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A Game-Theoretic Approach to Digital Marketing and Lead Generation for Duopoly Markets

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***Abstract* — Digital marketing has received much attention from most firms recently, and with the increasing competition and exigency, marketing managers' need for reliable and scientifically supported decision systems to assist them has never been greater. This paper presents a management model for estimating the quantity of online leads they should generate in a given period of time in order to achieve their goal, measured in contracts gained, in the most effective and efficient way possible. Through the application of Game Theory, the strategies of the rival firms are taken into account to provide marketing managers with a set of reliable possible decisions that can bring the firms competitive advantage. The applicability of the model is tested through two different scenarios, comparing the results with the actual data, and reaching the conclusion of the great potential that the proposed model evidences. Through the using of the model, the firms are able to benefit from it by operating at a more efficient level and by saving costs from the online lead generation and digital marketing budget.**

Keywords — digital marketing, lead generation, online leads, game theory

I. Introduction

Digital marketing's importance has never been greater, with much of the advertising effort carried out by firms going towards digital media such as the internet. Consumers tend to seek quality information, or in other words, the information they can gather, when planning to purchase new products [1]. In one hand, the emergence of digital marketing made the search of products or services by the consumers much easier and "with the Internet's growing popularity, online consumer reviews have become an important resource for consumers seeking to discover product quality" [1]. The Internet has become one of the most important marketplaces for transactions of goods and services [2]. Thus, it is extremely important and even vital in some cases, for firms to have a noticeable presence in the digital world and to take advantage of its benefits in order to reach out to increasingly more consumers to increase sales and gain competitive advantage over its rival(s). It is, however commonly stated that the presence of a firm in the Internet is no long a nice-to-have situation, but must-have situation. A 2015 study shows a clear rapid growth tendency that online ad spend is incurring [3]. The report shows that online ad spending grew by 15.6% year-over-year in 2014, from $42.8 billion to almost $49.5 billion. It's another record spending year, which is something that is becoming more and more common. Every firm searches for an effective and efficient way to reach their respective market and gain competitive advantage over its rivals, and one way to achieve this goal is through careful planning and implementation of advertising. A lead is generated "when a visitor registers, signs up for, or downloads something on an advertiser's site. A lead might also comprise a visitor filling out a form on an advertiser's site" [4]. To gather initial consumer interest and inquiries into a firm's product or service, leads are generated, and today's marketing managers struggle greatly in this process due to the lack of a solid, scientifically supported management model to assist them in the decision of the quantity of online leads to be generated at an efficient level, leading to waste of financial and human resources. Most marketing managers currently generate online leads according to their experience, or even "*gut feeling*". A

global survey by McKinsey & Co. reports "that companies tend to allocate marketing spending based on historical allocations and rules of thumb far more than quantitative measures" [5]. A study developed by Sirius-Decision Inc. in 2006, determined that on average, (just) "B2B firms spend 65% of their marketing budgets on activities such as trade shows, product seminars, cold-calling, data-base purchases, telemarketing" and other activities to attract new customers [6]. This is nothing more crucial than generating leads. These leads are considered offline leads but the principle and goal are the same, with firms shifting their attention to generating online leads. But there is still a lack of scientific articles that not only explain and provide information regarding this subject but also that develop methodologies for better ways to generate online leads. However, articles in business magazines and websites such [7, 8] have published very recent articles, has of 2015 providing evidence of the growing importance of online leads.

The application of game theory to the problem of lead generation and developing a management model revolving around this theory is natural and intuitive. Firms interact strategically with each other and try to understand what each one is doing in order to gain advantage - by making decisions based on their knowledge of the actions of its rivals - that can be measured in market share, revenue or profits. The decision of how many leads to generate each period is directly tied with how many leads does a firm believe that its rivals will generate, because it has an important effect on the firm's operations. Many authors have already proposed a number of game theory-based models that determine the allocation of advertising investment in a varied number of marketing channels [9], [10]. Some authors proposed a dynamic model [11] and others a static model [12]. Static modeling has great advantages because it proves to be useful in order to understand basic empirical results when innovations are introduced such as the case of the present paper, by providing a ready-to-use management model [13]. The study is made for one-period because it is assumed that there are no carry-over effects of the actions taken.

