Optimization of a Battery Pack in Plug-In Hybrid Electric Vehicles

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Abstract — One of the most important components of a Plug-In Hybrid Electric Vehicle (PHEV) is the battery pack. A large battery pack will allow the car to travel in charge-sustaining mode for a longer period of time, thereby reducing fuel consumption. However, a large pack will result in a high capital and operating cost. In this paper, the optimal number of batteries for the PHEV is determined. The vehicle model was developed using Autonomie and the optimization algorithm was carried out using the actual vehicle model. Due to the large computational time of the problem, the optimization was also executed using an empirical model. Comparisons are provided between the two approaches in terms of accuracy and computational time.

Keywords — Optimization, surrogate models, plug-in hybrid electric vehicle, multi-objective optimization

I. INTRODUCTION

Electric vehicles (EV) and plug-in hybrid electric vehicles (PHEV) have the potential to considerably decrease consumption of fossil fuels. Due to the increasing demand for alternative fuels, numerous car manufactures have looked into developing electric vehicles.[1] An important component of the plug-in hybrid electric vehicle is the battery pack. Different manufacturers use different energy storage systems. For example, the Chevrolet Volt extended range electric vehicle and the Nissan Leaf both use lithium ion batteries while the Toyota Prius and Honda Insight use nickel metal hydride batteries. However, lithium-ion technology has recently emerged as the main contender for applications in PHEVs due to its high specific energy and power. [2] Therefore, this project will consider optimization of a lithium ion battery pack for a PHEV.

The number of cells in the battery pack and their arrangements have a significant effect on the performance and efficiency of the car. Using a large battery pack can be beneficial in reducing CO2 emissions, however it can be expensive. Likewise, adding more cells in series (i.e. increasing the voltage) can increase acceleration performance and efficiency, but this will again incur a higher cost. This work will look at determining the optimal number of batteries that should be used in hybrid electric vehicles. It will examine both battery cells in series as well as in parallel. The goal to minimize emissions and cost.

II. BACKGROUND AND THEORY

This section will provide a brief overview of the important theory behind PHEV’s and the different vehicle architectures that are commonly utilized. It will then examine the important equations that are used in modeling a battery. Finally, a brief overview of multi-objective optimization will be provided along with a discussion of genetic algorithm and surrogate models.

A. Plug-In Hybrid Electric Vehicle

Plug-In Hybrid Electric Vehicles utilize an energy storage system as one method of generating energy. Once there is no more charge left in the battery, the vehicle can be plugged into the outlet in order to recharge the battery. When driving, the vehicle normally starts in charge depleting mode; a mode that uses only the battery. Upon reaching a certain state of charge, the gasoline engine will kick in for the rest of the trip.[3] This mode is referred to as charge sustaining mode and an example of this is shown in Fig. 1.

In Fig. 1, the vehicle starts at a particular state of charge and as the distance increases, the charge left in the battery decreases.
Upon reaching a certain critical value, the vehicle switches to charge sustaining mode, where the gasoline engine is used to keep a constant state of charge value. There are numerous control strategies that can be used in determining at what SOC the engine should kick in. However, for the purposes of this project, the battery will have a constant critical point.

Another important component of many plug-in hybrid electric vehicles is regenerative breaking. This is a mechanism that converts the kinetic energy of the vehicle into energy that can be stored in the battery. PHEVs use this type of energy to extend the range of the battery.

B. Vehicle Architecture

Before the car can be modeled, there are multiple vehicle architectures that are available. The two common architectures are the series hybrid and the parallel hybrid. Each has its own advantages and disadvantages and determining the optimal configuration depends on the needs of the user. First, let us consider the series hybrid architecture shown below.

In a series configuration, the plug-in hybrid vehicle uses only the electric motor to produce the torque to move the car. The internal combustion engine in this architecture is used only to charge the battery, which in turn provides the energy for the electric motor. The battery can be additionally charged using regenerative breaking.

