

Inventory Routing Problem for Deteriorating Items with Multi Tours

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Abstract—The design of a supply chain has been considered by many practitioners and researchers recently. One model that is used to optimize distribution from a single facility (depot) to a set of retailers over a given planning horizon and considering stock management decisions is known as Inventory Routing Problem (IRP). This model tries to optimize the trade-off between inventory and transportation costs. Designing IRP becomes more complex if the items are deteriorating. In this research, we develop IRP for deteriorating items where there is possibility that every vehicle has more than one route. Multi routing is suitable for deteriorating items since it minimizes delivery time for each item. We consider more than one vehicle at the depot and each vehicle has similar maximum capacity. Since the problem is an NP hard problem, we use Particle Swarm Optimization (PSO) method to solve the problem. A numerical example and sensitivity analysis are conducted to show the deteriorating rate effects at the vehicle and inventory, and also inventory and transportation cost to the total supply chain cost. The results show that the total cost/hour is more sensitive to different values inventory deteriorating rate than transportation deteriorating rate.

Keywords— Inventory, IRP, deteriorating items, multi tour, PSO

I. INTRODUCTION (*HEADING I*)

Inventory and transportation cost are two costs that significantly affect the supply chain management cost. Many companies try to reduce these two costs, however these costs are conflicting. Inventory routing problem (IRP) is a model that concern to minimize inventory and transportation costs simultaneously. In this model a single product is distributed from a single facility to a set of retailers over a given planning horizon.

Nowadays, research in the IRP area with consideration of many factors are developed continuously. Lei et al. [1] developed an integrated production, inventory and distribution model. In their model, they considered heterogeneous transporters with non-instantaneous traveling times and each customer have their own inventory capacity. Aghezzaf et al. [2] developed IRP with stable demand rates, economic order quantity policies and multi tours. Yu et al. [3] proposed a new algorithm to solve large scale IRP problems with split delivery and vehicle fleet size constraints. Liu and Lee [4] proposed a two-phase heuristic methods to solve vehicle routing problem with time windows (VRPTW) and inventory control decision simultaneously. Coelho et al. [5] wrote a comprehensive review of inventory routing problem during the last 30 years. They mentioned about varies product characteristics such as single product, two products and many products however they did not mention deteriorating as one of product characteristic. They did not present any papers about inventory routing problems for deteriorating items, although there are many products in practice have deteriorating characteristic.

Deteriorating items are the items that become decayed, damaged, evaporative, expired, invalid, and devaluation (Wee, [6]). Rau et al. [7] developed a multi-echelon inventory model for deteriorating items to get the optimal total supply chain cost. A production-inventory model for deteriorating items by considering multiple production setups and rework was developed Widyadana and Wee [8]. Wee and Widyadana [9] developed an economic production quantity model for deteriorating items by considering stochastic preventive maintenance time and rework process. Li et al. [10] reviewed studies of deteriorating items. In their paper, they stated some factors that have been considered in deteriorating items studies as deterministic and stochastic

demands, two echelons deteriorating models, and production-inventory models. They did not mention any research of deteriorating items by considering vehicle routing. Similar research has been taken by Baker et al. [11]. There are some interesting inventory deteriorating models by incorporating different factors such as price discounts, multi echelon inventory, and permissible delay in payment. However, this review does not mention any deteriorating inventory model by considering transportation cost.

Some papers show the importance of perishable or deteriorating items in IRP. Hwang [12] developed an inventory and distribution model for food distribution. Hsu et al. [13] developed vehicle routing problem with time windows by considering perishable items. In their model they consider vehicles cost, transportation cost, inventory cost, energy and penalty cost. They assumed an inventory cost of perishable items as the cost of lost inventory due to opening cargo and do not consider deteriorating rate during vehicles traveling time and period when products are stocked in customer warehouse. Similar research has been conducted by Osvald and Stirn [14]. They considered some food perish during the transportation process of the products, but they did not consider inventory in their model. Coelho and Laporte [15] applied three different selling priority policies to handle perishable products and proposed optimal joint decision model to replenish customer demands from one producer to some customers for perishable items by considering both inventory and transportation costs. Popovic et al. [16] developed an inventory routing problem solution for fuel delivery and Stalhane et al. [17] developed a solution of routing and inventory problem for liquefied natural gas (LNG). Although fuel and LNG can evaporate during transportation and stock time, but they did not deliberate deteriorating in their model.

