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Modified Cluster Boundary Search Technique for Improved Layout Designs

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***Abstract*—Published techniques for Facility Layout Problem (FLP) can be broadly classified as either constructive placement or iterative improvement techniques. For the former category, the quality of solution is dependent upon the ordering of facilities for placing them one at a time, while for the latter category the quality of solution depends on the initial design used for iterative improvement. This paper presents a constructive placement technique, based on modified cluster boundary search algorithm, to optimize the layout of unequal-area facilities on a continual planar site using a multiple firing order mechanism along with an efficient search procedure to minimize the impact of ordering. Since the dimensions of the facilities are modeled in the mathematical formulation of the problem, there is no overlapping of facilities at any stage of the optimization process. The effectiveness of the presented technique is determined in terms of the quality of optimal solution and its computation cost. Test results are presented for benchmark problems of VIP-PLANOPT, a well-known commercially available software package for obtaining optimal layout designs. It is shown that the technique generates high-quality layout designs with minimal computation cost, especially for large-size problems. For the benchmark problem involving 100 unequal-area facilities, the presented technique achieved a layout design with nearly 10% less cost as compared to that obtained by VIP-PLANOPT. Results are also presented for layout designs with specified upper bound on white space (area) in the final layout.**

Keywords—Facility layout problem; constructive placement; facilities planning; optimization; building-block layout.

I. INTRODUCTION

The Facility Layout Problem (FLP) of optimally locating equal-area facilities on a grid with discrete locations is a classic quadratic assignment problem (QAP) that is known to be an NP-hard problem [1]. This problem becomes even more complicated when facilities have unequal areas and they are to be placed on a continual planar site. In this case the problem is generally known as Unequal-Area Facility Layout Problem (UA-FLP). A number of successful heuristics have been developed over the past three decades to efficiently find near-optimal solutions for both FLP and UA-FLP. These include heuristics utilizing simulated annealing [2-5], genetic algorithms [6-10], Tabu search [11,12], particle swarm optimization [11,13], ant colony optimization [14], neural networks [15], and others [16-19]. Layout optimization of unequal-area facilities has also been attempted by formulating the problem as a graph theoretic problem [20,21]. Other approaches for solving this problem include linear integer programming [22], mixed integer programming [23] and non-linear programming [24]. While most techniques for solving FLP and UA-FLP attempt to minimize a single objective function based on weighted flow-distance metric, multi-objective techniques have also been developed [2,9,10,25]. For more detailed review of different techniques for layout design of facilities the reader is referred to [26-29].

Heuristic techniques for UA-FLP, in general, either ignore the dimensions of facilities or assume them to be all of the same unit-size during the optimization phase. However, for problems where the variation in the sizes of facilities does not justify the equal-area assumption, the optimized layouts will have significant overlaps. To remove these overlaps, the facilities are arbitrarily moved and the resulting overlap-free layout is no more an optimal layout. Some analytical techniques have also been presented for solving UA-FLP [30-33]. Analytical techniques that incorporate the actual dimensions of facilities in their mathematical formulation have the distinct advantage of producing optimal layouts without any overlaps. To combine the salient features of both heuristics and analytical techniques, some hybrid techniques for layout optimization have also been published [34-35].

Most techniques for solving FLP and UA-FLP can be broadly classified as either iterative improvement or constructive techniques. In an iterative improvement technique, an initial layout design is improved over a number of iterations while in a constructive technique the layout is constructed by placing facilities in a specific order, one at a time. The iterative improvement approach has the disadvantage that the quality of optimal layout depends on the initial layout, while in the case of constructive placement the quality of optimal layout depends on the order of placing the individual facilities one at a time. While a better ordering criterion will definitely contribute in obtaining improved layout designs, a number of different firing orders are still required in order to select the best layout among various designs achieved using these firing orders. However, if the number of firing orders is too large, the computation time becomes quite excessive. Thus, while increasing the number of firing orders increases the probability of finding a better layout design, it also significantly increases the computation cost.