This paper assumes, much like the literature, that the level of advertising of a firm directly affects its sales – this does not necessarily mean that the rhythm of increased advertising, matches exactly with the rhythm of increasing sales, because it is assumed (and it is realistic) that the effects of own and competitive advertising sales are subject to diminishing returns [14].

There is also a compelling idea, that relates to the particular application of the theory of games, that the advertising levels of one firm has implications on the rival firms' sales and own advertising levels [13]. One can intuitively think that the effect would be to hamper the sales of the competitors, but this might be an imprudent thought because the effect highly depends on several factors such as the industry, the competitive environment and the product or service commercialized.

Lastly, the focus of this paper is on duopoly markets. There exists, indeed a wide variety of industries that are characterized by competition primarily among two rival firms, but there are also market situations where multiple firms interact [14]. One could also adapt the situation of a duopoly to a market situation with multiple firms, simply by gathering the market data, such as the sales, the market share, market leads, and other indicators needed to the model (obviously excluding the firm's own data) and it becomes an interaction between the own firm and the remaining market (instead of interactions between all firms). Additionally, the data present in this paper is from two existing companies that operate in a duopoly, but this model can easily support the entrance of other companies with some changes (without the market aggregation). The idea of the proposed model is indeed richer in scope, but due to the lack of quality data for other markets, the focus of the present study is on duopoly markets.

The aim of this paper is to present a management model for estimating the quantity of online leads they should generate in a given period of time in order to achieve their goal, measured in contracts gained, in the most effective and efficient way possible. When compared to the literature, the model here proposed is static, and thus it can be easily applied in every firm, regardless of the competitive and market environment without the experience, knowledge and expertise that would be needed in order to apply a complex dynamic model. One other feature of the proposed model that is innovative when comparing to the models proposed by the previous mentioned authors is the fact that it is with the quantities of online generated leads and quantities of sales that the work is based, not with the monetary amounts of investment or revenue. In the previous works, it was always used the monetary values, but due to the great difficulty in estimating the monetary investment and revenue in this particular field of the rival firm(s), and the fact that it is much easier and more reliable to only estimate the quantity of online generated leads and contracts sold in a period, while the estimation of the profit per online contract can be gathered also with ease, the model presented in this paper does not have the monetary values as basis for its formulation and applications.

The paper is organized as follows: Section II presents several approaches to the sales model developed and compares them with each other. With the best fitted approach to the sales model selected, the mathematical formulation of the proposed management model is presented in Section III. In Section IV two application scenarios are introduced and their results are analyzed and discussed. At last, conclusion and recommendations for future work are presented in Section V.

II. METHODOLOGY

Our aim is to provide a management model that indicates the quantity of online leads that a firm should generate in order to maximize its profit. Firm A competes in the market with firm B, thus the decisions of firm A should take into account the

behavior of firm B. This way we model the behavior of the firms by means of a strategic game where both firms choose the online leads that maximize their profit, but both take into account that the sales of each other depends on the online generated leads of both firms.

Our first goal is to establish the sales function which should be a function of the leads generated by both firms (online leads and eventually offline leads). To choose the sales function model and the relevant variables in that function we first undertake an empirical study making use of a data set.

Table 1 represents a real market's data from firms A and B – it's a duopolistic Portuguese market where firm A has a market share of 45% and firm B has a market share of 55%. This is a consumer-based market where the consumers' buying decisions are rational and pondered instead of instinctive, though sometimes there are spot peaks due to, for example, news in the media. The data spans across 16 months, from January 2014 to April 2015 – monthly quantity of sold online contracts, online leads generated by both firms and the quantity of offline leads generated by firm A. The data from firm A is exact while the data from firm B is based on estimations, and although they are a great approximation of the reality, they only represent estimates from professionals from market intelligence. In a real world context, this is the case with most firms, i.e. a firm does not always know exactly what its competitors are doing but they have estimates (again, in this particular case the data from firm B is a very close estimation). For the purposes of the model formulation, this paper will follow the particular case of firm A, but the formulation for firm B is analogous.