Deteriorating items are common in reality and many items deteriorate faster during the transportation time than stocking period. However there are only few research considering deteriorating items in the inventory routing problem. This paper tries to extend the good research of Aghezzaf et al. [2] by considering deteriorating items and using Particle Swarm Optimization (PSO) as a method to solve the problem. We use PSO as a tool to solve the problem since PSO is a good tool for solving vehicle routing problem as shown by Ai and Kachitvichyanukul [18], Marinakis and Marinaki [19], Mir Hassani and Abolghasemi [20], and Moghaddam et al. [21]. Since vehicle routing problem is a general model of inventory routing problem, particle swarm optimization can be used to solve inventory routing problem as well. We divide this paper in four sections. The first section discusses the research motivation and literature review, then the mathematical model is developed in section two. Section three shows how the PSO method solves the problem and section four gives a numerical example and sensitivity analysis. The final section is the concluding remark.

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II. MATHEMATICAL MODELLING

In this problem a depot serves some retailers and each retailer has different demand rate and the demand rate is constant. There are more than one vehicle at the depot and each vehicle has same capacity. One retailer only be served by one vehicle and each vehicle can have more than one trip in one cycle time. Items are deteriorated directly when vehicles depart from the depot with constant deteriorating rate. Since condition in vehicles are different than in warehouses, deteriorating rate in vehicles is higher than the deteriorating rate in the warehouse. Items deplete continuously at the warehouse because of customer demand and deteriorated. Quantity of products that have to be delivered should be higher than the actual demand since the items are deteriorating. Inventory is set to become efficient so vehicles ready to replenish warehouse exactly when the stock is zero and shortage is not allowed. When one cycle is completed, similar route is used for the next cycle time. Loading and unloading time is small compared to transportation time, so we assume loading and unloading time is zero. The entire of this paper uses notations below.

Parameters

i	: Retailers 1,2, ..., I
R	: Depot
v	: vehicles 1, 2, ... V
d_i	: Demand rate/units/time at retailer i
Q'_i	: Total demand at retailer i
s_{ij}	: Distance between location i to j where $\{i,j\} \in S^+ = S \cup \{r\}$
ve_v	: Average speed of vehicle v

t_{ij}	: Transportation time between location i to j where $\{i, j\} \in S^+$, where $t_{ij} = s_{ij}/ve_v$
γ	: Deteriorating cost (€/unit)
δ_v	: Transportation cost for vehicle v (€/km)
ve_v	: Average speed of vehicle v
φ_i	: Handling cost at retailer i (€)
ψ_v	: Operational cost of vehicle v (€/unit time)
η_i	: Holding cost for retailer i (€/unit/unit time)
k_v	: Maximum capacity of vehicle v
T^v	: Cycle time of vehicle v
θ_1	: Deteriorating rate during delivery process
θ_2	: Deteriorating rate at retailer's warehouse
t_0	: Time to start of vehicle from DC
t_{1i}	: Time of vehicle arrive at retailer i
t_{2i}	: Time when inventory stock is zero at retailer i
I_i	: Average inventory at retailer i
T^{min}	: Minimum cycle time of vehicle v
T^{max}	: Maximum cycle time of vehicle v
Decision variables	
x_{ij}^v	: $\begin{cases} 1, & \text{if there is delivery from retailer } i \text{ to retailer } j \text{ using vehicle } v \\ 0, & \text{otherwise} \end{cases}$
y^v	: $\begin{cases} 1, & \text{if vehicle } v \text{ is used} \\ 0, & \text{otherwise} \end{cases}$
T^{EOQ}_v	: The optimal cycle time of vehicle v
z_{ij}^v	: Volume of items loaded by vehicle v from retailer i to retailer j
L_{rr}^{nv}	: Total vehicle capacity at sub-tour n in multi-tour v start from depot r back to depot r

Figure 1 shows an example of the routing problem. It shows two tours where each tour is conducted by one vehicle. Vehicle 1 serves retailers 8, 5, 2, 3 and 7 and vehicle 2 serves retailers 1, 6, 9 and 4. Vehicle 1 divide the tour into two sub-tours. In the first sub-tour vehicle 1 visits retailers 8, 5 and 2 and in the second sub-tour, it visits retailers 3 and 7. Since shortage is not allowed, the total sub-tour time must be less or equal to the cycle time.