Among the analytical techniques, a constructive placement technique based on cluster boundary search algorithm has been quite successful in efficiently solving large-size unequal-area facility layout problems [30]. This algorithm ensures that at any stage of the optimization process, an incoming facility will be placed, without any overlap, at its optimum position with respect to the already placed cluster of facilities. This is achieved by exploring all feasible search space to find the optimum location of new facility in the presence of already placed facilities [30]. As for any constructive placement technique, the ordering of facilities (or firing order) plays an important role in determining the quality of optimal solution achieved by the boundary search technique. Therefore, even though the new incoming facility will be placed at its optimum position, for a given set of placed facilities (cluster), the selected ordering criteria and its resulting firing order will affect the quality of optimal layout. In order to minimize the impact of ordering on the quality of optimal layout design, an Enhanced Constructive Technique (ECT), primarily based on cluster boundary search algorithm, was presented in [33]. However, ECT cannot be applied to layout designs requiring the 2-D layout to be restricted to a given enclosing rectangle or by specifying the maximum permissible white space (area) in the final layout. Also, ECT does not provide a multiple firing order mechanism to further reduce the impact of firing order. The presented technique is based on a modified cluster boundary search algorithm employing a multiple firing order mechanism along with an improved search procedure. Also, the presented technique allows layout optimization with or without the constraint of constructing the layout within an enclosing rectangle or by specifying an upper bound on white space (area) in the final layout design.

II. MATHEMATICAL FORMULATION

For the presented layout optimization technique, the objective is to place N rectangular facilities of unequal areas at their optimal positions in the Euclidean plane without any overlaps. The facilities are assumed to be of fixed shapes. The position of i^{th} facility is defined by the coordinates of its centroid (x_i,y_i). Let (L_i,W_i) denote the length and width of facility i along the X- and Y-axes, respectively. The weighted flow relationship between all pairs of facilities is given by the matrix $\{ f_{ij} \}$. The cost function C to be minimized is defined as follows:

$$C = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} f_{ij} d_{ij} \tag{1}$$

where, d_{ij} is the distance measured between the centroids of facilities i and j and could be either of the following three distance norms:

a) Euclidean distance: $$d_{ij} = ((x_i - x_j)^2 + (y_i - y_j)^2)^{1/2} \tag{2}$$

b) Squared Euclidean distance: $$d_{ij} = (x_i - x_j)^2 + (y_i - y_j)^2 \tag{3}$$

c) Rectilinear distance: $$d_{ij} = |x_i - x_j| + |y_i - y_j| \tag{4}$$

Subject to:

$$|x_\mathrm{i} - x_\mathrm{j}| \geq \frac{\mathrm{wi+wj}}{2} \quad \text{OR}$$

$$|y_\mathrm{i} - y_\mathrm{j}| \geq \frac{\mathrm{hi+hj}}{2} \tag{5}$$

Equation (5) ensures that there is no overlapping of facilities at any stage of the optimization process. The OR operator used here is logical OR function, that is, either one or both conditions are satisfied at each stage of the optimization process. In case neither of the two conditions given in Equation (5) is satisfied for facilities i and j, then these two facilities overlap each other.

III. OPTIMIZATION PROCEDURE

The optimization procedure used for solving the above stated problem is a constructive placement procedure primarily based on cluster boundary search algorithm [30] with two important modifications. Before explaining these modifications it is important to emphasize that the cluster boundary search algorithm ensures that at any stage of optimization process, an incoming facility will be placed at its optimum position with respect to the already placed cluster of facilities. With such a powerful search algorithm that explores all feasible search space for a new facility in the presence of placed facilities, the limitation in obtaining an optimal solution comes from not knowing the optimum firing order. Since the number of possible firing orders is of the order of N!, it is practically not feasible to try all possible firing orders. However, multiple firing orders can be tried but that requires the search algorithm to be extremely efficient. Otherwise, the computation time for multiple firing orders will become too excessive. The two modifications introduced in the boundary search procedure are: built-in multiple firing order mechanism along with significant enhancement in the computational efficiency of original search algorithm, and introducing a new constraint on developing the layout within a specified enclosing rectangle or by specifying an upper bound on permissible white space (area) in the final layout.