This type of configuration is useful for cars that have a constant duty cycle. In addition, using a series vehicle allows the internal combustion engine to run at maximum efficiency. The disadvantage of this architecture is that the electric motor must be powerful enough to meet the demands on the drive cycle.

Another common architecture that is often used in PHEVs is the parallel configuration, shown in Fig. 3. The parallel configuration uses both the electric motor as well as the internal combustion engine to generate the torque for the wheels. Unlike the series hybrid, the battery is charged only by regenerative breaking or by plugging the car into an outlet. There are multiple other architectures that can be used in building PHEVs but for the purposes of this project, only these two will be considered.

C. Energy Storage System Model

The energy storage model used in this project can be decomposed into three parts as shown in Fig. 4. Each part will be discussed individually. Block A describes the voltage calculation of the cell. The model calculates the open circuit voltage (i.e. the voltage without a current) and internal resistance based on the battery temperature and state of charge (SOC). These values are obtained using a look-up table in Simulink.
In order to determine the terminal voltage of the battery, a battery model must be utilized. These models can be either physics-based electrochemical models or equivalent circuit models. Although electrochemical models provide better accuracy, equivalent circuit models is used in vehicle models due to the computational intensity of electrochemical models. The equivalent circuit model used in Simulink is shown in Fig.5.

Once again, the term $V_{OC}$ refers to the open circuit voltage or the ideal voltage. $R_{charge}$ and $R_{discharge}$ is the total internal resistance for charge and discharging while $V_{bat}$ represents the actual voltage of the battery. Using the equivalent circuit, and the values from (2) and (3), the voltage of the battery can be calculated as:

$$V_{batt} = V_{OC} - R_{int}I_{ess}$$  \hspace{1cm} (3)

Block B is used in calculating the state of charge in the battery. The state of charge represents the amount of electric charge left in the battery. 100% SOC means that it is completely charged while 0% indicates that there is no charge left. In calculating SOC, the amount of useable capacity must first be determined as follows:

$$Cap_{used} = \int I_{in} \, dt + (1 - SOC_{int}) \times Cap_{max}$$ \hspace{1cm} (4)
Once the capacity is known, the absolute SOC can be determined as shown in (5).

\[
SOC_{\text{abs}} = \frac{\text{capacity}_{\text{max}} - \text{capacity}_{\text{used}}}{\text{capacity}_{\text{max}}}
\]  
(5)

Block C represents the calculation for the temperature of the battery. The model uses a lumped parameter thermal model to predict the average internal battery temperature. The obtained temperature is then used in calculating the SOC and battery voltage in blocks A and B.

D. Multi-objective Optimization

The problem that we are considering in this project, however, is multi-objective. The general expression for multi-objective optimization is given as:

\[
\min_x F(x) = [F_1(x), F_2(x), \ldots, F_k(x)]^T
\]  
(6)

Subject to:

\[
g_j(x) \leq 0, j = 1, 2, \ldots, m; \quad h_i(x) = 0, i = 1, 2, \ldots, n
\]  
(7)

where \( k \) is the number of objective functions, \( m \) is the number of inequality constraints and \( n \) is the number of equality constraints.

The biggest challenge in multi-objective optimization problems is that we need to find optimal solutions in the presence of conflicting objective functions.[6] In this project, increasing the number of cells will decrease the CO\(_2\) but will increase the cost. We will eventually run into a situation where we cannot improve one objective function without damaging or increasing the value of the other objective(s). To deal with this issue of trade-offs, the concept of Pareto optimality must be introduced. In essence, the Pareto optimal[6,7] is defined as follows:

A point \( x^* \) is Pareto optimal if and only if there does not exist another point in the sample space such that:

\[
f_i(x) \leq f_i(x^*), i = 1, 2, \ldots, n
\]  
(8)

A set of Pareto optimal solutions is referred to as the Pareto front. Mathematically, each solution that satisfies the Pareto condition is an acceptable solution to the multi-objective problem. Therefore using the Pareto front can give the user an indication of the trade-offs between the different objectives.