TABLE 1. DATA SET FROM EACH FIRM

Month	Online Contracts_A	Online Contracts_B	Online Leads_A	Online Leads_B	Offline Leads_A
Jan 14	98	155	852	780	525
Feb 14	110	200	534	1950	558
Mar 14	116	231	678	1950	681
Apr 14	114	256	825	1560	582
May 14	111	267	502	5850	685
Jun 14	198	501	227	7800	907
Jul 14	275	561	703	7800	785
Aug 14	210	267	574	780	565
Sep 14	127	299	571	1950	737
Oct 14	88	301	488	3900	1081
Nov 14	39	287	118	3900	1191
Dec 14	34	54	90	780	709
Jan 15	88	110	334	780	499
Feb 15	133	210	861	1950	519
Mar 15	144	202	1176	1950	598
Apr 15	80	176	1191	1560	589

The first sales model - M1 (based on [6]) - has the following sales response function:

$$\ln S_A = \ln \alpha_A + \beta_A \ln L_A + \gamma_B \ln L_B \tag{1}$$

where S_A are the sales (sold online contracts) of firm A in the studied period of time, α_A is a firm-specific intercept term, β_A and γ_B are the parameters that measure the sensitivity of firm A's sales to its own quantity of online generated leads and firm B's leads respectively, L_A and L_B represent the quantity of monthly online leads generated by firm A and B respectively, and finally, ln denotes the natural logarithm.

After performing the estimation of this sales response function from Table 1 (and the following models' sales response functions as well), it was evident that the firm's-specific intercept term was not significant (for this function and the other functions as well), and as a consequence this parameter was removed from every function and the estimations were performed without it. So, for the first case, the definite sales response function is the following:

$$\ln S_A = \beta_A \ln L_A + \gamma_B \ln L_B \tag{2}$$

The second sales model to be studied, M2, resorts to the square root instead of the natural logarithm to model the sales response of each firm:

$$\sqrt{S_A} = \beta_A \sqrt{L_A} + \gamma_B \sqrt{L_B} \tag{3}$$

where the significance of each parameter is the same as it was in M1.

Each of the above sales response functions, M1 and M2, does not use the offline generated leads of firm A. To thoroughly study which model and which individual and set of parameters better fit the real data to build a proper management model, one other version of the previous sales response functions was studied: the addition of the offline generated leads parameter.

For the model M3, the offline leads parameter is added to the model M1 – refer to (2):

$$\ln S_A = \beta_A \ln L_A + \gamma_B \ln L_B + \rho_A \ln O_A \tag{4}$$

where ρ_A is the parameter that measures the sensitivity of firm A's sales to its own offline generated leads, and O_A is the quantity of offline leads generated by firm A in a month.

For the model M4, the same line of thinking is followed, and its sales response function is as follows:

$$\sqrt{S_A} = \beta_A \sqrt{L_A} + \gamma_B \sqrt{L_B} + \rho_A \sqrt{O_A} \tag{5}$$

In Table 2 the results of the estimation of the sales response functions of M1, M2, M3, and M4 are presented. For each model, the value of each estimated parameter is presented, as well as its standard error and whether or not they are significant and if they are, at what level of significance.

The results clearly indicate an extremely good fit of the data to the proposed sales response functions. When analyzing the R^2 values, every model has very interesting values that establish confidence in the proposed response functions, although M1 has a slightly better R^2 value than the others. But the model M4 can already be excluded because the parameter ρ_A is not significant, and if we estimate M4 without this parameter, the resulting model and estimation would be the exact same as M2. That is, this result indicates that, in this model, the offline generated leads do not affect the sales of firm A. For the exact same reason, M3 is excluded, the parameter ρ_A is not significant, and without this parameter, the model would be equal to M1. Thus, due to these results the only models to be compared to each other are the models that don't take into account the offline generated leads, M1 and M2. Since the R^2 value of M1 is better than the R^2 value of M2, this last model is excluded. Therefore, the chosen model is M1, which is congruent with the idea already mentioned of diminishing returns that the natural logarithm provides.

