

Developing a Multi-objective Model for Locating a New Warehouse for Bama's Chain Store

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Abstract-The goal of this study is to improve the transportation management of chain store of Bama Company in Mashhad. Currently, there is only one warehouse that covers all the demands of these chain stores. Thus, transportation costs are very expensive and could not be responsive to the demands in a minimum amount time. Therefore, the objectives of this study are to keep transportation costs to a minimum and increase the speed of sales that would lead to a greater assessment of a successful trading business. This problem becomes a Location Routing Problem (LRP). We have proposed a multi objective model and used IBM ILOG CPLEX to solve the numerical case data to obtain the optimal result.

This model, includes the location of each store and their distance to the new proposed warehouses. Finally, we have compared transportation expenses and hidden costs to the company before and after finding the location of the new warehouse.

Keywords-Location Routing Problem; Multi- objective Programming; Chain Store; Cost Management.

I. Introduction

The new conception of the chain store runs concurrently with the industrial revolution of Europe in the seventeenth century. Chain stores are, by definition, a group of retail outlets owned by one firm and spread nationwide or worldwide such as body shops, K-Mart, Wal-Mart. It has been no longer than a few years that the chain stores and distribution issues have been brought forth for discussion in Iran [1]. The growth and development of chain stores in Iran is attributed to the reduction of the travel costs through the city, a way to purchase that saves time for citizens, decreases traffic congestion, reduces pollution as well as eases government monitoring on the retail sector performance [2]. Nowadays, the costs of transportation is one of the highest portions of logistics costs in several organizations. Major distribution costs normally consist of operating

vehicles and crews' salaries. Consequently, a small saving in the distribution costs can lead to a substantial total savings for an organization. The major problem in the transportation department in any industry is the vehicle routing problem (VRP). However, all the costs of production and services are primarily the result of management decisions on how to use the limited resources of the organization. Today the existence of many companies depends on their capability to increase their vigilance of their cost management system, which should focus on costs cutting measures throughout the process [3].

The organizational spending on commercial and retail activities is so significant and vital, even for the remunerative ones. Companies always try to increase their net profit by bring down operating costs, and also through increasing the profitability of the investment funds in a store's attractions, providing satisfactory services that basically result in an increase in the rate of sales, [3] and [4]. On the one hand, headquarter of chain stores has the significant duty to detect and purchase desirable goods from manufacturers and providers, and, on the other hand, it should be qualified enough to find the best way to distribute them among the stores. Therefore the supply and logistics system is the major issue for a chain store. Moreover, it is wise to state that the logistics network would play the role of the backbone for the store. According to the studies, about 10 to 35 percent of total sales revenue is allocated to cost distribution which is obviously a considerable number.

One of the major factors for chain stores to succeed is its location and central warehouse. A good opportunity not only results in welfare services for the customer, but it is of crucial importance to chain stores in terms of sales. In addition, the appropriate location of a warehouse contributes to the decline in the transportation costs, speeding up the goods sales. That makes the decision concerning the position of the chain store and its warehouse fundamental. Since a change in the location can be, at times laborious, notably causing great loss. A manager may err in his/her decision making about the prime pricing and the supply of goods, but these problems are remediable, while if the problems are because of the wrong choices of the warehouse location, the consequences are liable to be costly and irretrievable [1] and [4].

In this paper we appraise the Bama Company and its circumstances. In fact, it addresses a deterministic LRP that combines location problem (LP), the vehicle routing problem (VRP). We present a model which looks for the best new warehouse location.

II. Literature review

Location theory was first formally introduced in 1909 by Alfred Weber, who considered the problem of deciding a location in the plane to minimize the sum of distance from the distribution center to all demand consumers/retailers. LRP has been developed and studied only during the past four decades, a few extensive surveys can be already found in the literature, such as Madsen, Laporte, and Min et.al. [5] and [6]. Laporte was the first researcher who discussed and classified the LRP models. Min et.al [6]. Reviewed the LRP literature using a hierarchical classification based on the problem characteristics such as the number of depots, the capacity of depots and vehicles, the form of the objective function, etc. Nagy and Sodhi, also performed a comprehensive literature review on the LRP models, solution approaches, application areas and some future works [7].