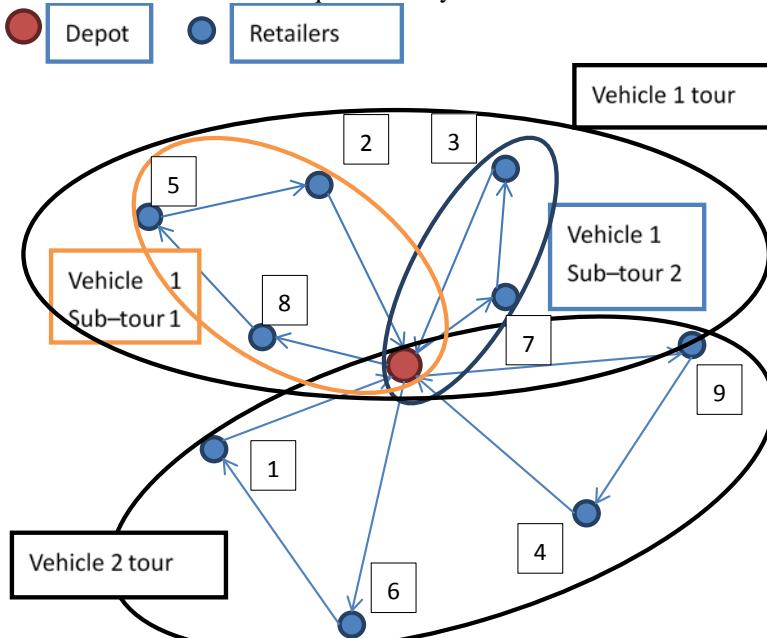


Figure 1. Multi tour and sub-tour

The inventory level for deteriorating items at retailer i can be modeled as:

$$\frac{dI(t)}{dt} + \theta_2 I(t) = -d_i \quad (1)$$

Through some calculation processes and simplifications, one has:

$$I(t) = (e^{\theta_2 t} - 1) \frac{d_i}{\theta_2} \quad (2)$$

Total demand at retailer i depend on demand and deteriorating rate during a cycle time. It can be modeled as:

$$Q_i = (e^{\theta_2 T} - 1) \frac{d_i}{\theta_2} \quad (3)$$

The volume of items delivered at each vehicle depend on demand during the replenishment period and deteriorating rate on vehicle and warehouse. Number of items deteriorated during transportation time can be formulated as:

$$\frac{dI(t)}{dt} + \theta_1 I(t) = 0 \quad (4)$$

Through some simplifications one has:

$$I(t) = \frac{Q'}{e^{\theta_1 t}} \quad (5)$$

When t_1 is a transportation time and quantity of item needed at each retailer Q , then quantity of items that has been brought by each vehicle (Q') is

$$Q'_i = Q_i e^{\theta_1 t_1} \quad (6)$$

So total quantities that have been delivered by each vehicle at each retailer can be derived from (3) and (6), so one has:

$$Q'_i = \frac{d_i (e^{\theta_2 T} - 1) e^{\theta_1 t_1}}{\theta_2} \quad (7)$$

Total quantity of items that are delivered by each vehicle can be figured out as Figure 2.

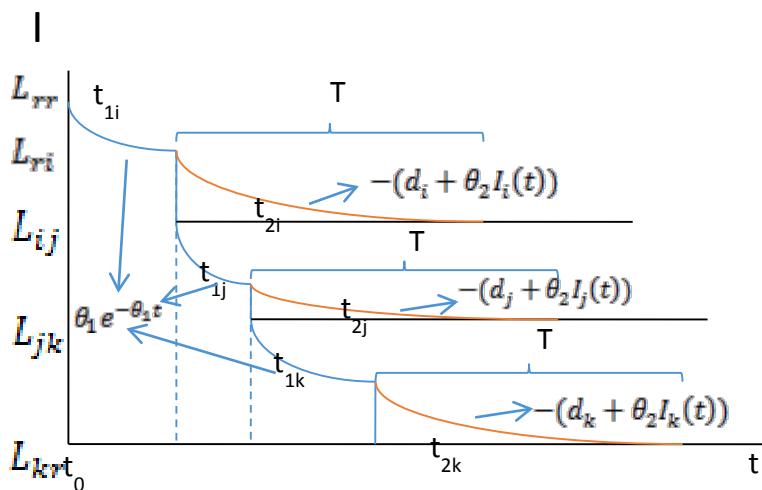


Figure 2. Inventory level in vehicle and warehouse

Total quantity that are delivered by each vehicle for a single tour can be modeled as:

$$L_{rr} = \sum_{i \in ST_n} \frac{(e^{\theta_2 T} - 1) e^{\theta_1 t_{1i}} d_i}{\theta_2} \quad (8)$$

The total cost of the problem consists of fix vehicle cost, transportation cost, handling cost, holding cost, and deteriorating cost. Vehicle cost can formulated as:

$$C_v = y^v \psi_v \quad (9)$$

Transportation cost/unit time can be modeled as:

$$C_T = \frac{1}{T^v} \left(\sum_{i \in S^+} \sum_{j \in S^+} (\delta_v t_{ij} x_{ij}^v) \right) \quad (10)$$