TABLE 2. PARAMETER ESTIMATES, (STANDARD ERRORS) AND "SIGNIFICANT AT THE LEVEL" FOR FIRM A

Parameter	M1	M2	M3	M4
β_A	0,446 (0,102) "0,1%"	0,260 (0,047) "0,1%"	0,468 (0,119) "1%"	0,240 (0,065) "1%"
γ_B	0,251 (0,083) "1%"	0,091 (0,022) "1%"	0,310 (0,170) "10%"	0,078 (0,0363) "10%"
ρ_A	-	-	-0,091 (0,225) "Not Sig."	0,045 (0,097) "Not Sig."
R^2	0,993	0,957	0,992	0,954

The parameters β_A and γ_B in M1 are significant at the 0,1 and 1 percent level respectively, and have interesting signs. In one hand, the parameter β_A's sign is as expected, but the parameter γ_B's sign is maybe counter intuitive, because a positive sign means that an increase in the number of online generated leads by firm B (L_B) will actually increase the sales of firm A by γ_B percent. This is a result that challenges many assumptions in the literature. It happens because of the particular characteristics of the industry and the product it commercializes. Because the product that both firms commercialize is something that a consumer never buys instinctively, but ponders greatly, an increase in the advertising from a firm has a

positive effect on the other firm's sales because the consumers also study their product and might want to analyze whether it has better characteristics, price or both. Additionally, the fact that this is duopolistic market, forces a consumer to only choose between two companies and that contributes to increase this effect. The data also shows that an increase in the number of online generated leads by firm A (L_A), leads to an increase in its own sales at a higher rate than the previous case. Table 3 presents the results of the estimation of firm B's sales response function (see (6)).

TABLE 3. PARAMETER ESTIMATES, (STANDARD ERRORS) AND "SIGNIFICANT AT THE LEVEL" FOR FIRM B

Parameter	Model_B
β_B	0,535 (0,063) "0,1%"
γ_A	0,214 (0,078) "5%"
R^2	0,997

$$\ln S_B = \beta_B \ln L_B + \gamma_A \ln L_A \tag{6}$$

Comparing the result of Table 3 with the result of Table 2's M1, it can be observed that firm B's online generated leads have a larger influence on firm A's sales than firm A's online generated leads on firm B's sales, because $\gamma_B > \gamma_A$. This result is logical due to the difference in market share already addressed.

Giving that the overall results of the estimation appear to have an adequate validity, it is acceptable to move forward and to present and discuss the proposed management model and its mathematical formulation.

III. PROPOSED MODEL

For every step of the formulation of the proposed management model, every equation for each firm (A and B) will be presented – with the equations for firm A above the equations for firm B. Thus, for the sales response function we have that:

$$S_A = (L_A)^{\beta_A}(L_B)^{\gamma_B} \tag{7}$$

$$S_B = (L_B)^{\beta_B}(L_A)^{\gamma_A} \tag{8}$$

Let Π_A and Π_B denote firms A and B's profit functions respectively. Then,

$$\Pi_A(L_A, L_B) = k_A M_A S_A - L_A \tag{9}$$

$$\Pi_B(L_A, L_B) = k_B M_B S_B - L_B \tag{10}$$

where k_A and k_B are firm-specific factors that standardize the units of each profit function, and they will be described in more detail and calculated ahead. M_A and M_B are called unit contribution, for firms A and B respectively, and they are calculated knowing each firm's cost structure. They are, within the scope of this paper, the profit per online contract, measured between 0 and 1. The quantity of leads to be generated in order to maximize the above functions is obtained by solving:

$$\frac{\partial \Pi_A}{\partial L_A} = 0 \tag{11}$$

$$\frac{\partial \Pi_B}{\partial L_B} = 0 \tag{12}$$

When the set of equations is solved, they yield the following reaction functions:

$$L_A(L_B) = (k_A M_A \beta_A {L_B}^{\gamma_B})^{\frac{1}{1-\beta_A}} \tag{13}$$

$$L_B(L_A) = (k_B M_B \beta_B {L_A}^{\gamma_A})^{\frac{1}{1-\beta_B}} \tag{14}$$

The idea behind reaction functions is to estimate what a firm should do given what its rival(s) did. Using (13) as an example, the idea is that firm A already has an estimation of how many online leads firm B will generate the following month thus, with this knowledge, it will use the reaction function to determine how many online leads it should generate for the same month.

