Abstract—Quality becomes one of the most important factors for sustaining company competitiveness in this intense industry competition. To control the quality of the ongoing production process, the common method used is Statistical Process Control (SPC), which is basically part of Statistical Quality Control (SQC). The primary tool used in SPC is control chart. Many companies conduct quality control by performing inspections of product defect with more than one defect attributes, or multi-attribute. Such conditions require the use of multi-attribute control chart. However, unlike the research developments in the field of multivariate control charts, little research has been done in the field of multi-attribute control charts. This research proposed a framework of statistical process control for some simultaneously correlated defect characteristics on bronze pipe fittings production process using multivariate np chart (MNP chart). The MNP chart consists of Phase I as the formulation of the initial control limits using the preliminary sample and Phase II as a reference for future statistical process control. The result of Phase I and II of the multi-attribute control chart application indicates that there were out-of-control processes, to then be identified and analyzed the main contributor of the cause.

Keywords—Quality Control, Correlated Defect, Multi-Attribute Control Chart

I. INTRODUCTION

In the era of globalization, competition among companies is getting tougher and makes quality becomes one of the most important factors for sustaining company competitiveness. Based on definitions from some researchers, quality of product or service is determined by its deviation from consumer’s desired specification [1][2], where the deviation of the specification will reduce the quality. To control the deviation, companies have to take significant effort on quality control.

On controlling the ongoing production process, the common method used is Statistical Process Control (SPC), which is basically part of Statistical Quality Control (SQC). During the production process, there is a possibility of variation resulting in low level of product consistency and product defect. The variations might be caused by common causes that unable to be identified, unable to be avoided and is the result of a not uniform process. In addition, there are variations due to assignable causes that able to be observed, identified and eliminated. The limits of variations can be determined using SPC. Then, the production process can be monitored to ensure the process does not run over the limits [3].
The primary tool used in SPC is control chart. Control chart is categorized into two types: variable control chart and attribute control chart. Control chart for quality characteristic that can be measured and expressed as a number on some continuous scale of measurement is called variable control chart, while control chart for quality characteristic that are not measured on a continuous scale or even a quantitative scale is called attribute control chart. In attribute control chart cases, each unit of product are judged as either conforming or nonconforming on the basis of whether or not it possesses certain attributes, or by counting the number of nonconformities (defects) appearing on a unit of product [3]. Both of variable and attribute control charts are divided into univariate/uni-attribute control chart and multivariate/multi-attribute control chart based on the number of variable/attribute concerned in the control chart.

Many companies conduct quality control by performing inspections of product defect with more than one attributes, or multi-attribute. Such conditions require the use of multi-attribute control chart. However, unlike the research developments in the field of multivariate control charts, little research has been done in the field of multi-attribute control charts [4], especially in Indonesia. Patel [5] was the first researcher who developed multi-attribute control chart with normal approximation, where multivariate binomial and Poisson distributions estimated as normal distribution. The research used time-dependent samples and limited on assumptions of normality and equal process variation. Later, Lu et al. [6] developed a multivariate version of np chart (MNP chart), which is a type of uni-attribute control chart, by plotting the number of defective products from the inspected sample. The MNP chart had been proven to be more sensitive in controlling a multi-attribute process than using multiple uni-attribute np charts at once.

Case study for this research was conducted in a metal industry in Karawang, Indonesia which produces bronze pipe fittings. The entire pipe fittings are then exported to foreign countries, such as US and UK. Therefore, statistical process control must be concerned. However, the quality control conducted in this company was only inspect and count the number of products that do not conform the specifications (defects) with seven characteristics of defect without further statistical processing. Those seven characteristics of defect are listed in Table I.

<table>
<thead>
<tr>
<th>No.</th>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>KD</td>
<td>Porous surface</td>
</tr>
<tr>
<td>2</td>
<td>KL</td>
<td>Rough surface</td>
</tr>
<tr>
<td>3</td>
<td>MG</td>
<td>Skewed</td>
</tr>
<tr>
<td>4</td>
<td>TR</td>
<td>Scratched by resibon</td>
</tr>
<tr>
<td>5</td>
<td>TN</td>
<td>Not connected or shrinkage product</td>
</tr>
<tr>
<td>6</td>
<td>TM</td>
<td>Scratched by grinding or bulge product</td>
</tr>
<tr>
<td>7</td>
<td>DR</td>
<td>Broken whorl</td>
</tr>
</tbody>
</table>

The average proportion of defects caused by those seven characteristics was 0.0676 or 6.76% with standard deviation of 0.0271 or 2.71%, which is below the quality standard set by Montgomery, i.e. three-sigma or defect proportion at 0.0668 or 6.68%. It indicated that the company’s quality control was not running well. Therefore, it is necessary to apply statistical process control using multi-attribute control chart in company’s production process.