Handling cost/unit time can be formulated as:

$$C_H = \sum_{i \in S} \left(\frac{\varphi_i}{T^v} \right) \left(\sum_{j \in S^+} x_{ij}^v \right) \quad (11)$$

Total deteriorating items at each retailer is equal to total quantity delivered by each vehicle at each retailer minus total demand. It can be modeled as:

$$D_i = \frac{(e^{\theta_2 T^v} - 1)e^{\theta_1 t_{1i}} d_i}{\theta_2} - d_i T^v \quad (12)$$

The total deteriorating items cost/unit time can be modeled as:

$$C_D = \sum_{i \in S} \left(\frac{\gamma_i D_i}{T^v} \right) \sum_{j \in S^+} x_{ij}^v \quad (13)$$

The total inventory cost/unit time can be formulated as follows:

$$C_S = \sum_{i \in S} \left(\frac{\eta_i d_i}{T^v \theta_2^2} (-1 - \theta_2 T^v + e^{\theta_2 T^v}) \right) \left(\sum_{j \in S^+} x_{ij}^v \right) \quad (14)$$

The objective function and the constraints of the model can be modeled as follows:

IRP_{IP}: Minimize

$$Z = \sum_{v \in m} \left[y^v \psi_v + \frac{1}{T^v} \left(\sum_{i \in S} \sum_{j \in S^+} (\delta_v t_{ij} x_{ij}^v) \right) + \sum_{i \in S} \left(\left(\frac{\varphi_i}{T^v} + \frac{\eta_i d_i}{T^v \theta_2^2} (-1 - \theta_2 T^v + e^{\theta_2 T^v}) + \left(\frac{\gamma_i D_i}{T^v} \right) \right) \left(\sum_{j \in S^+} x_{ij}^v \right) \right] \right] \quad (15)$$

Subject to:

$$\sum_{v \in V} \sum_{i \in S^+} x_{ij}^v = 1 \quad j \in S \quad (16)$$

$$\sum_{i \in S^+} x_{ij}^v - \sum_{k \in S^+} x_{jk}^v = 0 \quad j \in S^+, v \in V \quad (17)$$

$$\sum_{i \in S} \sum_{j \in S^+} t_{ij}^v x_{ij}^v - T^v \leq 0 \quad v \in V \quad (18)$$

$$\sum_{v \in V} \sum_{i \in S^+} z_{ij}^v - \sum_{v \in V} \sum_{k \in S^+} z_{jk}^v = \frac{d_j (e^{\theta_2 T^v} - 1) e^{\theta_1 t_{1j}} x_{ij}^v}{\theta_2} \quad i, j \in S \quad (19)$$

$$x_{rj}^v - y^v \leq 0 \quad v \in V, j \in S \quad (20)$$

$$z_{rj}^v \leq k(v) \quad v \in V, j \in S \quad (21)$$

$$x_{ij}^v \in \{0,1\}, z_{ij}^v \geq 0, y^v \in \{0,1\}, T^v \geq 0 \text{ for all } v \in V, i, j \in S^+$$

The objective function is represented by equation (15). The objective function consists of vehicle's operational cost, transportation cost, handling cost, inventory holding costs, and deteriorating cost. It derived from the summation of the equation (9-14). The first constraint is shown by the equation (16). This constraint assures that one retailer is served by one and only one vehicle. Equation (17) assures that once a vehicle enter a retailer, it will leave the retailer. The total transportation time of one vehicle cannot be higher than the cycle time is stated in equation (18). Equation (19) guarantees that volume of the items load in one vehicle is equal to total demand during one cycle, deteriorated items in a retailer's warehouse during one cycle and deteriorated items during transportation time. Equation (20) guarantees that the vehicle

is used when there is a delivery from the depot. Equation (21) assures that total demand and deteriorated items loaded in one vehicle cannot bigger than vehicle's capacity. Since the model is a non-linear model and it's an NP-hard model, PSO algorithm is used to solve the model.

III. PARTICLE SWARM OPTIMIZATION FOR SOLVING MTIRPDI

In this section, a PSO algorithm is proposed to solve multi tours inventory routing problem for deteriorating items (MTIRPDI). This section is divided into three parts. The first part discusses the PSO framework, the second shows the decoding method and the last part discusses the routing and sub-routing methods.

1.1. PSO framework

Particle swarm optimization is a population-based computation technique where each particle moves according to its own best position and the best position of the other particle. It is like a flock of birds collectively foraging for food, where the food location is represented by the fitness function. Detail of the PSO algorithm for solving multi tour inventory routing problem for deteriorating items are presented as Algorithm 1.

Algorithm 1.