The most recent review, classified LRP into two categories based on the problem perspective and solution methods. The problem perspective includes, for example, characteristics of facilities and vehicles, the nature of demand and supply (deterministic or stochastic), hierarchical planning levels, and planning horizon, while the solution methods include naturally exact and heuristic algorithms. Exact algorithms including branch and bound, integer programming, and dynamic programming can be found in many hazardous waste applications. On the other hand, most heuristics found in the literature, apply some combinations of four strategies, namely location-allocation-first, route-second; route-first, location-allocation-second; savings/insertion; and tour improvement/exchange. LRP is applicable to a wide variety of fields such as food and soft drink distribution, newspapers delivery, waste collection, bill delivery, military applications, parcel delivery, drug distribution and various consumer goods distribution [8] and [9].

Traditionally, location problem is a branch of operations research concerning itself with mathematical modeling and the solution of problems concerning optimal placement of facilities in order to minimize transportation costs, avoid placing hazardous materials near housing, outperform competitor, etc. Solutions of Location problems in supply chain management are to find the ideal locations for suppliers, manufactures, distribution centers and warehouses to achieve different objectives by using mathematical modeling, heuristics, and mathematical tools such as, ILOG CPLEX, Logic Net Plus and etc. Obviously, the above problem can be described as following the mathematical model:

$$\text{Geometric Solution} = \operatorname{argmin} \sum_{i=1}^{12} \|x_i - y\|_2, \quad y \in R^n \quad (1)$$

Where $x_1, x_2, x_3 \dots x_{12}$, are the points where the twelve customers are located, y is the position where the facility is located. Based on the simple model provided by Alfred Weber, researchers have proposed lots of models to describe complex location problems for different industries. In fact, location theory is not only a pure mathematical problem, but it comes from application, and it also has lots of applications in different industries, such as logistics, public fire protection, manufacture and etc. For example, when a supply retailer is thinking to open new outlets, he will consider customer demand and the related costs for different locations. When a manufacturer chooses where to position a warehouse, he will consider customer demand, cost, inventory and market trends of the targeted locations. When a city planner selects locations for fire stations, he will consider the requirements and constraints for fire fighting. Obviously, these problems are typical location problems. Although all of these problems are called location problems, there are many differences in constraints and objectives. These constraints and objectives are coming from factors/decisions for specific industries, Li et.al [10]. For different industries, the factors/decisions are different. For instance, customer demand, population shift and market trends will be considered by a logistics planner when he determines the locations for the distribution center, whereas minimum transportation time and district coverage rate will be thought by a city planner when he selects locations for fire stations. Because these factors will have impact on the constraints of the location model, they will result in lots of challenges for models and algorithms for various location problems.

In order to solve these problems, the researchers also proposed dozens of exact optimization algorithms and heuristics Brandeau & Chiu, Owen & Daskin, Rosing [11] and [12]. The most popular exact optimization algorithms are branch-and-bound, branch-and-cut, column generation, and decomposition methods. Branch-and-bound algorithms are sometimes combined with Lagrangian relaxation or heuristic procedures to obtain bounds. Normally, static and deterministic facility location problems are attractive to be solved by exact optimization algorithms. However, in the real world, the number of decision variables is large and the models are comparatively more complex; it is hard to obtain an optimal solution by exact optimization algorithms. Next there comes the heuristic method. Linear programming based on heuristics and meta-heuristics are among the most popular techniques. In fact, most of the time the dynamic location problems, stochastic location problems and problems with multiple objectives can only be solved with some specific methodology and/or heuristics. At the same time, researchers have created and built some useful and innovative tools to help us solve the location problem in a supply chain.

