

# Composition Methodology for Optimal Portfolio and Performance Measures Considering Indian SENSEX

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**Abstract**— India has one of the fastest growing stock market. However it seems that research of India stock market is woefully deficient. This paper develops the investment based on Markowitz's Portfolio Selection Theory using India historical stock return data. The entire experiment period holds nine years starting from the opening day in 2006 to the 2014 closing day. The research benchmarks Indian SENSEX of BSE (Bombay Stock Exchange). This process made comparison analysis of rate of SENSEX change, and rate of portfolio return. The investment category was chosen by top 30 on SENSEX market as of June 6, 2015, except in the case of five categories which lacks data. The portfolio was composed of eight weeks of investment period and eight weeks of rebalancing cycle. At this time the result displayed that rebalancing cycle influences the rate of return. Four weeks of rebalancing cycle performed outstanding return other than the eight and twelve weeks and rate of changes in SENSEX. In addition, this paper compares return on risk rate, also known as Sharpe ratio which measures portfolio performance.

**Keywords**—Portfolio; Markowitz; Indian stock market; SENSEX; Optimization theory; Sharpe ratio

## I. INTRODUCTION

Various methods are considered to trigger a financial investment action. Earning high return and avoiding the variable risk are common desire for the most investors, but there are all different sorts and conditions of investors. Some investors choose less risk investment while other investors choose risk taking investment. However, the ultimate goal of these investors is to form a portfolio that satisfies the minimum needs expected rate of return and minimizes the expected risk [1,2,3].

The conflicted connection of expected rate of return and risk was suggested by [4]. The efficiency of diversified investment was identified through 'Portfolio selection theory' in [5]. It decides the investment percentage of reducing the risk while getting highest return [6]. Markowitz 'Portfolio selection theory' has been a root and a representative of modern investment theory, and made a great ripple effect in financial market. Markowitz was approved around the world by receiving the Nobel Prize in economics.

This research practically analyzes the performance of India stock market by using the investment algorithm based on Markowitz portfolio selection. Indian Bombay stock exchange market was founded in 1875, which holds the longest history in Asia. Research CEO of Indian financial investment company Aditya Birla predicted that "India SENSEX index will grow twice in the next few years." According to [7] and [8]. Indian stock market keeps showing high potential, however, it is not sufficient in research arena that proves Indian markets' performance by using Markowitz model or by harmonizing the theory.

The investment period was set for nine years from the year of 2006 to 2014 to observe the sudden changes in market volatility. 2006-2007 takes up-phase before the subprime mortgage crisis, 2008-2009 takes rapid down-phase, and after 2010-2014 takes Recovery period [3,9]. Like this, the experiment period takes special fluctuating economic conditions. This research benchmarks SENSEX index, which includes 30th market capitalization of India Bombay Stock Exchange(BSE). When we measured the modeled performance, we added exchange stock commission and tax to be identical with reality in the process of rebalancing the portfolio. Therefore, frequent rebalancing will reflect the market conditions, but too high transaction cost will be required [3,5]. According to [5], a year of data reference period and 8 weeks of rebalancing cycle showed the best performance. [3] set rebalancing cycle following the result of [5]. Data reference period was selected as three years following [10]. However, possibility of error existed only to beg the rebalancing cycle as eight weeks.

To form an optimal portfolio, data reference period was fixed in three years and analyzed the portfolio that shows highest performance when the rebalancing cycle were four, eight, and twelve weeks. Also, this paper measured Sharpe ratio which examines the performance of investment by adjusting for its risk.

## II. PORTFOLIO SELECTION MODEL

Markowitz's optimal asset allocation theory is used to construct portfolio by India stock market. The portfolio selection model by [4] sets the objective function to minimize profit risk of the portfolio which shows the degree of the risk. In addition, it achieves minimum expected rate of return that investors want. And it follows the condition that it is non-linear programming that holds constraints of total sum investment percentage is one considering short stock selling is absent. The portfolio selection model by Markowitz is given below:

$$\begin{aligned} \text{Min } V &= \sum_{i=1}^n \sigma_i^2 \omega_i^2 + 2 \sum_{i=2}^n \sum_{j>1}^n \sigma_i \sigma_j \rho_{ij} \omega_i \omega_j \\ \text{s.t. } E &= \sum_{i=1}^n \mu_i \omega_i \geq K \\ \sum_{i=1}^n \omega_i &= 1 \quad \text{and} \quad \omega_i \geq 0 \quad (i = 1, 2, \dots, n) \end{aligned} \quad (1)$$

$n$  : Numbers of targeting investment target that can be include to portfolio

$\omega_i$  : Investing stock percentage at portfolio ( $i = 1, 2, \dots, n$ )

$\mu_i$  : Average rate of return of stock ( $i = 1, 2, \dots, n$ )

$\sigma_i$  : Standard deviation toward return rate of stock ( $j = 1, 2, \dots, n$ )

$\rho_{ij}$  : Correlation coefficient of stock and return that is required to portfolio

$K$  : Minimum expected rate of return that is required to portfolio

$V$  : Variance toward return rate of portfolio

$\sigma$  : Expected risk (standard deviation) toward expected return of portfolio

$E$  : Expected rate of return of portfolio

Under the condition that non-linear programming model above has no short stock selling, this research aims to find the optimal investment weight ( $\omega_1, \omega_2, \dots, \omega_n$ ) which can meet all the limited conditions and minimize variance ( $V$ ) of portfolio. The objective function of the model shows decreasing marginal return and it is quadratic programming which every constraint is linear. To find the global optimal solution, this paper used a commercial software such as Solver in Microsoft Excel [3,5].