1. Initialize particle by setting number of particles (pr), number of iterations (α) and some initial parameters. Set $\vec{v}_0 = 0$, personal best (pbest) $\vec{x}_{l_{ps}} = \vec{x}_{ps}$ and iteration $i=1$.
2. For $i=1,...,p$ decode \vec{x}_{ps} to a set vehicle route R_i .
3. For $i=1,...,p$. calculate the performance measurement of R_i as Z_i

Calculate optimal economic period using (22). The solution can be found by using Bisection method.

$$\text{Minimize: } Z_i = R_i \left(\psi_v + \frac{\sum_{i \in S} \sum_{j \in S^+} \delta_{ij} t_{ij}}{T_{EOQ}(C_v)} + \sum_{i \in S} \left(\frac{\varphi}{T_{EOQ}(C_v)} + \eta \bar{I}_i + \frac{\gamma(L_r^{iv} - d_i T_{EOQ}(C_v))}{T_{EOQ}(C_v)} \right) \right) \quad (22)$$

$v \in V$

subject to

$$T_{min}^v \leq T_{EOQ}^v \leq T_{max}^v \quad v \in V \quad (23)$$

4. Update pbest by setting $\vec{x}_{l_{ps}} = \vec{x}_{ps}$ if $Z_{x_{ps}} < Z_{x_{l_{ps}}}$
5. Update gbest by setting $\vec{x}_{g_s} = \vec{x}_{l_{ps}}$ if $Z_{x_{l_{ps}}} < Z_{x_{g_s}}$
6. Update the velocity and the position of each particle

$$\begin{aligned} \vec{v}_{ps}(i+1) = & w(i) \times \vec{v}_{ps}(i) + u[0,1] \times c1(i) \times (\vec{x}_{g_s} - \vec{x}_{ps}(i)) + u[0,1] \times c2(i) \\ & \times (\vec{x}_{l_{ps}} - \vec{x}_{ps}(i)) \end{aligned} \quad (24)$$

Update of the moment inertia using fitness distance ratio (FDR), and it can be shown as:

$$w(i) = w(F) + \left(\frac{i-F}{1-F} \right) (w(1) - w(F)) \quad (25)$$

Calculate the new position using (26)

$$\vec{x}_{ps}(i+1) = \vec{x}_{ps}(i) + \vec{v}_{ps}(i+1) \quad (26)$$

7. If the generation meet the stopping criteria , stop. Otherwise add generation by one and return to step 2.
8. Set gbest of the last solution as the best solution for multi route inventory routing problem for deteriorating items.

1.2. The decoding method

A particle is represented by three parts. The first part is the number of retailers, the second part is a continuous value from 0 to 1 and the third part is the sequence number of each retailer. The sequence procedure using Algorithm 2.

Algorithm 2. Decoding method

1. Generate random number from 0 to 1 for the x_{ps} values.
2. Sort in ascending order the value of x_{ps} and set the sequence of the retailers
3. Particle representation for nine retailers can be represented in Table 1.

Table 1. Particle representation

Retailers	1	2	3	4	5	6	7	8	9
x_{ps}	0.39	0.81	0.66	0.78	0.29	0.89	0.09	0.06	0.67
Sequence	4	8	5	7	3	9	2	1	6

Once a global route has been established, the next step is allocating the route to vehicles by considering vehicles capacity. Since items are deteriorating, quantity that should be brought by each vehicle consist of retailer demand and the quantity of deteriorated items during delivery time and stock period in the warehouse.

1.3. The routing and sub-routing method

Routing is set to get the balance of quantity to be delivered and the distance. The routing method is completely described in Algorithm 3. We calculate it using weights as shown in equation (27).

Algorithm 3. Routing method

1. For all i , calculate we_i using equation (27)

$$we_i = d_i(t_{(i-1,i)}) \quad i \in S \quad (27)$$

2. Calculate $W = \sum_{i \in S} we_i$
3. Set $i=1, ws_0=0, j=1$
4. Set $ws_i = we_i$ If $we_i > W$, go to 7
5. Set $i=i+1$
6. Calculate $ws_i = ws_{i-1} + we_i$, if $ws_i < W$ go to 5 otherwise go to 7
7. Calculate $|ws_{i-1} - W|$ and $|ws_i - W|$. If $|ws_{i-1} - W| < |ws_i - W|$, allocate 1 to $i-1$ into route j , otherwise allocate 1 to i into route j
8. Set $i=0, ws_0=0, j=j+1$ and go to 5.
9. If all retailers have been allocated then finish

The next step is setting how many sub routes should be allocated by one vehicle. The solution is done by using Algorithm 4.