A. Multiple Firing Order Mechanism

It is a two-step process as explained below:

Step #1 (Determining the Lead Facility)

Prior to starting the optimization process, an ordering of the facilities is determined since only one facility will be placed at a time at its optimal position. The first facility to be placed, called the Lead Facility, is the one with the highest value of ordering function φ_i, for i=1 to N, defined as follows.

$$\varphi_i = A_i^{\gamma} \sum_{j=1}^{N} f_{ij} \tag{6}$$

Where A_i is the area of the i^{th} facility (w_i x h_i) and γ specifies the user-defined weight on areas of facilities for determining the ordering function. This means that facilities that have higher flow relationship with other facilities will tend to have a larger value of ordering function. Also, if $\gamma > 0$ then larger size facilities will contribute towards a higher value of the ordering function and if $\gamma < 0$ then smaller size facilities will contribute towards a higher value of the ordering function. For $\gamma = 0$, the areas of facilities will have no role in determining the firing order, as is common in many constructive techniques.

The N values of the ordering function φ_i are sorted in the descending order. If only one firing order was used for optimization, then the first facility in the sorted list, that is the one with the highest value of ordering function, would be taken as the Lead Facility. However, for K firing orders, as used in this technique, K Lead Facilities are selected such that the i^{th} facility in the sorted list is the i^{th} Lead Facility.

Step #2 (Determining the Order of Follower Facilities)

For determining the order of Follower Facilities for each of the K firing orders, the ordering function, defined in Eqn. (6), is calculated for a given value of γ for all remaining facilities with one difference; the summation is carried over only the already placed facilities. Thus, for the first Follower Facility for a given firing order, the ordering function will be determined with respect to its Lead Facility only, and for the second Follower Facility the ordering function will be determined by doing the summation for the Lead Facility and the first Follower Facility, and so on. This process is repeated for each of the K firing orders.

For the built-in firing order mechanism introduced in the presented technique, five different values were specified for γ, that is -0.5, -0.25, 0, 0.25, and 0.5. Also, the number of firing orders K, for each specified value of γ, is input by the user. Thus, there will be n = 5K different firing orders for solving a layout problem.

B. Modified Cluster Boundary Search Procedure

Step #1 (Placement of Lead Facility and First Follower Facility)

Place the Lead Facility in the center of a two-dimensional continuous plane and then place the first Follower Facility at its optimal position. This optimal position will be somewhere along the edges (sides) of the Lead Facility because placing it away from the edges will increase the value of the cost function and placing it any closer will result in overlapping with the

Lead Facility. To determine the optimal position of the Follower Facility, one-dimensional (1-D) search is carried out, using the modified quadratic-fit procedure [36], along all the four sides of the Lead Facility and for both possible orientations of the Follower Facility. Among the eight optimal positions, the Follower Facility is placed at a position (with appropriate orientation) that corresponds to the minimum value of the cost function. Thereafter, the boundary of the cluster formed by these two facilities is determined. Also, an overlap-free search path along the cluster boundary is defined for the next facility (in order) to be placed.

Step #2 (Placement of Remaining Follower Facilities)

For the next Follower Facility in order, 1-D search is carried out along the overlap-free search path based on the previous cluster boundary. This improves the efficiency of the procedure as it does not require checking of overlaps following 1-D search. This process is repeated after changing the orientation of the new facility. The new facility is then placed at a position (with appropriate orientation) that corresponds to the minimum (best) value of the cost function. Now, instead of recalculating the boundary of the cluster after the placement of the new facility, as is carried out in the original cluster boundary technique [30], the previous cluster boundary is simply updated to reflect the addition of new facility. This significantly reduces the computation cost needed for recalculating the new boundary, especially for large-size problems, and thereby allows multiple firing orders to find a better quality layout without excessive computation cost. For the updated cluster boundary, the new overlap-free search path is defined for the next facility (in order) to be placed.