The most famous tools for the location problems are IBM ILOG Logic Net Plus XE and Watson Implosion Technology. IBM ILOG Logic Net Plus XE is a software for supply chain network optimization, supply chain design, and an off-the-shelf decision support solution for ongoing strategic planning. It is used for network design and production sourcing. IBM ILOG Logic Net Plus XE can solve the following typical applications:

1. "Distribution Network Design", determine the optimal number, location, and size of distribution facilities to meet customer service requirements at minimum cost.
2. "Manufacturing Network Design", determine the best number, location, and capacity of plants, lines, and processes to maximize asset utilization, minimize total cost, and align capacity with business growth projections.
3. "Manufacturing Sourcing Strategy", in a multi-plant environment, determine which product should be made at which plant, trading off manufacturing costs and economies of scale with transportation costs.
4. "Shipping Territory Realignment", determine the best service territory for each DC (Distribution Center) to improve service levels and reduce costs.
5. "Network Transition Planning", make the transition to a new supply chain configuration focusing on various asset, capacity, inventory and transportation lane requirements.
6. "Seasonal Supply Chain Design", in a highly seasonal business, determine the appropriate trade-off between capacity and inventory pre-build and the use of overflow facilities.
7. "Contingency Planning", understand how unexpected events in the supply chain will affect the costs, service levels, and potential revenues and develop plans to mitigate the risks.

III. Case Study

In the procurement system, programming is discussed within three main parts including: inventory, placement and transportation. The main purpose of making decision on three items is to reduce costs and increase the quality of customer services (two important indicators of physical distribution function). Consistently, the principle aim of procurement system is

to deliver the right product or service to the right place at the right time, based on the customer satisfaction while it validates the reliability of the company [13] and [14].

In this model, we have conducted a survey of Bama Company with 10 stores in 10 points in Mashhad. Bama Company is one of the largest Companies in the field of chain stores in Mashhad. In December 2011, Bama Company was established using international standards and its professional approach for the consumer, which were big steps for the Company to take at that time. The Company started with just two stores. In less than a year, the number of its stores increased up to 10 and their number is still continuing to grow throughout other Iranian cities.

An initiative aimed to concentrate on finding a new warehouse in order to have the goods distributed at the right time, reduce the transportation costs, depreciate the cost of operating vehicles and the other hidden costs. The consumptions in this model include:

1. Every customer should be visited just one time by one vehicle.
2. Every customer should be allocated only to one warehouse and visited only and just by one vehicle.
3. All the demands and the returns of the customers should not exceed more than the capacity of a vehicle.

After analyzing the following items:

1. Warehouse location based on the traffic in its area
2. Proximity of the warehouse to larger stores with higher sales
3. The base price for each square meter of the land
4. Several different possible directions to reach the warehouse

by using the AHP method, 100 locations are selected and among these 100 locations, 5 points, as the suitable points, are selected for the new warehouse. It should be noted that, one of the 5 selected candidate warehouse sites is randomly one of the Company stores. The proposed model is suggested due to the limited capacity of the vehicles and the warehouse as well as the time limitation that is faced. We have used the data such as order quantity, number and location as well as the time requirement of customers, transportation costs and the number of trucks. Also, we have supposed that all potential distribution center locations can be used for storing products and while we open a distribution center it will have a capacity for transportation. It will be consumed for each transportation. If the warehouse is not opened, the capacity is zero which prevents the products to be transported to this warehouse. Perfect locations of warehouse are near to more customers or larger customers, and they are influenced by the production supply from a factory, demands from customer and transportation fees. Our multi-objective model can optimize the warehouse location given all this information.

Parameters:

- c_{ij} : Transportation costs from i to j
 d_i : Store's demand i
 p_i : Amount of the returned goods from the store i
 CV : Vehicle Capacity

Collections:

- N : The set of all the stores and warehouses
 N_c : The set of all the stores
 N_0 : The Set of all the warehouses
 K : The Collection of the all vehicles

Decision Variables:

$$x_{ijk} = \begin{cases} 1, & \text{if there is a rout between the store } i \text{ and } j \text{ by the vehicle } k \\ 0, & \text{Otherwise} \end{cases}$$

$$z_{ik} = \begin{cases} 1, & \text{If the store } i \text{ is visited by the vehicle } k \\ 0, & \text{Otherwise} \end{cases}$$

V_{ijk} : The amount of returned goods of the stores to the store j after serving the store i traversed by the vehicle k .