## III. EXPERIMENT METHODOLOGY

### A. Investment Target Selection

This research benchmarks the Indian stock market for an experiment object model. Indian financial market has settled the systematic system throughout the long history. Indian stock market started from Bombay stock exchange (BSE), which is the first stock market in Asia in 1875. BSE is one of the fifth world stock exchanges [11].

India has recorded high levels of economics growth from 2003 to present, except for 2008 which had subprime mortgage [12]. In 2010, Indian stock market recorded 3.2 trillion dollars in comparison with 2004 increased 4.3 times. Since GDP ratio went over 100%, it also showed a rapid recovery after the drop in 2008. In 2004, it recorded the end of 2010 193% [12]. According to [7] and [8], Korean researchers who studied about Indian economic situation and stock market are insisting that the lack of data showed from India which is compared to the importance of the Indian market. Therefore, this study emphasizes the possibility of investment of importance occurred and possibility of Markowitz portfolio selection model of the Indian market. This research selected Indian SENSEX which represents ratio of Indian BSE. Through this process, analysis of the past data was preceded using investment selection model mentioned earlier. The investment category is the 30th market capitalization of Indian SENSEX basis on June 6, 2015, except 5 lacks on data (COAL INDIA, TATA CONSULTANCY SERVICES, MARUTI SUZUKI INDIA, BAJAJ-AUTO, NTPC were crossed out in TABLE I).

TABLE I. INVESTMENT CATEGORY OF INDIAN SENSEX ON JUNE 6, 2015.

No.	Company Name (Simplified Form)	Company name (Full)
1	SUNPHARMA	Sun Pharmaceutical Industries
2	COALINDIA	Coal India
3	TATAMOTORS	TATA Motors
4	SBIN	State Bank of India
5	TATASTEEL	TATA steel
6	ICICIBAN	ICICI Bank
7	RELIANCE	Reliance Industries
8	INFY	Infosys
9	LT	Larsen Toubro
10	ITC	ITC
11	AXISBANK	Axis Bank
12	TCS	TATA Consultancy Services
13	ONGC	Oil And Natural Gas Corporation
14	HDFCBANK	HDFC Bank
15	MARUTI	Maruti Suzuki India
16	BHARTIARTL	Bharti Airtel
17	CIPLA	Cipla
18	HINDALCO	Hinadlco Industries
19	HDFC	Housing Development Finance Corporation
20	HINDUNILVR	Hindustan Unilever
21	BAJAJ-AUTO	Bajaj Auto
22	BHEL	Bharat Heavy Electricals
23	VEDL	Vedanta Limited
24	HEROMOTOCO	Hero MotoCorp
25	WIPRO	Wipro
26	DRREDDY	Drreddys Laboratories
27	M&M	Mahindra Mahindra
28	NTPC	NTPC
29	GAIL	Gail (INDIA)
30	TATAPOWER	TATA Power

### *B. Experiment Period Selection*

The whole experiment period includes nine years starting from the opening day in 2006 to the 2014 closing day. From the beginning of 2006 to at the end of 2007, before sub-prime mortgage came, was defined as the upturn. There was a global financial crisis in between the earl year of 2008 and at the end of 2009. This period was defined as the slump machine, i.e., subprime mortgage. After the global financial crisis in 2010, the recovery period was observed until 2011. The time from 2012 to 2014 was the flat period. The flat period was added to measure the latest investment performance.

Fig. 1 depicts Indian SENSEX movement for recent nine years from January 2, 2006 (9,390 point) to December 31, 2014 (27,499 point). During this period, Markowitz's portfolio selection model was used to precede the simulation experiment. In the process of the experiment, Indian SENSEX showed 292% of high growth rate for nine years. This research analyzed portfolio model's utility in the various periods and particular economic conditions [3].

