE

A simple example of a hypothetical wind farm, whose sales are strongly influenced by the fluctuation of demands (Electricity of housing) and the storage level, is not perfectly known. We suppose that the failure time of wind turbine has a degradation law characterized by a Weibull distribution, tries to develop an aggregated production/maintenance plans which minimizes total costs over a finite planning horizon: \( H=12 \) periods.

In this example, we left on the use of such a wind turbine type WT6000 having the following characteristics:

- Power: 6000 W to 12 m/s
- Startup speed: 2.5 m/s
- Nominal speed: 12 m/s
- Survival speed: 65 m/s
- Rotor diameter: 5.5 m
- Mean wind speed: 6 m/s
- Battery Voltage: 12V, Battery Capacity: 100Ah

The remainder of the input data is presented below: Production cost of a KW of energy: \( C_p =3 \mu \) (monetary units)

Storage cost of 1 KW: \( C_s =6 \mu \)

Customers’ satisfaction degree (required service level): \( \theta_i=100\% \)

The random demand of electricity is characterized by a Normal distribution with mean and variance given respectively by

\( \hat{P}_d \) and \( \hat{V}_d = 10 \). From the average wind speed (6 m/s), we established the Rayleigh distribution of wind using the formula of Weibull probability density with parameters \( A=6 \) and \( k=2 \).

Fig.1 shows the average power random demand during the finite horizon.
For the maintenance strategy, the Weibull scale and shape parameters are respectively $\beta = 100$ and $\alpha = 2$ and the cost associated with a corrective and preventive maintenance action are respectively $M_c = 3000$ monetary and $M_p = 500$ monetary.

We recall that in the case of Weibull distribution we have:

$$\lambda(t) = \frac{\alpha}{\beta} \left( \frac{t}{\beta} \right)^{\alpha-1}$$

Thus, the failure rate of wind turbine is given:

$$\lambda_i(t) = \lambda_i(t_0) + \sum_{j=1}^{i-1} \frac{w_j(i)}{w_{\text{max}}} \cdot \frac{\alpha}{\beta} \left( \frac{\Delta t}{\beta} \right)^{\alpha-1} + \frac{w_i(i)}{w_{\text{max}}} \cdot \frac{\alpha}{\beta} \left( \frac{t}{\beta} \right)^{\alpha-1}$$

Thus applying the numerical procedure we obtained simultaneously the optimal production plan, the amount of stored energy and the optimal maintenance period that are presented in figures 2, 3 and 4.
Simultaneously with the optimal output power production plan, we find the optimal maintenance period. Figure 4 presents the curve of the average total maintenance cost, $\phi(j)$, as function to $j$. We obtain the optimal number of preventive maintenance action that equals to $j^*=4$. It means that over the finite horizon $H$ of 12 periods. A preventive maintenance actions must be down every $T^*=j^* \Delta t$, with a minimal average total cost of maintenance action equals to $\phi^*=21.63934 \mu$. 

![Fig. 3. Average output power of the wind turbine per period](image1)

**IV. CONCLUSION**

This paper described a constrained stochastic production and maintenance planning problem for a wind turbine power generation system in order to satisfy a random electricity demand under service level. Wind turbine is subjected to randomly failure. A minimal repair is practiced at every failure.

Given the technical characteristics of a wind turbine as well as its different operating and maintenance costs respecting the service level, we have developed an analytical production and a maintenance model completed by jointly optimization in order
to obtain an optimum production plan and an economical maintenance planning should be performed on the wind turbine taking into account the influence of the production rate on the wind turbine failure rate.

REFERENCES


