APPENDIX

Discretization of (16):
It's known from calculus that the differential equation of the form (A.1) has a solution on the form (A.2):

$$\dot{y}\,(t) = -\lambda\, y(\mathrm{t}) + \alpha\, x(t) \tag{A.1}$$

$$y(t) = e^{-\lambda\,(t-t_0)} y(t_0) + \int_{t_0}^{t} e^{-\lambda\,(t-s)}\, \alpha\, x(s)\, ds \tag{A.2}$$

By applying a uniform discretization on (A.2) :

$$y(t_{i+1}) = e^{-\lambda\,(t_{i+1}-t_i)} y(t_i) + \int_{t_i}^{t_{i+1}} e^{-\lambda\,(t_{i+1}-s)}\, \alpha\, x(s)\, ds \tag{A.3}$$

By integration and putting $\Delta t = t_{i+1} - t_i$:

$$y(t_{i+1}) = e^{-\lambda\,(\Delta t)} y(t_i) + \frac{\alpha\, x(t_i)}{\lambda}(1 - e^{-\lambda\,(\Delta t)}) \tag{A.4}$$

By analogy, (16) becomes:

$$y(t_{i+1}) = e^{-\lambda\,(\Delta t)} y(t_i) + \frac{k\,(v(t_i) - v_{th})}{\lambda}(1 - e^{-\lambda\,(\Delta t)}) \tag{A.5}$$

Similarly, (3) becomes:

$$v(t_{i+1}) = e^{-\gamma\,(\Delta t)} v(t_i) + \frac{u(t_i)}{\gamma}(1 - e^{-\gamma\,(\Delta t)}) \tag{A.6}$$

Eauation (5) becomes:

$$\sum_{i=0}^{n-1} \Delta t\;\; v(t_i) \;\le\; v_{cum} \tag{A.7}$$

Constants:

$$a = e^{-\lambda\,(\Delta t)}$$
$$b = \frac{k}{\lambda}(1 - e^{-\lambda\,(\Delta t)}) \tag{A.8}$$
$$c = e^{-\gamma\,(\Delta t)}$$
$$d = (1 - e^{-\gamma\,(\Delta t)})$$