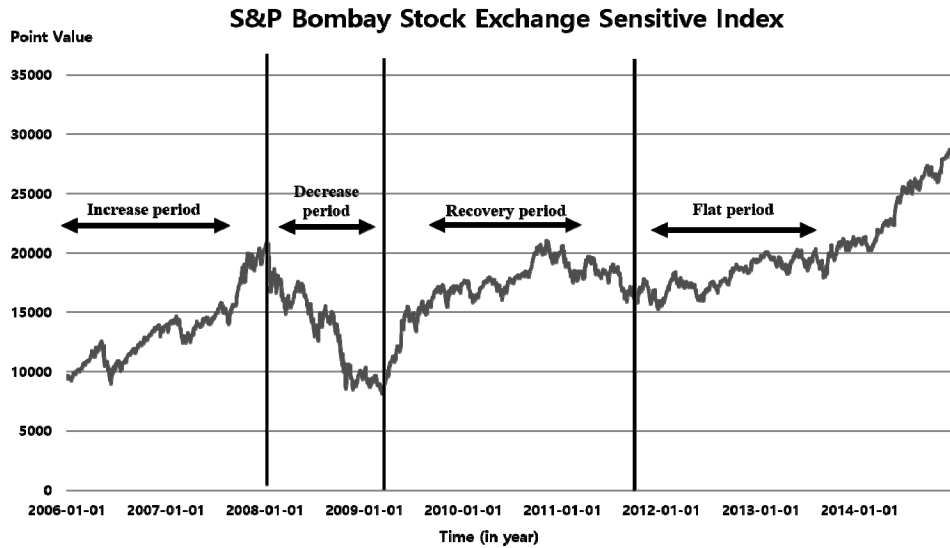


Fig. 1.  $\Delta$  in Indian SENSEX from January 1, 2006 to December 31, 2014.

C. Investment Analysis Methodology

This experiment constituted imaginary portfolio considering categories in Indian SENSEX. Furthermore, applying Markowitz portfolio selection model, scientific and automatic investment has been done to manage the portfolio under consideration. In sequence, we compared the performance in Indian SENSEX investment with the imaginary fund [9].

According to [10], as figuring out covariance using the past data on return, it reveals that the expected figures of covariance is creditable when the number of data on return on investment is greater than the numbers on investment. This research selected the reference data as three years of period for constant use of the past data. As shown in TABLE II, the expected values, standard deviations and covariances of rate of return were computed and displayed using past date on daily return from January 2003 to December 2005 for constructing the initial portfolio.

TABLE II. THREE YEARS(2003.01.01-2005.12.30) OF RETURN, COVARIANCE, AND STANDARD DEVIATION AT INDIAN SENSEX

Covariance	SUNPRA REAL	TATAMOTORS	SBID	TATASTEEL	WICI BANK	RELIANCE	INFY	ITC	AXIS BANK	CHOC	HDFC BANK	SHARDA RTL	CELA	INDIAN CO	HDFC	INDIAN LIFE	BIEL	VEDL	HEXAGON TODD	WIPRO	DRREDDY	MMI	GAIL	TATA POWER		
SUNPRA	0.0006	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0001	0.0000	0.0001	0.0001	0.0001	0.0001	0.0000	0.0001	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	
TATAMOTORS	0.0000	0.0005	0.0002	0.0002	0.0002	0.0001	0.0001	0.0002	0.0000	0.0002	0.0001	0.0001	0.0000	0.0002	0.0001	0.0000	0.0002	0.0002	0.0001	0.0001	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001
SBID	0.0000	0.0002	0.0015	0.0002	0.0001	0.0000	0.0002	0.0003	0.0002	0.0001	0.0001	0.0000	0.0002	0.0001	0.0000	0.0002	0.0002	0.0002	0.0001	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0002
TATASTEEL	0.0001	0.0002	0.0002	0.0005	0.0002	0.0000	0.0002	0.0000	0.0002	0.0001	0.0001	0.0001	0.0000	0.0003	0.0001	0.0000	0.0002	0.0003	0.0001	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0002
WICI BANK	0.0000	0.0002	0.0002	0.0002	0.0012	0.0002	0.0000	0.0003	0.0000	0.0003	0.0002	0.0002	0.0000	0.0002	0.0002	0.0000	0.0002	0.0002	0.0001	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0002
RELIANCE	0.0000	0.0001	0.0001	0.0002	0.0002	0.0003	0.0000	0.0001	0.0000	0.0002	0.0001	0.0001	0.0000	0.0002	0.0001	0.0000	0.0002	0.0002	0.0001	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001
INFY	0.0000	0.0001	0.0000	0.0000	0.0000	0.0007	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ITC	0.0001	0.0000	0.0002	0.0002	0.0003	0.0001	0.0000	0.0003	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0001	0.0000	0.0003	0.0002	0.0001	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0002
AXIS BANK	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
CHOC	0.0000	0.0001	0.0002	0.0001	0.0002	0.0000	0.0001	0.0000	0.0002	0.0004	0.0001	0.0001	0.0001	0.0002	0.0002	0.0000	0.0003	0.0002	0.0001	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001
HDFC BANK	0.0001	0.0001	0.0001	0.0001	0.0002	0.0001	0.0000	0.0001	0.0001	0.0002	0.0001	0.0002	0.0001	0.0000	0.0001	0.0000	0.0001	0.0001	0.0001	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001
SHARDA RTL	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0004	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001
CELA	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0001	0.0001	0.0000	0.0002	0.0001	0.0001	0.0000	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
INDIAN CO	0.0001	0.0002	0.0002	0.0003	0.0002	0.0002	0.0000	0.0002	0.0001	0.0002	0.0001	0.0001	0.0001	0.0006	0.0001	0.0000	0.0003	0.0004	0.0001	0.0000	0.0000	0.0001	0.0000	0.0000	0.0001	0.0002
HDFC	0.0001	0.0001	0.0001	0.0001	0.0002	0.0001	0.0000	0.0001	0.0001	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0003	0.0000	0.0002	0.0001	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001
INDIAN LIFE	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
BIEL	0.0000	0.0002	0.0003	0.0002	0.0003	0.0002	0.0000	0.0003	0.0000	0.0003	0.0002	0.0001	0.0002	0.0001	0.0002	0.0000	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000	0.0001	0.0002	0.0002	0.0002
VEDL	0.0001	0.0002	0.0002	0.0003	0.0002	0.0002	0.0001	0.0002	0.0000	0.0002	0.0001	0.0001	0.0001	0.0004	0.0001	0.0002	0.0006	0.0001	0.0001	0.0001	0.0000	0.0001	0.0001	0.0001	0.0001	0.0002
HEXAGON TODD	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000	0.0001	0.0000	0.0001	0.0001	0.0001	0.0000	0.0001	0.0001	0.0000	0.0001	0.0001	0.0003	0.0000	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001
WIPRO	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
DRREDDY	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MMI	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000	0.0001	0.0000	0.0001	0.0001	0.0001	0.0000	0.0001	0.0001	0.0000	0.0001	0.0001	0.0001	0.0000	0.0000	0.0000	0.0003	0.0001	0.0001	0.0001
GAIL	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000	0.0001	0.0000	0.0001	0.0001	0.0001	0.0001	0.0000	0.0001	0.0000	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000	0.0001	0.0002	0.0001	0.0001
TATAPOWER	0.0000	0.0001	0.0002	0.0002	0.0001	0.0000	0.0002	0.0000	0.0000	0.0002	0.0001	0.0001	0.0001	0.0002	0.0001	0.0000	0.0002	0.0002	0.0001	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0002
Rate of Return	22.22%	19.72%	-7.17%	6.74%	9.29%	9.25%	0.12%	13.32%	23.47%	11.21%	14.29%	28.75%	2.66%	24.12%	14.21%	22.87%	22.42%	9.22%	12.15%	18.11%	13.27%	22.92%	22.47%	8.07%	2.69%	
Variance	0.06%	0.25%	0.12%	0.25%	0.12%	0.25%	0.07%	0.25%	0.22%	0.14%	0.24%	0.22%	0.24%	0.22%	0.26%	0.22%	0.27%	0.26%	0.23%	0.23%	0.22%	0.22%	0.23%	0.23%	0.23%	
Standard Deviation	2.37%	2.14%	3.51%	2.21%	3.51%	1.59%	2.62%	2.31%	1.42%	3.62%	1.84%	1.49%	2.05%	1.47%	2.46%	1.62%	1.62%	1.67%	1.67%	1.66%	1.37%	1.68%	1.71%	1.71%	1.51%	