The factors k_A and k_B are determined by solving (13) and (14) for k_A and k_B, respectively:

$$k_A = \frac{\overline{L_A}^{(1-\beta_A)}\overline{L_B}^{(-\gamma_B)}}{\beta_A M_A} \tag{15}$$

$$k_B = \frac{\overline{L_B}^{(1-\beta_B)}\overline{L_A}^{(-\gamma_A)}}{\beta_B M_B} \tag{16}$$

Equations (15) and (16) are reached through solving (13) and (14) for k_A and k_B, respectively. $\overline{L_A}$ and $\overline{L_B}$ are the average quantity of online generated leads for the period's available data. This is not the only to calculate these factors. Instead of the average quantities of online generated leads, each firm can select its own way to calculate the factors, by for example, choosing to select the triple amount of online leads than the rival(s) and calculate its factor accordingly.

Finally, to calculate the Nash equilibrium - at the equilibrium, no firm as an incentive to change the quantity of online leads to generate - the first order conditions are written as a system, a natural logarithm is applied to each (13) and (14), and the system is solved by using Cramer's rule, thus:

$$L_A = (k_A M_A \beta_A)^{\frac{1-\beta_B}{(1-\beta_A)(1-\beta_B)-\gamma_A\gamma_B}}(k_B M_B \beta_B)^{\frac{\gamma_A}{(1-\beta_A)(1-\beta_B)-\gamma_A\gamma_B}} \tag{17}$$

$$L_B = (k_A M_A \beta_A)^{\frac{\gamma_B}{(1-\beta_A)(1-\beta_B)-\gamma_A\gamma_B}}(k_B M_B \beta_B)^{\frac{1-\beta_A}{(1-\beta_A)(1-\beta_B)-\gamma_A\gamma_B}} \tag{18}$$

Additionally, if both firms are symmetric, i.e. $\beta_A = \beta_B = \beta, \gamma_A = \gamma_B = \gamma, M_A = M_B = M$, the Nash equilibrium is as follows:

$$L = (kM\beta)^{\frac{1}{(1-\beta)+\gamma}} \tag{19}$$

Then, $L_A = L_B = L$.

IV. Application Scenarios

Firms A and B establish a goal of gained online contracts in the beginning of every month, then they estimate (through a process with very little rigor) how many online leads they should generate in order to achieve their goal. This management model helps the marketing managers in this process.

To perform an initial validation of the model, we estimated how many online contracts firms A and B would gain through the proposed model and compare them with the actual gained online contracts during the 16 months. The online generated leads from the 16 months studied from firms A and B presented in Table 1, are inserted into (7) along with the estimations from Table 2's M1, where we use the estimated values of β_A and γ_B and substitute in their respective places into (7), as well as the value of M_A which is 0,2, i.e. 20% of profit for each online contract sold. For firm B, the process is identical, where both firms' leads, the estimations from Table 3's M1, using the values of β_B and γ_A, and the value of M_B, which is 0,1, are inserted into (8). With the results computed, we compared the contracts effectively gained by firm A each month and the model's estimations through the error percentage (see (20)):

$$Error\ (\%) = \frac{|Estimated\ Value - Exact\ Value|}{Exact\ Value}.100\% \tag{20}$$

According to the estimations in Table 4, firm A would sell 130 less online contracts with the application of the proposed model, while firm B would sell 181 less online contracts, when compared with the actual values. This difference can be explained by the fact that some of the data used in the parameters' estimations is not completely exact, the fact that 16 is not a high number of observations that can inspire a model with absolute confidence, although that is one of the reasons why the bootstrap technique was used. Still it is important to state that the more observations one can gather, the better and more trustworthy the model will be, and also the fact that there is no clear pattern when observing the evolution of the monthly quantity of online leads that each firm generates and the quantity of monthly online contracts sold.

The average error incurred is 27,9%, for firm A, which might seem somewhat high at first, but one has to acknowledge certain limitations such as the few number of observations and the fact that the monthly quantity of firm B's online generated leads is an estimate, and thus the final result is not as accurate as ideally would be required. For this firm, the months of Aug 14, Oct 14, Nov 14, and especially Apr 15 have very high percentage error relative to their monthly averages and they can be explained by some unforeseen events such as trends and news in the media, and advertising campaigns for example. Other months, such as Jun 14 and Jul 14 have high percentage errors, evidencing a clear susceptibility to seasonality (and interestingly when most of the advertising campaigns are carried out by both firms.