A common method of solving optimization algorithms involves combining the different optimization functions into one expression. This can be accomplished using a weighted sum method, where each objective function is multiplied by a corresponding weight.[8] It is one of the simplest methods to implement and the method is shown below:

\[
\min_x \sum w_i f_i(x) \quad i = 1, 2, \ldots, n
\]  
(9)

Subject to: \( x \in X \)

where, \( X \) represents the sample space. The weights are chosen according to the user’s requirements.
In many instances, the different objectives will have different orders of magnitude. With respect to the current project, fuel consumption will have much lower values than the cost of battery packs. Therefore, the weights must be normalized in order to obtain accurate results.[9] Multiple methods have been proposed in literature on how these methods can be normalized. One approach is to determine the maximum value for each objective function and normalize with respect to this value[10]. The advantage of this method is that the values for each objective function will be between 0 and 1.

One of the problems with using weighted sums is how to select the weights. It is sometime difficult to physically interpret what the weights signify with respect to the different objective functions[11]. The optimization can be solved using different weights for cost and fuel consumption in order to get a better idea of the trade-offs between these two outputs.

E. Surrogate Models

In engineering problems, the governing equations for a given system can be very complex and the computational time required to run these problems can be large. This is especially true for the PHEV model that was created in Simulink; a model which needs to simulate the entire drive cycle of a car for each iteration. One method of reducing the computational time is to use surrogate based analysis and optimization (SBAO).[10] This method has been used extensively in the literature; for example, it has been used by numerous authors in the development of aerospace systems.[11,12,13]. Using SBAO, the actual model of the PHEV can be approximated by an empirical model. The optimization procedure is then carried out on this new model. The process involves first designing the experiments, determining the output at these selected values, obtaining the empirical model and finally conducting an optimization. The steps are shown in Fig. 6, which is followed by a brief discussion of each step.

The first step in the optimization process is the design of experiments. There are numerous approaches that can be taken when determining what is the best location to run the experiments. One of the most basic experimental designs is the 2k design, where k represents the number of factors.[14] Although the approach is simple, the number of runs grows exponentially as the number of factors increases. In situations where a large number of runs are required, a fractional factorial design can be utilized. However, these designs cannot be used in generating a second order polynomial model.

\[ Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \beta_4 X_1^2 + \beta_5 X_2^2 \]  

(10)

![Diagram](image).

When polynomial models are considered, Central Composite Designs (CCD) and Box-Behnken Designs are commonly employed. Central Composite Designs are 2k designs that are augmented with central points and star points. The Box-Behnken Designs are a fraction of the 3k, which only use three levels instead of five. Both these designs can be used to generate models that take into account curvature of the response surface.[14]

Numerous other design methods have been proposed in literature. For example, Latin Hypercube Designs have been proposed in order to sample large design spaces.[15] This project will involve using a central composite design in order to
generate the required model. The Simulink model will be evaluated at each of the designed experiments in order to generate a data set.

Once the data is generated, the next step is to select the appropriate model. There are numerous methods available in generating the surrogate models. Examples of these include, Kriging models, Artificial Neural Networks and Support Vector Machines to name a few.[10] In this optimization, response surface methodology will be used for determining the empirical models.

The response surface methodology, discussed in greater detail by Box & Draper (1987),[16] involves searching the design space for optimal solutions. Using an initial starting point, a first-order model is proposed and the experiments are designed using a two-level factorial design. If the first order model is not adequate, then a higher order model is proposed. Linear regression can be applied in order to determine the model parameters. The equation for determining the parameter is shown below:

$$\beta = (X^T X)^{-1} X^T y$$  \hspace{1cm} (11)

Once the model has been identified, a method of steepest ascent is normally used to move to an area with a larger response. If the problem is a minimization problem, the method of steepest descent can be applied. The new optimal point is used as the next starting value for generating the new model. Commonly, polynomial models, such as the model shown in equation 10, are utilized in modeling the response surfaces.[21]

### III. MODEL DEVELOPMENT

This section will first discuss how the vehicle was modeled in Autonomie and how the Simulink files were generated. It will provide a general description of the drive cycles used in the project and how the car battery pack was designed.