This study developed a framework of statistical process control using multivariate np chart (MNP chart) in bronze pipe fittings production process to find out whether or not the process is out-of-control. The MNP chart consists of Phase I as the formulation of the initial control limits using a preliminary sample and Phase II as a reference to the process of statistical process control in the future. If it is detected that the production process is out-of-control, there will be investigation on the main contributor of the out-of-control process and analysis of the cause.

The rest of the paper is organized as follows: Section 2 describes the research methodology. Next section presents the results and discussion of Phase I and Phase II development of the MNP chart, and also identification of out-of-control process contributor. Finally, Section 4 presents the conclusions and suggestion for further research.
II. RESEARCH METHODOLOGY

This section explains the data acquisition, Phase I and Phase II development of MNP chart, and data analysis method.

A. Data Acquisition

The study began by collecting secondary data from historical inspection since November 2013 until January 2014 of a metal industry company located in Karawang. Because the company conducts quality control based on production schedule, the inspection data is not recorded every day. So, a total of 57 samples obtained during the three months period, in which each sample contained 3,000 inspected products.

B. Phase I Development

Once the data were obtained, the data were processed with a framework of Phase I and Phase II. The Phase I of control chart is the formation or initial formulation of control chart when the vector of defect product proportion \( p \) and the correlation matrix \( \Sigma \) were not known in advance. This phase used historical data of November 2013 that as many as 22 samples. This number of samples are in initial sample size criteria [6]. The first step of Phase I is to calculate the correlation coefficient of seven characteristics by (1). \( \delta_{ij} \) is the correlation coefficient between characteristics \( i \) and \( j \); \( c_{ij} \) is the count of nonconforming units with respect to characteristics \( i \) in a sample. This step is performed to determine whether there is any correlation between defect characteristics.

\[
\delta_{ij} = \frac{\text{Cov}(c_i, c_j)}{\sqrt{\text{Var}(c_i)\text{Var}(c_j)}}
\]

\[
= \frac{4\Sigma^k_{i=1}c_{ih}c_{jh} - (\Sigma^k_{i=1}c_{ih})^2(\Sigma^k_{i=1}c_{jh})^2}{\left[\Sigma^k_{i=1}(c_{ih}^2)(\Sigma^k_{i=1}c_{jh})^2\right]^2}
\]

(1)

The next step is to estimate the model parameter which will be the element in calculating control limits in MNP chart. To estimate the parameter, \( k \) number of initial sample was taken within sample size \( n \). Then, \( c_{ij} \) is calculated as number of defective products with respect to characteristics \( i \) in sample \( j \), where \( i = 1, 2, \ldots, m \) and \( j = 1, 2, \ldots, k \). The parameter is defined as the mean of defect proportion \( p \) on every defect characteristic as estimated as follows.

\[
\hat{p}_i = \frac{\sum_{j=1}^k c_{ij}}{kn} = \left( \frac{\sum_{j=1}^k c_{i1}}{nk}, \frac{\sum_{j=1}^k c_{i2}}{nk}, \ldots, \frac{\sum_{j=1}^k c_{im}}{nk} \right)
\]

(2)

After obtaining the estimated mean of defect proportion vector, the next step is to calculate the upper control limit, lower control limit, as well as the center line by (3), (4), and (5). The last step of Phase I is the calculation of the X statistics introduced by Lu et al. [6]. X statistic is a total of the number of defective products that were weighted for each quality characteristic in a sample. X statistics which are calculated by (6) will be the points that are within the control chart.

\[
UCL = n \sum_{i=1}^m \hat{p}_i + 3 \sqrt{n \left( \sum_{i=1}^m (1 - \hat{p}_i) + 2 \sum_{i < j} \delta_{ij} (1 - \hat{p}_i)(1 - \hat{p}_j) \right)}
\]

(3)

\[
CL = n \sum_{i=1}^m \sqrt{\hat{p}_i}
\]

(4)

\[
LCL = n \sum_{i=1}^m \sqrt{\hat{p}_i} - 3 \sqrt{n \left( \sum_{i=1}^m (1 - \hat{p}_i) + 2 \sum_{i < j} \delta_{ij} (1 - \hat{p}_i)(1 - \hat{p}_j) \right)}
\]

(5)
\[ X_j = \frac{\sum_{i=1}^{m} c_{ji}}{\sqrt{p_i}} \] (6)

\( X_j \) is the value of X statistic on sample \( j \); \( c_{ji} \) is the sum of defect product on sample \( j \) for defect characteristic \( i \); and \( p_i \) is the mean of defect proportion for defect characteristic \( i \).

Once all these elements are calculated, the X statistics are plotted on the control limits that have been calculated, to see whether there is any point which is out-of-control. If there is no out-of-control point in the MNP chart of Phase I, then Phase II can use that control limit, but if there is any out-of-control point, those points should be eliminated and MNP chart is recalculated until all the points are in-control condition.