TABLE 4. ONLINE CONTRACTS ESTIMATION FOR FIRMS A AND B

Month	Online Contracts_A	Estimated Online Contracts_A	Error_A (%)	Online Contracts_B	Estimated Online Contracts_B	Error_B (%)
Jan 14	98	108	10	155	151	2,6
Feb 14	110	110	0	200	219	9,5
Mar 14	116	122	5	231	231	0,0
Apr 14	114	126	11	256	215	16,0
May 14	111	141	27	267	384	43,8
Jun 14	198	106	46	501	372	25,7
Jul 14	275	176	36	561	481	14,3
Aug 14	210	90	57	267	138	48,3
Sep 14	127	113	11	299	222	25,8
Oct 14	88	126	43	301	308	2,3
Nov 14	39	67	71	287	223	22,3
Dec 14	34	39	16	54	90	66,7
Jan 15	88	71	18	110	122	10,9
Feb 15	133	136	2	210	244	16,2
Mar 15	144	156	8	202	262	29,7
Apr15	80	149	86	176	234	33,0

The studied market is an extremely volatile – to news in the media, to trends and mainly to advertising campaigns - and seasonal market which contributes for the difference of the "*conversion rate*" – the rate at which online leads are converted into online contracts - in certain months. We can see that during the summer months (namely June, July and August) and the fall months (namely November and December) the conversion rate is much higher. The recommended solution is to add a factor to (7) and (8) to correct these estimations. The value to be assigned to the factor will be determined by the knowledge and experience of the digital marketing manager of the firm that is using the model. This factor, which varies from 1 to a non-fixed maximum, represents a percentage increase that relates to the expected change in the conversion rate. For example, for a month in which the conversion rate is stable and in, approximately, accordance to the standard model's estimations, the factor will be 1, but if an advertising campaign is incurred and the digital marketing manager forecasts an increase in the conversion rate of 50%, then the value for the factor to be used is 1,5. With the addition of this factor, it is expected that the average monthly percentage error for the estimations to decrease substantially, providing more confidence to the proposed model. Another possibility is to consider a separated model for these months, but due to the low number of observations this solution cannot be developed as of now. The factor will also be used in order to improve the estimation of the gained online contracts when different events such as news in the media occur that can potentially affect the sales.

After an analysis of the factor with the assistance of the marketing manager of firm A, we again estimated the monthly quantity of online contracts gained and the error improved to an average of 16%, which in the view of the authors and the manager is a very acceptable result.

Finally, the month April15, can potentially be viewed as an outlier, and if we ignore that month, the initial average error would be 24% which is better than the original error.

For firm B, the average error incurred is 22,9%, a decrease in 5% when compared to firm A's result. The months of May 14, Aug 14, Dec 14, and, Apr 15, have very high percentage errors relative to the monthly average. The error incurred in the month of Dec 14, 66,7% is especially high, but also explained by the unusual conversion rate pattern presented in that month. To correct these discrepancies, the same methods can be used as explained for firm A.

The total percentage error, for the 16 studied months was 6,6% and 4,4% for firm A and B respectively. These results are interesting due to the fact that the data from firm B is not exact because it was gathered with the assistance of experts in the studied market but not from the actual marketing manager of firm B, and the data from firm A is exact.

Another target application of the model is described as follows: with the estimated values presented in Table 2 and Table 3, and with the monthly estimations (or known data) of firms A and B's online leads, we calculate firms B and A's reaction functions respectively, to determine the quantity of leads that should be generated each month. With this data we can then estimate the quantity of online contracts firms A and B would gain. The results are presented in Table 5.

TABLE 5. ONLINE LEADS AND CONTRACTS ESTIMATION FOR FIRMS A AND B

Month	Estimated Online Leads_A	Estimated Online Contracts_A	Estimated Online Leads_B	Estimated Online Contracts_B
Jan 14	340	72	3321	322
Feb 14	514	108	2658	258
Mar 14	514	108	2979	289
Apr 14	465	98	3271	317
May 14	843	177	2581	250
Jun 14	959	202	1769	171
Jul 14	959	202	3031	294
Aug 14	340	72	2752	267
Sep 14	514	108	2745	266
Oct 14	702	148	2547	247
Nov 14	702	148	1295	125
Dec 14	340	72	1138	110
Jan 15	340	72	2126	206
Feb 15	514	108	3338	323
Mar 15	514	108	3873	375
Apr15	465	98	3896	377