Algorithm 4. Sub-routing method

1. Set $sr=1, k=1$
2. Calculate $T_{min} = \sum_k T_k$
3. Calculate $T_{max} = \frac{K_v}{\sum_{k \in ST_n} \frac{(e^{\theta_2 T_{-1}}) e^{\theta_1 t_{1k} d_k}}{\theta_2}}$, if $T_{min} > T_{max}$ go to 5
4. Set $k=k+1$ and go to 2
5. Put 1 to $k-1$ into sub routing sr . If all retailers in routing have been allocated then go to 6, otherwise go to 1.
6. Calculate the fitness function
7. Set K as number of vehicles and $k=1$
8. Set discrete random variable from 1 to number of retailers ($n=U(1..N)$).
9. Allocate the first n retailers to vehicle k
10. If $k < K$, then $k=k+1$ and go to 9, otherwise go to 11
11. Calculate the fitness function. If $K_v < \sum_{k \in ST_n} \frac{(e^{\theta_2 T_{-1}}) e^{\theta_1 t_{1k} d_k}}{\theta_2}$ then fitness function = fitness function + penalty cost, where penalty cost is a big value.
12. Choose sub-routing with the best fitness function

IV. A NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

In this section, a numerical example is derived to check validity of the model and performance of the PSO algorithm. A set data from Aghezzaf (2006) is used in this section, where 15 retailers are supplied from one depot. Distance between depot to retailers and distance between retailers is shown in Table 2. Table 3 shows demand rate at each retailer where each retailer has different demand rate. The other parameters used in this numerical example are vehicle capacity is equal to 100 units, vehicle average speed 50 km/hour, fixed operating cost €50/hour, transportation time €1/km, inventory holding cost €0.1/unit/hour and fixed handling cost is equal to €50. We assume there are two vehicles available.

Table 2. Distance between retailers and depot (km)

r	r	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
r	-	270	480	490	330	550	430	140	260	240	150	240	360	200	430	320
1	270	-	740	560	580	680	650	420	530	440	210	410	450	260	400	170
2	480	740	-	500	490	410	190	350	370	320	540	590	750	660	600	800
3	490	560	500	-	630	160	310	480	630	290	390	720	850	670	180	710
4	330	580	490	630	-	770	590	290	160	470	490	190	300	350	750	520
5	550	680	410	160	770	-	220	540	630	310	490	760	910	740	340	810

6	430	650	190	310	590	220	-	340	430	210	440	610	770	630	430	740
7	140	420	350	480	290	540	340	-	160	230	250	270	420	310	470	450
8	260	530	370	630	160	630	430	160	-	340	400	230	380	350	630	510
9	240	440	320	290	470	310	210	230	340	-	240	450	600	440	320	540
10	150	210	540	390	490	490	440	250	400	240	-	370	480	280	290	320
11	240	410	590	720	190	760	610	270	230	450	370	-	160	170	650	330
12	360	450	750	850	300	910	770	420	380	600	480	160	-	210	770	310
13	200	260	660	670	350	740	630	310	350	440	280	170	210	-	570	170
14	430	400	600	180	750	340	430	470	630	320	290	650	770	570	-	570
15	320	170	800	710	520	810	740	450	510	540	320	330	310	170	570	-

Table 3. Demand rate at retailers

Retailer	Demand rate (units/hour)
1	0.109
2	0.326
3	0.322
4	0.478
5	0.134
6	0.429
7	0.381
8	0.503
9	0.187
10	0.123
11	0.953
12	0.638
13	0.247
14	0.188
15	0.441

The algorithm is run up to 1000 iterations and the average solution in each iteration is figure out in Figure 3. Figure 3 shows that the average solution will be convergence from around iteration 950.