Investment was made through the way of plugging past data in Fig. 1 into Markowitz portfolio selection model and setting minimum expected rate of return (K). To consider various investors' inclination, minimum expected rate of return, K, was set to 10, 15, 20, 25, and 30%. TABLE III shows optimal solution whose investment weight is sum to one while the initial setting condition for K is satisfied as well as each variance is minimized. A weight of individual stock changes when K is set in five ways as presented in TABLE III. The reason why weights for each stock change is simply in that the differences among each individual stock's data listed in TABLE II [9].

TABLE III. INVESTMENT WEIGHT, VARIANCE, AND ACTUAL RETURN DURING FIRST PERIOD (JANUARY 2, 2006 – MARCH 27, 2006)

Weight	10%	15%	20%	25%	30%
SUNPHARMA	6.33%	6.62%	6.33%	6.09%	5.82%
TATAMOTORS	0.00%	0.00%	0.00%	0.00%	0.00%
SBIN	0.00%	0.98%	0.00%	0.00%	0.00%
TATASTEEL	0.00%	0.00%	0.00%	0.00%	0.00%
ICICIBANK	0.00%	0.00%	0.69%	3.34%	4.58%
RELIANCE	0.00%	0.00%	2.02%	4.32%	4.68%
INFY	4.94%	5.56%	5.13%	4.56%	3.99%
LT	3.39%	4.42%	5.10%	5.51%	5.72%
ITC	5.05%	4.44%	3.65%	2.93%	2.44%
AXISBANK	0.00%	0.00%	0.00%	0.00%	0.61%
ONGC	0.00%	3.92%	5.63%	6.31%	7.47%
HDFCBANK	0.00%	0.12%	2.82%	4.72%	5.51%
BHARTIARTL	0.00%	0.00%	0.00%	0.16%	2.85%
CIPLA	3.74%	2.81%	1.75%	0.59%	0.00%
HINDALCO	8.45%	7.22%	6.21%	5.09%	4.24%
HDFC	2.76%	8.95%	11.30%	13.08%	13.78%
HINDUNILVR	33.97%	29.60%	25.30%	20.45%	17.02%
BHEL	0.00%	0.00%	0.00%	0.00%	0.00%
VEDL	0.00%	0.00%	0.00%	0.00%	0.00%
HEROMOTOCO	0.00%	0.00%	2.18%	4.07%	5.09%
WIPRO	4.75%	2.36%	0.69%	0.00%	0.00%
DRREDDY	26.62%	23.93%	21.16%	18.22%	16.04%
M&M	0.00%	0.00%	0.00%	0.00%	0.18%
GAIL	0.00%	0.00%	0.00%	0.00%	0.00%
TATAPOWER	0.00%	0.00%	0.00%	0.00%	0.00%
SUM	100.00%	100.00%	100.00%	100.00%	100.00%
Variance	0.02%	0.02%	0.02%	0.01%	0.01%
Rate of Return	-25.11%	-24.05%	-22.70%	-21.22%	-20.44%

Through the process of investment, purchasing and selling the stock arises transaction fee. This paper selected the trade commission as 0.3% that works as a global market minimum investment commission. Selling stock is directed to encashment of stock. South Korea takes extra commission of 0.3% for tax. Therefore, this research calculated 0.6% commission for selling the stocks. The initial portfolio's commission was calculated as 0.3% because the total sum of capital was used for only buying the stocks [9].