Step #2 is repeated for all remaining facilities, one at a time, until all the facilities have been placed at their optimal positions with optimal orientations. This completes the optimization procedure if the only constraint was non-overlapping of facilities. However, if a constraint on placing the facilities within an enclosing rectangle is specified either directly or by specifying the maximum permissible white area (additional area in the enclosing rectangle other than the summation of actual areas of all facilities), then the following procedure is also included in step #2.

If at any stage, placement of a new facility violates the enclosing rectangle constraint, that is, the facility is placed outside the permitted area of enclosing rectangle, a penalty function is applied and accordingly that placement position is excluded from further consideration. Furthermore, to minimize the accumulation of white space (area), if two or more placements of a new facility produce cost functions whose difference is within a specified relative threshold, then the placement with the least contribution of white space (area) is given preference.

The above-mentioned procedure was implemented in a computer program written in C++ and C#. The modified Cluster Boundary Technique will be referred to as mCBT in the discussion of test results.

IV. TEST RESULTS AND DISCUSSION

The presented technique (mCBT) was tested with UA-FLP problems taken from published benchmark or test problems. For the first set of test problems, benchmark problems presented by a general-purpose block layout design package, VIP-PLANOPT [37], were tried and compared with its optimal layouts. VIP-PLANOPT is a well-known commercially available software package for general-purpose block layout optimization. Its optimization technique is based on a pseudo-exhaustive search procedure.

The performance of the presented technique was compared with three benchmark problems of VIP-PLANOPT involving 8, 28, and 100 blocks (or facilities) of unequal-areas. For these benchmark problems the only constraint was non-overlapping of facilities. The results are shown in Table 1 below. As can be observed from this table, mCBT always produced a better quality optimal layout as compared to the best results obtained by VIP-PLANOPT. Also, the improvement in cost function increases as the number of facilities increase. For the 100-facility problem, there is 9.77% improvement in the cost function value as compared to the best value achieved by VIP-PLANOPT. This shows that the presented technique is especially suited for solving large-size layout problems. For the 100-facility problem, mCBT took only 56s on Acer laptop using Intel Core i5 CPU running at 2.27 GHz and having 4GB RAM. The layout designs obtained by mCBT for the three benchmark problems are given in Figures 1 to 3.

The data for another layout problem involving 30 unequal-area facilities were taken from [11] where it was solved by using individual Tabu search and particle swarm optimization techniques as well as by hybrid techniques combining both Tabu search and particle swarm optimization. The layouts were obtained without imposing any constraint on enclosing rectangle or specifying any upper bound on white area. The same problem was solved by mCBT with three different values (15%, 25%, 40%) of upper bound on white area in the final layout. It may be mentioned here that the objective function defined in [11] is twice the cost function defined by Eqn. (1). Accordingly, the cost obtained by mCBT was doubled for

comparison purposes. The results obtained by mCBT are shown in Table 2. As expected, the objective function value decreases as upper bound on white area is relaxed. For the purpose of comparison, the best objective function value obtained in [11] for this problem without considering any upper bound on white area is 474,724 which is higher than the values obtained by mCBT even after imposing white area constraints of 40% and 25%. The layouts obtained by mCBT for 15% and 25% white area constraints are shown in Figure 4(a) and 4(b), respectively.
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Table 1: Comparison of optimal solutions obtained by VIP-PLANOPT and mCBT

Test No.	**Number of Blocks (Facilities)**	**Cost Function Value**		**Improvement in Cost Function**
		VIP-PLANOPT [37]	**mCBT**	
1	8	692.5	669	3.39%
2	28	6447	6331	3.48%
3	100	538193	485598	9.77%

Table 2: Results for 30-facility problem [11]

#	**Upper Bound on White Area**	**Objective Function Value by mCBT**
1	15%	476,996
2	25%	458,550
3	40%	440,896

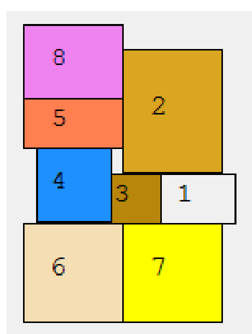

Figure 1: Optimal layout for 8 block VIP-PLANOPT benchmark problem L8

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