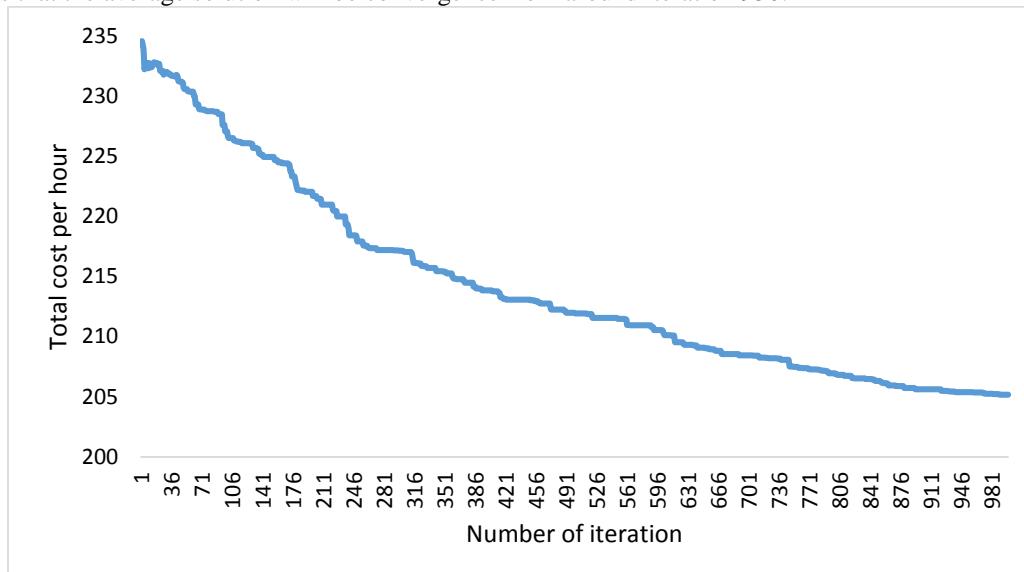


Figure 3. The average total cost/hour of PSO at every iteration

Performance of PSO algorithm is calculated using comparison the optimal result of the last iteration and the first iteration. After running the PSO algorithm for five times, the results are shown in Table 4.

Table 4. PSO improvement performance

Running					Average
1	2	3	4	5	
15.03%	10.92%	11.96%	13.66%	10.31%	12.37%

Table 4 shows that in average the performance improvement of the PSO method compares to the initial solution is 12.37%. The best solution from the Table 4 result in routing solution as shown in Table 5. Vehicle 1 has cycle time (T^1) = 102.34 hours and vehicle 2 has cycle time (T^2) = 101.94 and results in the total cost equal to €186.29/hour. Using the same set of data, the total cost derived by Aghezzaf (2006) is equal to €186.90/hour. It means that the PSO algorithm does not give worse result than the calculation procedure of Aghezzaf (2006). Since the PSO algorithm result in good solution for the inventory routing problem with multi-route without deteriorating rate, we can use the algorithm to solve IRP problem with deteriorating rate.

Table 5. Vehicles route

PSO		Aghezzaf (2006)	
Vehicle	Route	Vehicle	Route
1	r -> 9 -> 5 -> 3 -> 14 -> r r -> 6 -> 2-> r r -> 7 -> r r -> 12 -> r	1	r -> 2 -> 6 -> 9 -> r r -> 4 -> 8 -> r r -> 11 -> r
2	r -> 11 -> r r -> 13 -> 15 -> 1 -> 10 -> r r -> 8 -> 4 -> r	2	r -> 7 -> 13 -> r r -> 10 -> 1 -> 15 -> r r -> 5 -> 3 -> 14 -> r

When we set deteriorating rate at vehicles and retailers equal to $\theta_1 = 0.5\%$ per hour and $\theta_2 = 0.25\%$ per hour, we have the same route as Table 5 with vehicle 1 has cycle time = 99.59 and vehicle 2 has cycle time = 88.09. The optimal total cost is equal to €196.05/hour and cost comparisons between no deteriorating rate and with deteriorating rate is shown in Table 6.

Table 6. Solution for $\theta_1 = 0.5\%$ and $\theta_2 = 0.25\%$ Vs $\theta_1 = \theta_2 = 0$

		$\theta_1 = 0.5\%$ $\theta_2 = 0.25\%$ (a)	$\theta_1 = \theta_2 = 0$ (b)	Percentage of difference $\left(\frac{(a - b) * 100\%}{b} \right)$
Cost	Total cost/hour	196.05	186.29	5.24%
	Fix operating cost/hour	Vehicle 1 50.00	50.00	0.00%
		Vehicle 2 50.00	50.00	0.00%
	Travel cost/hour	Vehicle 1 34.34	20.83%	18.52%
		Vehicle 2 24.81	18.71%	16.55%
	Handling cost/hour	Vehicle 1 4.02	20.72%	18.38%
		Vehicle 2 3.97	15.74%	16.37%
	Holding cost/hour	Vehicle 1 14.12	-9.89%	-8.53%
		Vehicle 2 13.55	-6.87%	-7.50%
	Deterioration cost/hour	Vehicle 1 0.98	---	---
Cycle time	T	93.84	111.14	-15.57%
	Tmax	93.84	111.14	-15.57%
	Tmin	55.50	55.50	0.00%
Load per hour (units/hour)	Vehicle 1	3.10	2.61	18.77%