Economic situation in change could be considered when the period of rebalancing cycle is short. However, this way has a drawback in that it generates a transaction surcharge for each period. According to Kim and Kim [9], this research selected rebalancing period as eight weeks. For the portfolio established in January 2, 2006 and rebalanced after eight weeks, March 27, 2006, the net return can be calculated after putting the rate of return data for each category during the past eight weeks in the model and subtracting tax and fee. Portfolio returns for each expected rate of return of the first period are shown TABLE III.

In this way, the oldest eight week of reference data is replaced with the latest eight week data. Therefore, three years of covariance, average and variance for rate of return is used to compose a new portfolio (see TABLE IV.). If a category weight increases, a transaction fee is applied. Both transaction fee and tax are calculated if a weight decreases due to monetary liquidation. New portfolio composed in the second section (March 3, 2006 - May 23, 2006) is in TABLE V.

TABLE IV. THREE YEARS (FEBRUARY 26, 2003-2FEBRUARY 27, 2006) OF RETURN, COVARIANCE, AND STANDARD DEVIATION

Covariance	SUNPHA RMA	TATAMO TORS	SBIN	TATASTE EL	ICICI BANK	RELIANCE	INFY	LT	ITC	AXIS BANK	ONGC	HDFC BANK	BHARTIA RTL	CIPLA	HINDAL CO	HDFC	HINDUNIL VR	BHEL	VEDL	HEROMO TOCO	WIPRO	DRREDDY	M&M	GAIL	TATA POWER		
SUNPHARMA	0.0007	0.0002	0.0001	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0001	0.0001	0.0002	0.0002	0.0001	0.0002	0.0001	0.0002	0.0002	0.0002		
TATAMOTORS	0.0002	0.0006	0.0003	0.0004	0.0002	0.0003	0.0002	0.0002	0.0002	0.0002	0.0003	0.0002	0.0003	0.0003	0.0002	0.0002	0.0002	0.0003	0.0004	0.0003	0.0004	0.0003	0.0002	0.0003	0.0004	0.0003	
SBIN	0.0001	0.0003	0.0005	0.0004	0.0003	0.0003	0.0002	0.0001	0.0002	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002	0.0003	0.0004	0.0005	0.0003	0.0003	0.0001	0.0003	0.0003	0.0004	0.0003	
TATASTEEL	0.0002	0.0004	0.0005	0.0004	0.0002	0.0003	0.0002	0.0002	0.0003	0.0003	0.0003	0.0002	0.0003	0.0003	0.0003	0.0002	0.0003	0.0004	0.0005	0.0003	0.0003	0.0002	0.0003	0.0004	0.0004	0.0003	
ICICIBANK	0.0001	0.0002	0.0003	0.0002	0.0003	0.0002	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0002	0.0003	0.0002	0.0002	
RELIANCE	0.0001	0.0003	0.0003	0.0004	0.0002	0.0003	0.0002	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0003	0.0003	0.0003	0.0003	0.0001	0.0002	0.0003	0.0003	0.0002	
INFY	0.0001	0.0002	0.0002	0.0002	0.0001	0.0002	0.0015	0.0000	0.0002	0.0001	0.0001	0.0002	0.0002	0.0002	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002	0.0003	0.0001	0.0002	0.0002	0.0002	
LT	0.0001	0.0002	0.0001	0.0002	0.0001	0.0001	0.0000	0.0012	0.0001	0.0000	0.0001	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0001	0.0000	0.0001	0.0001	0.0001	0.0001	
ITC	0.0001	0.0002	0.0002	0.0003	0.0002	0.0002	0.0002	0.0001	0.0015	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0002	0.0001	0.0002	0.0002	0.0002	
AXISBANK	0.0001	0.0002	0.0003	0.0003	0.0002	0.0002	0.0001	0.0000	0.0001	0.0010	0.0002	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0002	0.0001	0.0002	0.0002	0.0003	0.0003
ONGC	0.0001	0.0003	0.0003	0.0003	0.0002	0.0002	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0003	0.0003	0.0003	0.0002	0.0003	0.0001	0.0002	0.0003	0.0003	0.0002	
HDFCBANK	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0000	0.0002	0.0003	0.0002	0.0005	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0003	0.0002	0.0003	0.0003	0.0002	
BHARTIARTL	0.0001	0.0003	0.0002	0.0003	0.0002	0.0002	0.0002	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0007	0.0002	0.0002	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0003	0.0003	