C. Phase II Development

Phase II of control chart is a phase of controlling the on-going production process by calculating the X statistics by (6). By this step, it will be known whether the production process is running in-control or not towards control limits that have been obtained from Phase I. In this framework, Phase II used historical inspection data in December 2013-January 2014 so that a total of 35 samples were obtained. If MNP chart of Phase II find a out-of-control point, it is necessary to identify the main contributor towards the score of \( Z \) statistic in (7).

\[ Z_t = \frac{c_{ji} - np_i}{\sqrt{p_i}} \] (7)

If out-of-control point is above upper control limit, then the defect characteristic with the most positive \( Z \) score becomes the main contributor of out-of-control process. Otherwise, if out-of-control point is below lower control limit, then defect characteristic with the least negative \( Z \) score becomes the main contributor of out-of-control process.

D. Data Analysis

After obtaining the main contributor of out-of-control process, the next step is to find the cause of them. In this study, the analysis was conducted by observation and brainstorming with QC manager and his officer. Furthermore, fishbone diagram was used to find the root cause of the problem.

III. RESULTS AND DISCUSSION

This section explains the data processing using MNP chart, analysis of the results of data processing, and also analysis of the factors that could potentially be the cause of the out-of-control process.

A. Phase I Development

The first conducted data processing was calculating the correlation coefficient of the seven defect characteristics. The purpose of this calculation is to determine whether there is any relationship between the variation of number of defects in a characteristic to another characteristic. In addition, the calculation of the correlation coefficient will also be used later in the calculation of MNP control chart limits. In this study, the calculation of correlation coefficient and the correlation matrix \( \Sigma \) used Minitab software which the result can be seen in Fig. 1.

The Minitab result shows two values, which the upper value is the value of the correlation coefficient between the two defect characteristics, and the under value is the \( P \) value of the correlation. If the \( P \) value is less than \( \alpha = 0.05 \), the correlation or relationship between the two characteristics is significant. The calculation of the correlation coefficient matrix shows that nothing is valued zero. This means that all defect characteristics are mutually correlated with each other, although not all of the correlations between the characteristics is significant. From the correlation coefficient matrix, there are five pairs of defect characteristics which has significant correlation, i.e.: (1) porous surface (KD) with rough surface (KL), (2) skewed (MG) with not connected and shrinkage product (TN), (3) scratched by grinding or bulge product (TM) with skewed (MG), (4) scratched by grinding or bulge product (TM) with scratched by resinbon (TR), (5) scratched by grinding or bulge product (TM) with not connected or shrinkage product (TN).
Of the five pairs of defect characteristics, the pair with the strongest correlation is not connected and shrinkage product (TN) with the scratched by grinding or bulge product (TM). In addition, scratched by grinding or bulge product (TM) has the most numbers of significant correlation with other defect characteristics. Furthermore, skewed (MG) and not connected and shrinkage product (TN) are significantly correlated, and both were correlated with scratched by grinding or bulge product (TM). This indicates a causal loop between those three defect characteristics, in which all of the correlation is positive that form pattern of reinforcing process.

The next step is to estimate the model parameters which is the mean of defect proportion of each defect characteristics listed in Table I. The means of defect proportion are shown in Table II. After obtaining the estimated mean of defect proportion vector, the next step is to calculate the upper control limit, lower control limits, as well as centerline. The calculation of the three values incorporate elements of the correlation matrix Σ and the mean of defect proportion vector p that have been obtained previously. From the results of these calculations, the values obtained as follows: upper control limit = 3,346.7; centerline = 1,525.525; lower control limit = 0.

**TABLE II. MEANS OF DEFECT PROPORTION**

<table>
<thead>
<tr>
<th>p</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>p1</td>
<td>0.02217</td>
</tr>
<tr>
<td>p2</td>
<td>0.000801</td>
</tr>
<tr>
<td>p3</td>
<td>0.019719</td>
</tr>
<tr>
<td>p4</td>
<td>0.000561</td>
</tr>
<tr>
<td>p5</td>
<td>0.00686</td>
</tr>
<tr>
<td>p6</td>
<td>0.003275</td>
</tr>
<tr>
<td>p7</td>
<td>0.002146</td>
</tr>
</tbody>
</table>
From the value of the control limits and centerline that has been obtained, the control chart was made of the x-axis as the sample and y-axis as statistical values that has been calculated, while the indicators of the process in-control or out-of-control is the X statistic which has also been calculated previously. The MNP chart can be seen in Fig. 2.

Results of Phase I MNP chart indicates that 22 samples of the initial observational data located in the in-control area because there is no out-of-control point. Random pattern lines also formed, so there is no indication of a variation of out-of-control process. Therefore, the upper control limit, lower control limits and center line on the first phase can be used for Phase II, which is the process of monitoring whether of the production process beyond the control in the future.