	Vehicle 2	3.30	2.85	15.79%
	Average	3.20	2.73	17.22%

Table 6 shows that deterioration affect total inventory and transportation cost/hour. By setting deterioration rate 0.5% during transportation time and 0.25% in the warehouse, the total cost/hour increase 5.32%. The model can show some management insights. A company can reduce cost of deteriorating items by reducing the cycle time. In this example, cycle time is reduced up to 15.57%. Since the cycle time is reduced, then the order quantity is reduced. Similar as the economic order quantity (EOQ) model, when the ordering quantity decreases, the inventory holding cost/hour will decrease and the travel time cost/hour and the handling cost/hour increase. This situation occurs since less cycle time means more transportation frequency. In this example, travel cost/hour of deteriorating items on average are 17.5% higher than the travel cost/hour of non-deteriorating items. The handling cost/hour of deteriorating items on average are 17.4% higher than the handling cost/hour of non-deteriorating items. The example shows that deteriorating rate should be considered by a company who has items with deteriorating characteristics. A sensitivity analysis is used to show how the model works in different situations. The sensitivity analysis is conducted by varying inventory cost, transportation cost, the deteriorating rate in transportation and the deteriorating rate in inventory data from -40% to 40%. The sensitivity analysis of the total cost/hour is shown in Table 7.

Table 7. Sensitivity analysis of the total cost/hour

	Percentage parameter changes				
	-40%	-20%	0%	20%	40%
Inventory Deteriorating rate (θ_2)	194.01	194.40	196.05	198.12	200.11
Transportation Deteriorating rate (θ_1)	194.05	195.80	196.05	196.90	198.30
Inventory cost (η)	184.37	190.70	196.05	202.91	206.41
Transportation cost (γ_v)	170.64	183.49	196.05	208.23	221.14

Table 7 shows that all parameters affect the total cost. The sensitivity analysis shows that the total cost increase as deteriorating rate, inventory cost and transportation cost increase. This condition is common in inventory problems, so we can conclude that the model and solution are valid.

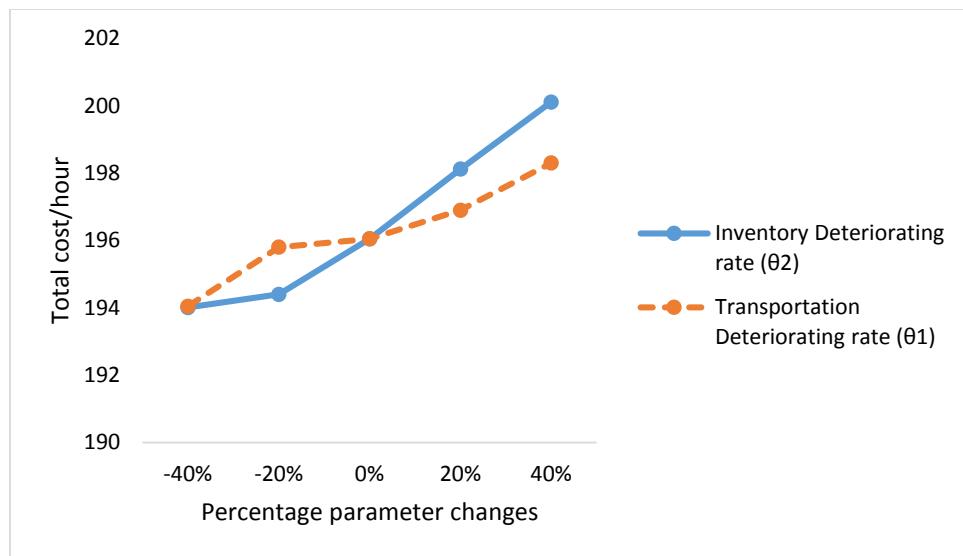


Figure 4 Total cost/hour for different values of inventory and transportation deteriorating rate

It is interesting when Figure 4 shows that the total cost/hour is more sensitive in different value of an inventory deteriorating rate than transportation deteriorating rate even though in this case the transportation deteriorating rate two times bigger than the inventory deteriorating rate. It means that the inventory deteriorating rate is more important to be considered than transportation deteriorating rate. This finding shows that is more important to consider deterioration items in inventory than only consider deterioration items in transportation period that have been modeled by previous papers.

V. CONCLUSION

In this paper, a particle swarm optimization method has been used to solve inventory routing problem with multi route for deteriorating items. A numerical example and sensitivity analysis are conducted to verify the model. The results show that optimal replenishment time is shorter when the items deteriorated compare with items without deterioration. The sensitivity analysis shows that deteriorating rate affects the total cost. Total cost increases as deteriorating rate increases. This paper tries to consider items deteriorate/perish not only during the transportation time such model from previous paper, but also considers items deteriorate in inventory. The sensitivity analysis shows that the total cost is more sensitive in different value of an inventory deteriorating rate than the transportation deteriorating rate. It makes the model is more practical. Although this paper can show how to model and solve inventory routing problem for deteriorating items, it can be improved by considering stochastic demand and deteriorating rate instead of constant demand and deteriorating rate.

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