CIPLA	0.0002	0.0003	0.0002	0.0003	0.0002	0.0002	0.0002	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0018	0.0002	0.0002	0.0002	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002	0.0003	0.0003	
HINDALCO	0.0002	0.0003	0.0002	0.0003	0.0002	0.0002	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0016	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0003	0.0002	
HDFC	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002	0.0001	0.0002	0.0016	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0002	0.0003	0.0002	
HINDUNILVR	0.0001	0.0002	0.0002	0.0003	0.0002	0.0002	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002	0.0001	0.0002	0.0001	0.0002	0.0004	0.0002	0.0002	0.0002	0.0003	0.0001	0.0002	0.0002	0.0002	0.0002	
BHEL	0.0002	0.0003	0.0002	0.0004	0.0002	0.0003	0.0002	0.0002	0.0002	0.0003	0.0002	0.0002	0.0003	0.0003	0.0002	0.0002	0.0006	0.0003	0.0002	0.0002	0.0003	0.0002	0.0003	0.0003	0.0003	0.0003	
VEDL	0.0002	0.0004	0.0003	0.0003	0.0002	0.0003	0.0002	0.0002	0.0002	0.0003	0.0002	0.0002	0.0003	0.0003	0.0002	0.0002	0.0002	0.0017	0.0002	0.0002	0.0001	0.0003	0.0003	0.0003	0.0003		
HEROMOTOCO	0.0001	0.0003	0.0002	0.0003	0.0002	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0002	0.0003	0.0002	0.0002	
WIPRO	0.0002	0.0003	0.0003	0.0003	0.0002	0.0003	0.0003	0.0001	0.0002	0.0003	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002	0.0003	0.0002	0.0002	0.0002	0.0011	0.0002	0.0002	0.0004	0.0003	0.0003	
DRREDDY	0.0001	0.0002	0.0001	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0009	0.0001	0.0002	0.0002	0.0002	
M&M	0.0002	0.0003	0.0003	0.0003	0.0002	0.0002	0.0002	0.0001	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0002	0.0003	0.0003	0.0003	0.0002	0.0001	0.0003	0.0003	0.0003	0.0003	
GAIL	0.0002	0.0004	0.0003	0.0004	0.0003	0.0003	0.0002	0.0001	0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0004	0.0002	0.0003	0.0004	0.0002	0.0003	0.0003	0.0004	0.0004	
TATAPOWER	0.0002	0.0003	0.0003	0.0004	0.0002	0.0003	0.0002	0.0001	0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0003	0.0003	0.0003	0.0004	0.0004	
Rate of return	31.30%	39.23%	33.48%	30.90%	44.24%	38.09%	3.16%	84.07%	11.02%	87.04%	28.59%	42.78%	88.88%	-2.42%	-3.47%	38.29%	14.02%	65.18%	67.21%	34.38%	-0.99%	-1.75%	44.57%	23.22%	42.39%		
Variance	0.07%	0.08%	0.02%	0.09%	0.02%	0.12%	0.12%	0.12%	0.10%	0.02%	0.02%	0.07%	0.18%	0.01%	0.18%	0.02%	0.04%	0.17%	0.02%	0.12%	0.09%	0.08%	0.09%	0.08%	0.08%		
Standard deviation	2.74%	2.49%	2.22%	2.97%	2.30%	2.23%	3.81%	3.41%	3.91%	3.13%	2.22%	2.11%	2.47%	4.28%	4.02%	2.22%	2.09%	2.33%	4.16%	2.30%	3.80%	2.92%	2.90%	2.92%	2.94%		

TABLE V. INVESTMENT WEIGHT, VARIANCE, AND ACTUAL RETURN FOR SECOND PERIOD (MARCH 28, 2006-MAY 23, 2006)

Weight	10%	15%	20%	25%	30%
SUNPHARMA	0.00%	0.46%	4.52%	6.12%	7.01%
TATAMOTORS	0.00%	0.00%	0.00%	0.00%	0.00%
SBIN	0.00%	0.00%	0.00%	0.00%	0.00%
TATASTEEL	0.00%	0.00%	0.00%	0.00%	0.00%
ICICIBANK	0.00%	0.00%	0.00%	0.00%	0.00%
RELIANCE	0.00%	0.00%	3.90%	4.54%	4.62%
INFY	5.05%	7.51%	6.94%	6.46%	6.05%
LT	0.00%	0.00%	0.00%	0.00%	0.00%
ITC	4.87%	6.03%	5.13%	4.29%	3.60%
AXISBANK	0.00%	0.00%	0.00%	0.00%	0.00%
ONGC	0.00%	0.00%	2.94%	5.31%	6.51%
HDFCBANK	0.00%	0.00%	0.00%	2.58%	4.63%
BHARTIARTL	0.00%	18.25%	0.00%	0.00%	0.00%
CIPLA	0.00%	3.78%	2.90%	2.11%	1.59%
HINDALCO	18.47%	11.76%	9.37%	7.82%	6.65%
HDFC	0.00%	0.00%	4.47%	7.35%	8.85%
HINDUNILVR	28.42%	35.90%	31.13%	27.28%	24.18%
BHEL	0.00%	0.00%	0.00%	0.00%	0.00%
VEDL	0.00%	0.00%	0.00%	0.00%	0.00%
HEROMOTOCO	0.00%	0.00%	0.00%	0.00%	0.00%
WIPRO	25.73%	7.97%	4.52%	2.46%	1.18%
DRREDDY	17.46%	26.58%	24.19%	21.43%	18.75%
M&M	0.00%	0.00%	0.00%	0.00%	0.00%
GAIL	0.00%	0.00%	0.00%	0.00%	0.00%
TATAPOWER	0.00%	0.00%	0.00%	0.00%	0.00%
Sum	100.00%	100.00%	100.00%	100.00%	100.00%
Variance	0.03%	0.02%	0.02%	0.02%	0.01%
Rate of Return	6.68%	4.79%	5.80%	6.62%	7.14%