#### A. Vehicle Modeling Approach

The vehicle modeling approach that will be used in this project is described in Fig. 7. Two design variables will be used in this project: the number of cells in parallel and the number of cells in series. As shown in Fig. 7, these two inputs will be passed into the cost model in order to calculate the cost of the battery pack. The inputs will also be used to calculate the fuel consumption of the PHEV by using a Simulink model. This is a black box model, where the number of cells is changed resulting in a corresponding change in fuel consumption. It should be noted that the inputs into the vehicle Simulink model contains multiple parameters and the model generates multiple outputs. However, in the optimization algorithm, all inputs expect the number of cells is kept constant. The goal is to minimize the cost of the battery pack while also minimizing fuel consumption. This is a multi-objective problem, which will be converted to a single objective function using a weighted sum.

![Fig. 7 Vehicle model developed in Autonomie for a series PHEV](image)
Before the optimization process can be carried out, the vehicle black box model, described in Fig. 7, must be first built in Autonomie. The software provides a graphical user interface that allows users to combine different components of the car’s powertrain. It then runs the car through various drive cycles and provides an output of the car’s performance (i.e. fuel consumption, engine efficiency etc…). As described in section 2, there are various vehicle architectures that can be employed in constructing the power train for the plug-in hybrid electric vehicle. For this project, the series architecture was chosen since it is one of the simplest ones to construct. The developed model in Autonomie is shown below:

![Vehicle model developed in Autonomie](image)

**Fig. 8 Vehicle model developed in Autonomie for a series PHEV**

The model above uses two electric motors, one for the front axle and the other for the rear axle. Note that in this architecture, there is no mechanical connection between the engine and the wheels. Using a generator, the mechanical energy from the internal combustion engine (blue box), is used to produce electricity that is stored in the battery (red box). The battery then drives the electric motor which in turn propels the vehicle (green box). The software Autonomie uses multiple equations for each of the vehicle components. It then generates a Simulink file that can be used in the optimization algorithm. Although the battery section is the only component that is being optimized, all the other components of the vehicle must be considered in the optimization process in order to determine the fuel consumption of the vehicle.

### B. Battery Pack Design

Once the vehicle model has been generated, the next step is to determine the design of the battery pack. The cells can be connected in parallel or series depending on the required specifications. In order to increase the battery capacity, the cells need to be connected in parallel. A higher battery capacity means that the vehicle will spend more time in charge sustaining mode and therefore will need less fuel. Increasing the battery voltage, on the other hand, requires the cells to be connected in series. A higher voltage means that there will be less current required to provide the needed torque, and likewise will result in a lower usage of fuel. In order to get a better idea of how the battery configuration affects the voltage and capacity, consider Fig. 10.

The picture on the left of Fig. 10 presents a battery pack that is connected in parallel. The voltage does not change; however, the capacity is doubled. In a similar way, Fig. on the right shows a series connection where the voltage is doubled but the capacity stays constant. The battery specifications of a typical A123 cell are shown in Table 1.

![Cell configuration](image)

**Fig. 9 Configuration of cells connected in parallel (left) and series (right)**

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IV. RESULTS AND DISCUSSION

The optimization of the model described in section 2 will be carried out using two different approaches. The first approach will use the actual vehicle model that was generated using Autonomie. The second approach will look at using response surface methodology in order to generate an empirical model for the vehicle.