B. Phase II Development

After obtaining the control limits that is in-control, then the next step is to control the quality of the production process in the future. X statistical calculations in Phase II did not differ from Phase I. Once calculated, the value is inserted into the control chart of Phase I as in Fig. 3.

MNP chart of Phase II shows that there are 5 out-of-control points, i.e. sample 7, 10, 11, 15, dan 18. So it was indicated that there was out-of-control process which has to be identified and analyzed.

C. Identification of Out-of-Control Contributor

Results of Phase II MNP chart show that there are out-of-control points at 5 samples. Therefore, the next step is to identify the major contributor to cause of the shift to out-of-control condition by calculating the Z statistic score of each defect characteristic for each point that is outside the control limits. Z statistic scores were obtained as in Table III.
Because of all the points that are out-of-control is above the upper control limit, then the defect characteristic which is being the main contributor in the out-of-control process is the defect characteristic with the largest positive $Z$ statistics score. So, it can be determined that: (1) the main contributor to the out-of-control sample 7 is TN / not connected and shrinkage product, (2) the main contributor to the out-of-control sample 10 is KD / porous surface, (3) the main contributor to the out-of-control sample 11 is KD / porous surface, (4) the main contributor to the out-of-control sample 15 is DR / broken whorl, (5) the main contributor to the out-of-control sample 18 is MG / skewed.

In this study, fishbone diagrams is used to find out the factors that cause the main contributor of the out-of-control process in Phase II. The identification of the factors derived from direct observation and interviews/brainstorming with expert and officer. Fig. 4 to Fig. 7 explain the causal factors of each defect characteristic identified as the main contributor in the process out of control in Phase II.

![Fishbone diagram of not connected and shrinkage product](image-url)
Fig. 5. Fishbone diagram of porous surface inward

Fig. 6. Fishbone diagram of broken whorl

Fig. 7. Fishbone diagram of skewed
The potential material factor causing not connected and shrinkage product that able to be controlled is too frequent of molten bronze pouring. Meanwhile, cold air and wet sand mold factors unable to be controlled. From the branch of method, there is a potential problem of unmatched mold dimension and molten bronze volume. From those two controllable problems, can be concluded that the potential root cause of not connected and shrinkage product is no calculation of frequency optimization on bronze pouring liquid volume used as standards in the production process.

The potential factors of porous surface are the existence of impurities in the molten metal (sand, resin, or other elements) and the existence of metal tensile strength. Therefore, the process that need to be evaluated is the bronze smelting process which can be contaminated by unwanted elements.

Based on Fig. 6, the potential factors of broken whorl are the poor quality of tapping machine lubricant, worn-out whorl gauge and shaky tapping knife. The maintenance system of the company’s machine, especially the tapping machine, should be concerned. Company need to find out the most effective and efficient system that can prevent tear or irregularities on the machine.

The potential factors of skewed product are not asymmetric mold pattern, loose stopper, shifting sand mold in the area of bronze smelt foundry, unfit sand mold, and broken sand mold when pouring molten bronze. Of the factors causing skewed product, the important factor is the sand mold used to form the products. In addition, the factors that could be the root causes are the lack of standard quality control on the sand mold and lack of method to prevent the production of unsatisfactory sand mold.

IV. CONCLUSIONS AND FUTURE RESEARCH

This study used MNP chart for multi-attribute quality control (seven correlated defect characteristics) that were divided into two phases, i.e. Phase I and Phase II. In the first phase, it was obtained in-control condition, so that the control limit of MNP chart of Phase I can be used in the Phase II. In Phase II, it was found five out-of-control points which indicates there is out-of-control process. Therefore, it is necessary to identify the defect characteristics that is being the main contributor by calculating the Z statistic score. From the calculation results, the defect characteristics obtained as the main contributor by selecting the largest positive Z score for each out-of-control sample.

From the defect characteristics that have been analyzed, the factors that need to be considered to keep the company’s production process remain in-control conditions are as follows: (1) the need of frequency optimization calculations along for bronze pouring liquid volume to be used as standards in the production process, (2) evaluation of bronze smelting process that is still accessible to unwanted elements, (3) effective and efficient machines maintenance system to prevent tear or irregularities on the machine, (4) the need of specific quality control in the sand forming to make sand molds as per the specifications and prevent such nonconformance affects the subsequent production process.

The problem on certain assumptions such as the possibility of autocorrelated processes, as often happens in the case of time series data, is still ignored in this study. Therefore, further studies are expected to incorporate these assumptions into research and anticipate the negative effect of autocorrelation in the control chart. Further research is also expected to compare several multi-attribute control chart that already exists in terms of effectiveness and sensitivity in identifying the out-of-control.

REFERENCES

BIOGRAPHY

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