Totally 54 experiments were conducted for 9 years (January 2, 2006 - December 31, 2014) at eight weeks of rebalancing cycle. Required rate of return for each K is shown in TABLE VI.

TABLE VI. ACTUAL RETURN OF PORTFOLIO AND  $\Delta$  INDIAN SENSEX IN EACH PERIOD

Minimum Expected Rate Of Return(K)	Rate of Return					
	Increase Period	Decrease Period	Recovery Period	Flat Period	All period (Average)	All period (Accumulate)
10%	-19.26%	49.00%	25.65%	55.46%	27.71%	110.85%
15%	-8.33%	50.46%	20.97%	53.65%	29.19%	116.75
20%	-6.73%	52.87%	13.82%	52.66%	28.15%	112.62%
25%	3.18%	48.95%	5.31%	48.74%	26.54%	106.17%
30%	11.56%	53.88%	-6.25%	42.08%	25.32%	101.27%
$\Delta$ Indian SENSEX	104.50%	-49.61%	59.18%	59.94%	43.50%	170.01%

#### IV. COMPARISON OF PERFORMANCE

Fig. 2 shows comparisons among portfolios' returns based on portfolio selection theory and SENSEX index. The composed portfolio showed lower return than SENSEX index during the increase period. However, it showed higher return at the decrease period and lower return in the recovery and flat period. Also, it showed low return at average and cumulative return in the entire section. 20% on average return and 70% for cumulative return deficiency were found.

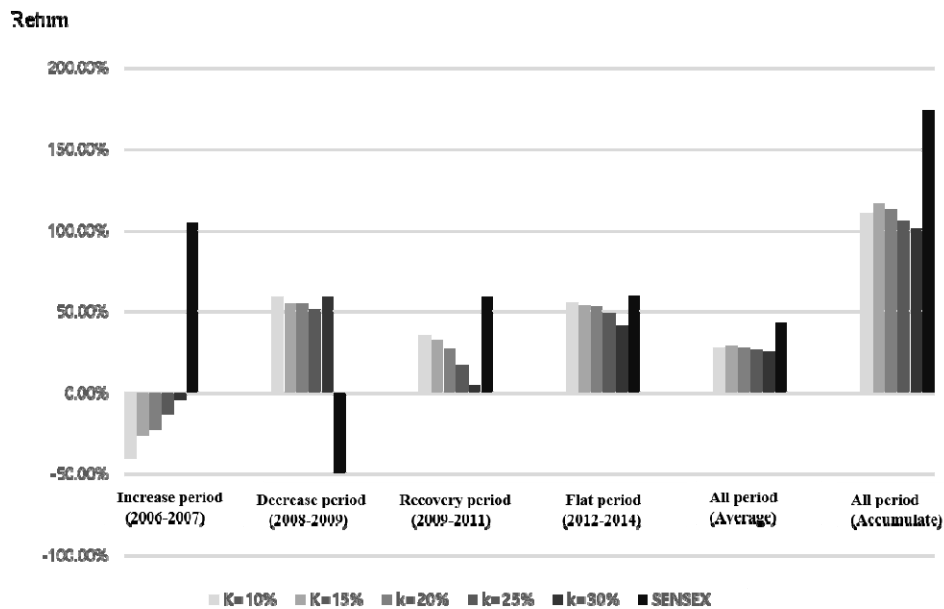


Fig. 2. Comparison of performance at 8 weeks

The short period of rebalancing cycle could reflect the market's variability by frequent up date. In other hand, the long term of rebalancing cycle could reduce the return with commission and tax. Consequently, this paper assumed that 8 weeks of rebalancing cycle is not an optimal cycle and made an additional experiment by setting up the extra rebalancing cycle which is 4 and 12 weeks. The actual rate of return during the rebalancing cycle for 4 and 12 weeks is like TABLE VII and TABLE VIII.