Table 1: Battery specification of an A123 cell

<table>
<thead>
<tr>
<th>Battery Components</th>
<th>Battery Specs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost per Cell ($)</td>
<td>35</td>
</tr>
<tr>
<td>Battery Mass (kg)</td>
<td>0.5</td>
</tr>
<tr>
<td>Mass Packing Factor</td>
<td>1.25</td>
</tr>
<tr>
<td>Battery Dimensions (mm)</td>
<td>7.25x160x227</td>
</tr>
<tr>
<td>Volume Packing Factor</td>
<td>1.25</td>
</tr>
<tr>
<td>Voltage (V)</td>
<td>3.3</td>
</tr>
<tr>
<td>Cell Capacity (Ah)</td>
<td>19.5</td>
</tr>
</tbody>
</table>

A. Optimization using Actual Vehicle Model

1) Development of the Multi-Objective Function

The amount of fuel consumed by an electric vehicle is an important parameter when determining the quality of the hybrid car. A car that uses a large amount of gasoline will not only incur a higher operating cost but will only generate a large amount of CO2 emissions. Using a larger battery pack can lower the fuel consumption as the car can travel for a longer period of time without needing to use the gasoline engine. However, if the battery pack becomes too large, the increased weight can result in a higher rate of fuel consumption. Therefore, the first objective of this project will be to minimize fuel consumption. Note that the value for fuel consumption can be obtained using the approach shown in Fig. 7; the fuel value here represents the total amount of gasoline consumed during the entire drive cycle.

Another variable that will change during the optimization is the mass of the car. As the number of batteries increases, so will the mass of the car. A car that is heavier will consume a large amount of fuel. The Simulink model uses a pre-defined mass and therefore, for each iteration, the mass of the battery pack must be determined and inputted into the Simulink model. The mass of a battery can be calculated as follows:

\[
M_{battery} = (Mass \ Per \ Cell)^* (Packing \ Factor) * NumCells \quad (12)
\]

The second objective that needs to be considered is the cost. The Simulink model does not calculate the cost of the battery pack and therefore a separate model is required. Assuming that the cost of each cell is $35, the cost of the battery pack can be calculated by multiplying the number by the total number of cells used. In order to make the problem slightly more complicated, it was assumed that the wiring for cells in parallel is less expensive than the wiring for cells in series. Wiring for parallel cells will cost $5 per cell while wiring for series cells will be $10 per cell. The final model for the cost objective function is:

\[
Cost = 35 * x(1) * x(2) + 10 * x(1) + 5 * x(2) \quad (13)
\]

Where \(x(1)\) and \(x(2)\) represent the number of cells in series and parallel. Using the weighted sum method, the two objectives can be written as a single objective:

\[
Obj = (Fuel \ Consumption) \frac{w_1}{a_1} + (Cost) \frac{w_2}{a_2} \quad (14)
\]

where \(w_1\) and \(w_2\) are the weights of the function and \(a_1\) and \(a_2\) are the normalization factors.

One method of normalizing the objective function is to determine the maximum value of fuel consumption and cost. Using this approach, the values for each objective function will be between 0 and 1. The maximum value of fuel consumption was determined to be 7.2 kg of gasoline while the maximum cost was estimated to be $3725.

Now that the objective function has been defined, the constraints for the design variables will be developed.
2) Development of design variable constraints

The inequality constraints are determined by the volume constraints of the vehicle as well as the voltage range of the electric motor. Let us first consider the volume constraints. Although a large battery pack will yield lower fuel consumption, there is a limit to the size of the battery pack. To determine the volume of the battery pack the following equation can be utilized:

\[ V_{\text{battery}} = (\text{Length} \times \text{Width} \times \text{Height} \times \text{Packing Factor} \times \text{NumCells} \]  \hspace{1cm} (15)

The packing factor refers to the additional wiring and packaging that is required in a car battery. For example, lithium-ion battery packs normally require a battery management system to maintain constant voltage and temperature. The packing factor takes into account this additional volume. The numerical value shown in Table 1 is obtained by Autonomie software and will be used in the optimization.