U_{ijk} : The amount of entering goods to the store j after serving the store i traversed by the vehicle k .

t_{ik} : The time that the product is delivered to the store i traversed by the vehicle k .

L_k : Delivery time of the product to the last store by the vehicle k .

According to the defined parameters and sets and in order to minimize products delivery time and transportation costs, as multi-objective functions, the following model is proposed:

$$F1 : \min \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^K c_{ij} x_{ijk} \quad (1)$$

$$F2 : \min \left(\max_{k \in K} (L_k) \right) \quad (2)$$

S.t

$$\sum_{j=1}^N \sum_{k=1}^K x_{ijk} = 1 \quad \forall i \in N_c \quad (3)$$

$$\sum_{j=1}^N x_{ijk} - \sum_{j=1}^N x_{jik} = 0 \quad \forall i \in N \quad (4)$$

$$\sum_{k=1}^K z_{ik} = 1 \quad \forall i \in N_c \quad (5)$$

$$x_{ijk} \leq z_{ik} \quad \forall i, j \in N, \forall k \in K \quad (6)$$

$$x_{jik} \leq z_{ik} \quad \forall i, j \in N, \forall k \in K \quad (7)$$

$$x_{ijk} + z_{ik} + \sum_{m \in K \& m \neq k} z_{jm} \leq 2 \quad \forall i, j \in N_c, i \neq j, \forall k \in K \quad (8)$$

$$\sum_{j=1}^N U_{jik} - \sum_{j=1}^N U_{ijk} = d_i z_{ik} \quad \forall i \in N_c, \forall k \in K \quad (9)$$

$$\sum_{j=1}^N V_{ijk} - \sum_{j=1}^N V_{jik} = p_i z_{ik} \quad \forall i \in N_c, \forall k \in K \quad (10)$$

$$V_{ijk} + U_{ijk} \leq CV \cdot x_{ijk} \quad \forall i, j \in N, \forall k \in K \quad (11)$$

$$\sum_{j \in N_c} U_{ijk} = \sum_{j \in N_c} z_{jik} d_j \quad \forall i \in N_0, \forall k \in K \quad (12)$$

$$\sum_{j \in N_c} U_{jik} = 0 \quad \forall i \in N_0, \forall k \in K \quad (13)$$

$$\sum_{j \in N_c} V_{jik} = \sum_{j \in N_c} z_{jik} p_j \quad \forall i \in N_0, \forall k \in K \quad (14)$$

$$\sum_{j \in N_c} V_{ijk} = 0 \quad \forall i \in N_0, \forall k \in K \quad (15)$$

$$U_{ijk} \leq (CV - d_i)x_{ijk} \quad \forall i \in N_c, \forall j \in N, \forall k \in K \quad (16)$$

$$V_{ijk} \leq (CV - p_i)x_{ijk} \quad \forall i \in N, \forall j \in N_c, \forall k \in K \quad (17)$$

$$U_{ijk} \geq d_i x_{ijk} \quad \forall i \in N_c, \forall j \in N, \forall k \in K \quad (18)$$

$$V_{ijk} \geq p_i x_{ijk} \quad \forall i \in N, \forall j \in N_c, \forall k \in K \quad (19)$$

$$t_{jk} \geq t_{ik} + c_{ij} - M(1 - x_{ijk}) \quad \forall i, j \in N, \forall k \in K \quad (20)$$

$$L_k \geq t_{ik} \quad \forall i \in N, \forall k \in K \quad (21)$$

$$x_{ijk}, z_{ik} \in \{0,1\} \quad \forall i, j \in N, \forall k \in K \quad (22)$$