TABLE VII. ACTUAL RETURN OF PORTFOLIO AND  $\Delta$ INDIAN SENSEX IN EACH PERIOD AT 4 WEEKS

Minimum Expected Rate of Return(K)	Rate of Return					
	Increase Period	Decrease Period	Recovery Period	Flat Period	All period (Average)	All period (Accumulate)
10%	62.64%	78.57%	-15.27%	123.18%	62.28%	249.13%
15%	72.18%	106.76%	5.62%	112.44%	74.25%	297.02%
20%	74.66%	120.20%	27.32%	116.55%	86.18%	344.75%
25%	50.51%	129.33%	55.78%	107.85%	85.87%	343.48%
30%	65.24%	134.62%	74.13%	97.09%	92.77%	371.10%
$\Delta$ Indian SENSEX	104.50%	-49.61%	59.18%	59.94%	43.50%	170.01%

TABLE VIII. ACTUAL RETURN OF PORTFOLIO AND  $\Delta$ INDIAN SENSEX IN EACH PERIOD AT 12 WEEKS

Minimum Expected Rate of Return(K)	Rate of Return					
	Increase Period	Decrease Period	Recovery Period	Flat Period	All period (Average)	All period (Accumulate)
10%	-1.40%	29.50%	-6.84%	36.87%	14.53%	58.13%
15%	5.87%	40.82%	2.34%	39.56%	22.15%	88.61%
20%	11.94%	46.78%	13.27%	47.65%	29.91%	119.66%
25%	16.98%	48.14%	25.90%	43.02%	33.51%	134.05%
30%	22.13%	48.48%	26.31%	31.29%	32.05%	128.22%
$\Delta$ Indian SENSEX	104.50%	-49.61%	59.18%	59.94%	43.50%	170.01%

Data reference period is fixed in 3 years at every rebalancing cycle. SENSEX index showed the highest performance at increase period. Besides, 4 weeks of rebalancing and 20% K showed highest performance in the portfolio model. At decrease period every portfolio showed a higher performance than SENSEX index especially when rebalancing cycle is 4 weeks and K is 30%. Due to the whole experiment result, the portfolio composed by 4 weeks of rebalancing cycle showed the best return.



To measure the portfolio's performance from various perspectives, this paper used Sharpe ratio. Sharpe ratio is the measure for calculating risk-adjusted return. This ratio has become the industry standard for such calculations. The Sharpe ratio is often used to compare the change in a portfolio's overall risk-return characteristics when a new asset or asset class is added to it [6, 13]. In order to obtain Sharpe ratio, risk-free rate of return of benchmarked stock market from the return rate of portfolio was adjusted, and then it was divided by standard deviation. Risk-free rate of return was set up as 7% due to cheapest deliverable bond. First of all, the Sharpe ratio of benchmarked SENSEX index is 2.38. The Sharpe ratio of portfolio based on portfolio selection theory is presented in TABLE IX.

TABLE IX. SHARPE RATIO OF PORTFOLIO WHICH WAS BASED ON PORTFOLIO SELECTION THEORY

Rebalancing Cycle	Minimum Expected Rate of Return(K)				
	10%	15%	20%	25%	30%
4 Weeks	1.54	1.42	1.39	1.10	0.89
8 Weeks	1.03	1.09	0.99	0.84	0.67
12 Weeks	0.08	1.31	1.73	1.81	1.58

12 weeks of rebalancing cycle and 25% K showed the highest Sharpe ratio in the composed portfolio. On the other hand, 4 weeks of rebalancing cycle and 30% of K showed the lowest Sharpe ratio which was the portfolio that showed the highest performance at return rate. Furthermore every portfolio showed lower Sharpe ratio than SENSEX index, which means utility of risk-adjusted return was insufficient. In other words, composed portfolio performed high outcome in return rate but high risk made the utility of Sharpe ratio decrease.

## V. CONCLUSION

This paper comprised the portfolio on the basis of portfolio selection theory and then evaluated portfolio assuming that is applied to Indian stock market. The whole experiment period holds 9 years starting from the 2006 opening day to 2014 closing day. The research benchmarks Indian SENSEX index of Bombay Stock Exchange, fiducially June 6th except 5 lack of data. Based on the past data, it is able to compute variance, standard deviation and covariance. After the process portfolio is composed with 3 setups. Minimum expected rate of return K is divided into 10%, 15%, 20%, 25%, 30%. Rebalancing cycle was divided into 4 weeks, 8 weeks and 12 weeks. In addition, 3 years of data reference period was applied to the latest date from the first investment point. Due to the result, 4 weeks of rebalancing cycle performed higher return rate at every section and K more than SENSEX rate of change. In other hand, SENSEX index showed higher Sharpe ratio than each portfolio at every section. The highest Sharpe ratio was shown in portfolio was 12 weeks of rebalancing cycle and 25% of K, the lowest Sharpe ratio was shown in portfolio was 4 weeks of rebalancing cycle and 30% of K. Therefore, in terms of risk-adjusted return, the optimal portfolio is 12 weeks of rebalancing and 25% K. The optimal portfolio in terms of return rate is 4 weeks of rebalancing cycle and 30% K.

In conclusion, the investment portfolio has been constructed using the past data on an investment profit for the study. The study draw the optimal solution for portfolio selection, and the proposed methodology showed the productive investment outcome.

The performance of the portfolio's selection model had shown a disparate result, such as lower return at increase period and higher return at decrease period. This result was expected by the following explanation. The investment target was experimented by setting the 30th market capitalization of Indian SENSEX index on the basis of June 6, 2015. It is not by setting the 30th market capitalization of Indian SENSEX index at each rebalancing point. Also, indefinite reference period and rebalancing cycle could have influenced the return rate of portfolio.

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