The upper bounds on the number of cells in series and parallel are restricted based on the volume of the battery pack. The PHEV modeled in this study will be assumed to have a maximum battery volume of 0.13 m³. The value was chosen based on the battery pack size of the Chevrolet Volt, which has a size of 0.1 m³. Using (15), it can be shown that the maximum number of cells that can be present in the battery pack must be 390 cells. If we assume that there can only be a maximum of 13 parallel modules, then the upper limits on the two design variables are given by:

\[ x_1 \leq 30 \text{ and } x_2 \leq 13 \]

In addition, to volume, the motor of the hybrid is normally rated for a particular voltage range. Therefore there must be a minimal amount of voltage supplied in order for the engine to function efficiently. For this project, it will be assumed that the battery pack must be able to generate a minimum of 20 volts. Therefore, the lower bound in the number of cells in series is 6.

\[ x_1 \geq 6 \]

There is no lower bound for the number of cells in parallel since the capacity of one cell is sufficient. To summarize, the upper and lower limits of the design variables are:

\[ 6 \leq x_1 \leq 30 \text{ and } 1 \leq x_2 \leq 13 \]  \hspace{1cm} (16)

3) Optimization Results

The problem presented in this paper represents an integer programming problem since the number of cells in a battery can only take integer values. Since a black box model is being applied, it cannot be assumed that the problem is convex. Therefore genetic algorithm was used in finding the optimal minimum. One of the biggest limitations with running the actual vehicle model is the computational time. Therefore as the population size increases and the number of generations increases, the program will not converge in a reasonable time. Therefore, the population size as well as the number of generations was lowered in the optimization algorithm. A population size of ten was utilized along with five generations.

Table 2 shows the optimal number of cells in series and parallel for a particular weight used in the objective function. As previously mentioned, when using a weighted sum approach, it is difficult to determine exactly what weights should be used. However, it appears that when a lower weight is used for fuel such as 0.2 and 0.5, the optimization algorithm converges at the lower bound for both the number of cells in series and parallel. This indicates that for this particular problem, the cost is a significant factor.

When a higher weight of 0.8 was placed on fuel, the optimization converged to a solution that had a larger number of cells. In this scenario, using a large amount of fuel will incur a larger penalty in the optimization due to the higher weight. Ideally the optimization should be run using many different weights in order to get a better idea of the Pareto front. Only three simulations were carried out in this project due to time limitations.
Table 2: Optimization results for different weights

<table>
<thead>
<tr>
<th>Weight</th>
<th>Number of Cells in Series</th>
<th>Number of Cells in Parallel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel Weight ($\beta_1$)</td>
<td>0.5</td>
<td>6</td>
</tr>
<tr>
<td>Cost Weight ($\beta_2$)</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Fuel Weight ($\beta_1$)</td>
<td>0.2</td>
<td>6</td>
</tr>
<tr>
<td>Cost Weight ($\beta_2$)</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>Fuel Weight ($\beta_1$)</td>
<td>0.8</td>
<td>7</td>
</tr>
<tr>
<td>Cost Weight ($\beta_2$)</td>
<td>0.2</td>
<td></td>
</tr>
</tbody>
</table>

B. Optimization using Surrogate Models

Optimization using surrogate models will use the same procedure as shown in Fig. 7, with one exception. Instead of using the Simulink model, an empirical model will be used instead. Genetic algorithm will once again be used in the optimization step.

1) Development of the Surrogate Model

Central composite design of experiments was used in generating the empirical model. In order to determine the value of the variable in non-coded form, the following equation can be applied.

$$X = \bar{X} + X' \left( \frac{R}{2} \right)$$ (17)

Where, $\bar{X}$ represents the mean value, $X'$ represents the coded value and $R$ represents the range. Using (17) and the constrains in (16), the designed experimental runs are shown in Table 3. The values of the above design can be used to generate the model outputs. The weights were set to $\beta_1 = 0.5$ and $\beta_2 = 0.5$. A similar procedure as shown in Fig. 7 was used. The only difference between the two methods is that the actual vehicle model is replaced now by an empirical model. Using linear regression, the following model was determined.

$$Y = 0.6977 - 0.0169X_1 - 0.0433X_2 + 0.0433X_1X_2 + 0.000373X_1^2 + 0.0022X_2^2$$ (18)

The term $X_1X_2$ are used to represent the interaction between the two factors while the squared terms are required to capture the curvature of the model. In order to determine the accuracy of the model predictions, the predicted value from the empirical model is compared with the actual value from the vehicle model. The results are summarized in Table 4.