$$V_{jik}, U_{ijk} \geq 0 \quad \forall i, j \in N, \forall k \in K \quad (23)$$

In this model, the first objective function (F1) minimizes the travel costs and the second objective function (F2) minimizes the products delivery time to the last store before returning to the warehouse. Constraint (3) guarantees that there should be a way from the store i to the store j or warehouse j by vehicle k . Constraint (4) shows the continuity of path. Constraint (5) indicates that, every store should be just only be allocated to one warehouse. Inequalities (6) and (7) show that the store allocated to a warehouse can be served by vehicle k . Constraint (8) guarantees that if there is a path between two stores, both of them must belong to the same warehouse. Constraint (9) shows that the difference of the amount of goods in two respective stores must be equal to the demand of the store. Constraint (10) indicates that the difference of the amount of the returned goods between two respective stations should be equal to the amount of the returned goods of determined station. Constraint (11) states that the amount of the input and the output goods at each station should not exceed the capacity of the vehicle. Constraint (12) ensures that the amount of the goods carried by the vehicle from the warehouse along the way should be equal the demand of the stores. Constraint (13) indicates this issue, that the amount of the goods on the vehicle of the last store to the warehouse should be zero. Constraints (14) and (15) show the amount of the returned goods at the beginning and at the end of the way. Constraints (16) and (18) show the minimum and maximum amount of the input and the output goods at each station, respectively. Constraints (17) and (19) show the minimum and maximum amount of the input and the output returned goods in each station, respectively. Constraint (20) represents the delivery time of the goods to the stores, and constraint (21) calculates the longest delivery time of the goods to the last store and finally constraints (22) and

(23) determine the type of the variables. The weighted multi- objective function is solved by IBM ILOG CPLEX 12.3, where weighting method is as follows:

$$\min F^{Total} = \min((1-\alpha)F_1 + \alpha F_2) \quad (24)$$

IV. Result

The results are shown in Table 1. The first objective function (total travel costs) is for the situation that we have just one warehouse (the central warehouse), which is 138 in the optimized mode, and the second objective function (minimum distance of the warehouse to the longest distance of the store), which is 53 in the optimized mode. Thus, according to the results, if we establish another warehouse, the first and second objective functions will be minimized.

TABLE1 -THE VALUES OBTAINED FOR THE OBJECTIVE FUNCTIONS WITH DIFFERENT PRIORITIES

	α	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Central warehouse and place(1)	*Minimum distance	51	51	51	51	51	51	51	51	49	48	48
	*Cost	106	106	106	106	106	106	106	106	114	120	120
Central warehouse and place(2)	Minimum distance	50	50	50	50	50	50	50	50	48	48	48
	Cost	103	103	103	103	103	103	103	103	121	121	121
Central warehouse and place(3)	Minimum distance	50	50	50	50	50	50	48	48	48	48	48
	Cost	104	104	104	104	104	104	108	108	108	108	108
Central warehouse and place(4)	Minimum distance	52	51	51	51	51	51	46	45	45	45	45
	Cost	107	107	107	107	107	107	123	127	127	127	127
Central warehouse and place(5)	Minimum distance	50	50	50	50	50	50	50	50	44	44	44
	Cost	108	108	108	108	108	108	108	132	132	132	132
Central warehouse and zanbagh store ,place(6)	Minimum distance	44	44	44	44	44	44	44	44	44	44	44
	Cost	104	104	104	104	104	104	104	104	104	104	104

* The Units of the length and the cost are based on the minute. The first objective function (F1) related to the cost and the second objective function (F2) is related to the greatest length.

Table 1 shows that the optimal locations for a new warehouse must be one of the two following places: The place (6) (that is Zanbagh store) and the place (2) (which is a new place). Fig.1 and Fig.2 show the values of F₁ and F₂, respectively.