By observing the Tables 1 and 5, the only months where the difference between the actual quantity of online generated leads and its estimation for the presented model for firm A, are somewhat similar are Feb 14, Sep 14 and Jan 15, although the difference in Sep 14 can be viewed as a little high. During these months the difference in the quantity of online sold contracts varies, where in Feb 14 it is estimated that firm A would have sold 2 online contracts less, in Sep 14 19 less and in Jan 15 16 less. There are some months with results that may seem counterintuitive because, for example the estimation might suggest an inferior quantity of online leads to be generated and then the estimated quantity of online sold contracts would be higher (or the inverse), such is the case of Jul 14, Jan 15 and Apr 15. This might happen due to the fact that in these months the conversion rate from online lead to online contract follows an extremely unusual pattern when compared to the generality of the other months, and because there is a high degree of uncertainty when it comes to this rate. It is important to point out that throughout the 16 months, according to the model it is estimated that firm A would have generated 699 less online leads than it actually did and it would have gained less 64 online contracts than it actually did, for a change in profit of -0,04% compared to the actual profit verified.

For firm B, although there are no months in which the estimated quantity of online generated leads is similar the actual values, there are a couple of months that have very interesting results, namely Aug 14 and Sep 14. For Aug 14, although the difference between estimated online generated leads and its actual value is 1972 more estimated online generated leads, the difference in the quantity of online sold contracts is zero. This can be, again counterintuitive because with such an increase in the estimated quantity of online generated leads, it would be expected an increase in the quantity of online gained contracts, but this doesn't happen. And in Sep 14, an increase in the estimated quantity of online generated leads actually leads to a decrease in the estimated quantity of online sold contracts. This situation can also be explained by the uncertainty of the monthly conversion rate as it was in the situation regarding firm A. The estimated total quantity of online generated leads is much less, 1920 less to be precise, than the actual value, while the estimated total quantity of online sold contracts is higher, by 120 units than the actual value. This means that, for firm B, these decisions can be made in a much more efficient and effective manner providing an increase in profit of 12,14% when compared to the actual profit.

V. CONCLUSION

The contribution of this paper is a proposed management model based on game theory to estimate the quantity of online generated leads and the gained online contracts based on what the competitor is doing – the online leads the rival generated during a period of time. After analyzing the results of the estimated selling of online contracts compared to the actual data for the 16 months, for both firms A and B the results verified was that the average monthly errors were 27,9% and 22,9% for firms A and B respectively. Thus, it was suggested the addition of a factor, varying every month and chosen by the marketing manager recurring to his/hers experience, and for firm A, that would diminish the error, fixing it at 16%, which is considered a very interesting and acceptable result for a predictive model in such a volatile market. The annual error was fixed at 6,6% and 4,4% for firms A and B respectively, which although not perfect, they were acceptable and served to verify discrepancies and vulnerabilities in the model. Finally, it was studied the predicted evolution of the online leads and contracts sold, if firms A and B would have employed this model from the first month of the study. The results were extremely encouraging despite a very slight profit decrease in the case of firm A (-0,04%) because it was demonstrated that, with the assistance of this model, firm B would have needed to generate 4,2% less online leads to achieve a profit increase of 12,14% from the selling of online contracts, thus making the decision more effective and efficient.

Another very interesting result was derived from the parameters' estimations that challenge some statements in the literature, and what some might think as intuitive. It is the fact that we demonstrated that an increase in the quantity of a firm's online generated leads has a positive impact on the sales not only of that firm, but also on the sales of the competitor.

A game-theoretic based approach to develop a model to support the marketing managers' decisions on lead generation such as the one proposed by the present paper is very useful for virtually any firm, with the obvious exception of a monopoly market, since it takes into account the strategies that its rival(s) is/are employing. It is an excellent source of benchmarking and competitive advantage, by increasing a firm's efficiency and effectiveness in its performance. The results also show the potential that this model has in this particular task.

For future work, and since this paper covers a part of the Master's dissertation of the primary author, it is recommend a greater data set to more accurately estimate the parameters, and a large scale application of the model to prove its efficiency and utility throughout most firms and markets, and the determination and analysis of the Nash and Stackelberg equilibriums.

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