Table 3: Experimental design coded and non-coded values for the number of cells in parallel and series

<table>
<thead>
<tr>
<th>Coded Value ($x_1$)</th>
<th>Coded Value ($x_2$)</th>
<th>Actual Value ($x_1$)</th>
<th>Actual Value ($x_2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1</td>
<td>28</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>28</td>
<td>12</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>1.21</td>
<td>0</td>
<td>30</td>
<td>7</td>
</tr>
<tr>
<td>-1.21</td>
<td>0</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>0</td>
<td>1.21</td>
<td>18</td>
<td>13</td>
</tr>
<tr>
<td>0</td>
<td>-1.21</td>
<td>18</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>18</td>
<td>7</td>
</tr>
</tbody>
</table>
Table 4: Comparison between predicted and actual value at different conditions

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>$x_2$</th>
<th>Predicted Value</th>
<th>Actual Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>5</td>
<td>0.8282</td>
<td>0.8135</td>
</tr>
<tr>
<td>17</td>
<td>6</td>
<td>0.8248</td>
<td>0.8159</td>
</tr>
<tr>
<td>13</td>
<td>7</td>
<td>0.7780</td>
<td>0.7783</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>0.7472</td>
<td>0.7444</td>
</tr>
</tbody>
</table>

Table 4 shows that for three different starting values, the predicted value is in good agreement with the actual value. Therefore, the actual model can be replaced with the surrogate model for the purposes of this optimization.

2) Optimization Results and Discussion

The results from both the actual model and the empirical model are shown in Table 5 for different weights of fuel and cost. The Table shows that when weights of 0.5 and 0.2 are used for fuel, the same values are obtained for both approaches. At a higher fuel weight, however, there are discrepancies between the two approaches. One possible reason for the different values could be the small number of populations and generations used for the actual model optimization. Using too low of a number might result in a poor estimate of the optimal. It should be noted however, that the trend obtained from both models is the same. When a larger weight was placed on the fuel, the size of the battery pack increased. Therefore, replacing a theoretical model with an empirical one appears to be a strong alternative for optimization problems where computational time is an issue.

Table 5: Comparison between predicted and actual value at different conditions

<table>
<thead>
<tr>
<th>Weight</th>
<th>Actual Model Results</th>
<th>Surrogate Model Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Fuel Weight (\beta_1)$</td>
<td>0.5</td>
<td>[6,1]</td>
</tr>
<tr>
<td>$Cost Weight (\beta_2)$</td>
<td>0.5</td>
<td>[6,1]</td>
</tr>
<tr>
<td>$Fuel Weight (\beta_1)$</td>
<td>0.2</td>
<td>[6,1]</td>
</tr>
<tr>
<td>$Cost Weight (\beta_2)$</td>
<td>0.8</td>
<td>[6,1]</td>
</tr>
<tr>
<td>$Fuel Weight (\beta_1)$</td>
<td>0.8</td>
<td>[7,12]</td>
</tr>
<tr>
<td>$Cost Weight (\beta_2)$</td>
<td>0.2</td>
<td>[6,9]</td>
</tr>
</tbody>
</table>

V. CONCLUSIONS

The number of cells in series and parallel for a battery pack in a PHEV was optimized. The results from the optimization showed that when reducing cost was a priority, the smallest possible battery pack should be used with 1 set of cells in parallel and 6 in series. However, when fuel consumption is important, the optimal solution is 7 cells in series and 12 in parallel.

The computational time however, was an issue and therefore an empirical model was used to reduce the computational time. Using response surface methodology, a second order model was used to model the vehicle. Comparison of the actual value with the predicted value showed that the surrogate model was adequate. Although the empirical model obtained slightly different values when compared to the actual model, it was able to accurately capture the trend. Therefore, this work shows that surrogate model optimization is a viable option when computational time is an issue.

REFERENCES


**BIography**

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