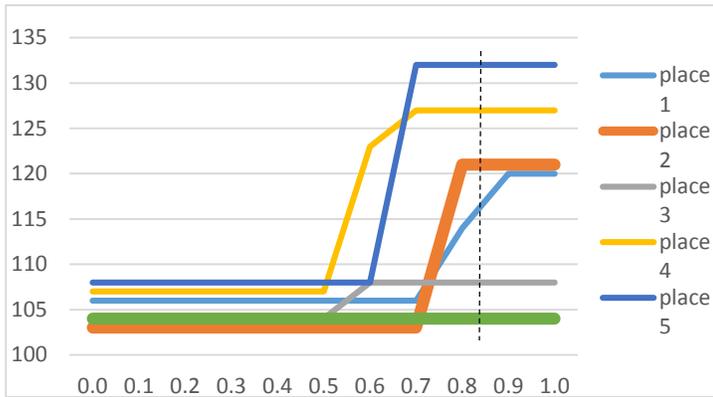


Fig. 1. Value of F_1 function

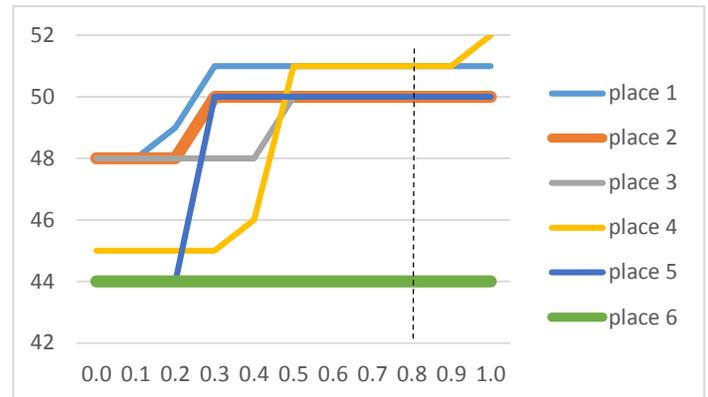


Fig. 2. Value of F_2 function

Now, based on the equation (24), Fig. 1 and Fig. 2, when α is greater than 0.8, it means that the second objective function (that minimizes the longest distance) is more important. In this case, we only establish a warehouse in the place (6) (an optimal place). Additionally, it's notable that, in this mode, the first objective function in the place (6) which is again, lower than the Place (2). Therefore when α is greater than 0.8, the values of both objective functions in the place (6) are lower than the place (2).

When α is smaller than 0.8, (except equality case that happened in 0.5), the first objective function (that minimizes the transportation costs) is more important. In this case, the transportation costs in the place (6) is lower than the place (2), while the value of the second objective function in the place (2) is more lower than the place (6). Moreover, according to the hidden costs for the establishment of the new warehouse, we reach the result that, the place (6) (Zanbagh store), could be selected as the optimal choice.

Finally, with this proposal, Bama Company could decrease 20% of the products delivery time while decreasing 35% the transportation costs that basically would be an adequate result for the Company.

V. Conclusion

Numerous researchers and engineers have completed a great amount of work for location routing problems. The American Mathematical Society (AMS) even created specific codes for these kinds of problems. In this chapter, we firstly reviewed the traditional location problem in supply chain management from the following three views: modeling, solving algorithms and mathematical tools. Then a mathematical model is proposed to describe the distribution center location. Based on this model, we illustrate the suitable center location warehouse impact on the transportation cost and product delivery time by using the IBM Watson Implosion Technology tool. The first objective function (F_1) minimizes the travel costs and the second objective function (F_2) minimizes the products delivery time. The case study has shown that the proposed technique can lead to significant savings in logistics costs. Nevertheless, reflecting a realistic distribution of goods within the location planning process can provide the opportunity to obtain good solutions.

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VII. Biography

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Reza Atefi earned B.S. in Industrial Engineering from Birjand University of Technology, Iran, Masters in Industrial Engineering from Ferdowsi University of Mashhad, Iran. He has published conference papers. His research interests include manufacturing, optimization, scheduling, manufacturing, queuing, location and statistic.

Sohrab Effati received a B.S. degree in applied mathematical from Birjand University, Birjand, Iran, in 1992; the M.S. degree in applied mathematics from Tarbiat Moallem University of Tehran, Tehran, Iran, in 1995; and the Ph.D. degree in control systems from Ferdowsi University of Mashhad, Mashhad, Iran, in 2000. Since 2005, he has been an Associate Professor with the Department of Applied Mathematics, Ferdowsi University of Mashhad. His research interest include control systems, optimization, ordinary differential equation and partial differential equations, and neural networks and their applications in optimization problems.