Production Optimization Problem under Carbone emission Constraint

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Abstract—This paper treaties the production problem of manufacturing and remanufacturing system under Carbone emission constraint. We have developed a production policy for a manufacturing and remanufacturing system composed by a two machines. The problem consists on a production system, unable to satisfy a random demand over a finite time horizon under given service level and take into account the product quantity returned by customer. Other hand, manufacturing and remanufacturing system engenders harmful emissions to the environment and may be sanctioned by an environmental tax. In order to assure an economical production planning a best combination of production rate and inventory level is made which minimize the total production, inventory and carbon tax costs for the manufacturing and remanufacturing units. The key of this study is to consider the influence of the product returned, Carbone quantity and service level on the optimal production planning. An analytical study and a numerical example are presented in order to prove the developed approach.

Keywords—production; corbone tax; service level; manufacturing; remanufacturing; optimization

I. INTRODUCTION

Recently, companies in service industries have begun to adopt new optimization methods to reduce waste in service production and delivery and to more efficiently meet customer requests. At the emotion of successful implementation efforts lies an operations-based, employee-involved, continual improvement-focused waste elimination culture. While environmental wilds (e.g., hazardous wastes, air emissions, wastewater discharges…) are seldom the explicit targets for lean implementation efforts, case study and empirical evidence shows that the environmental benefits resulting from lean initiatives are typically substantial. Although not explicitly targeted, environmental benefits are embedded in creating this smooth and rapid flow of products through the production process with minimal defects, inventory, downtime, and wasted movement. In the context, for example, reducing defects eliminates the environmental impacts associated with the materials and processing used to create the defective product, as well as the waste and emissions stemming from reworking or disposing of the defective products. Theoretically, the link between production and environmental improvement is strong. In fact, the fundamental objective of production systems is the systematic elimination of waste by focusing on production costs, product quality and delivery, and worker involvement. In the context, we can cite the works of [4], [5] and [6], they presented a mathematical formalism in the context of environment control approaches (taxes, emission) in order in order to minimize the total cost function and to determine the effect of environmental policy on production and inventory decisions. Concerning the interaction between production/maintenance and environmental aspect, [9] proposed a production and maintenance strategy take into account the deteriorating items and the consideration of an emission tax and pollution. [3] presented a mathematical model in order to determine an optimal maintenance period by minimizing the total maintenance cost and the environment emissions. [2] presented a joint production and maintenance problem with emissions control for a manufacturing system subject to degradation. In order to decrease the effect of the system degradation, they proposed a feedback strategy to control simultaneously production and emission rates as well as the maintenance rate.

In an industrial environment, a number of approaches have been proposed concern the production and maintenance optimization. In the context of production and inventory optimization problem, [1] proposed a quadratic model of production and inventory costs take into account both the rupture and backorders of inventory. [8] dealt with combined production and maintenance plans for a manufacturing system satisfying a random demand over a finite horizon. In their model, they assumed that the failure rate depends on the time and the production rate. [7] developed a maintenance strategy integrating a subcontracting constraint. They treated a production system represented by a machine producing a single product type to satisfy a constant demand during time.
Indeed, the originality of this paper is to study the impact of the quantity of Carbone and service level on the optimal production plan for the manufacturing and remanufacturing units.

This paper is organized as follows. The next section describes the production planning problem, the used notation and the adopted production policy. In section III, the mathematical model is presented expressing the total expected cost determining the production plan. The section IV dedicated to the numerical example to show the proposed approach efficiency. Finally, the conclusion is given in Section V.

II. PRODUCTION PLANNING PROBLEM

A. Notations

We used the following notations in this paper:

- $\Delta t$: length of a production period
- $H$: number of production periods $k = 0, 1, \ldots, H$.
- $H, \Delta t$: finite time horizon.
- $\hat{d}(k)$: average demand during period $k = 0, 1, \ldots, H$.
- $V_{d_k}$: variance of demand during period $k = 0, 1, \ldots, H$.
- $\alpha$: fraction of returned product that is sent back to second stock $W_2$.
- $\tau_r$: time to return the product to the second stock $W_2$.
- $U_m(k)$: manufacturing rate during period $k = 0, 1, \ldots, H$.
- $U_r(k)$: remanufacturing rate during period $k = 0, 1, \ldots, H$.
- $W_1(k)$: level of principal stock
- $W_2(k)$: level of second stock
- $C_{pm}$: unit manufacturing cost
- $C_{pr}$: unit remanufacturing cost
- $C_{sm}$: inventory cost of principal stock $W_1$
- $C_{sr}$: inventory cost of second stock $W_2$
- $Q_m(k)$: remaining amount of carbon at each period $k$ for the manufacturing unit
- $Q_r(k)$: remaining amount of carbon at each period $k$ for the remanufacturing unit
- $Q_{m}^0(0)$, $Q_{r}^0(0)$: quantity of carbon credits allocated by the authorities for the manufacturing unit.
- $UL_{max}$: maximal production rate for manufacturing unit
- $UL_{min}$: minimal production rate for manufacturing unit
- $UR_{max}$: maximal production rate for remanufacturing unit
- $UR_{min}$: minimal production rate for remanufacturing unit

B. Problem Description

The production system studied is composed of a manufacturing unit, which produces one type of product, in order to meet a random demand from a stock $W_1$ according to a prescribed service level. The law of random demand characterized by a Normal distribution with a normal mean $\hat{d}_k$ and standard deviation $\sigma_d$. Other hand, part of the products delivered to the customer noted by noted by the percentage $\alpha$ is eventually returned to the second stock $W_2$ after $\tau$ time periods. After check, the part of returned product noted by $\beta$ will be for destruction and $(1- \beta)$ will be remanufacturing by a second remanufacturing unit.

Points of view reliability, the remanufacturing unit is subject to a random failure and a fixed preventive maintenance actions every $h.Tr$ with $Tr=x.\Delta t$ and $h:1…H/Tr$ with fixed time duration $t=y.\Delta t$ characterized by a uniform law with parameters $[a,b]$. 
Other hand, the production units are subject to the regulations in force concerning the carbon tax. In this case, \( Q_{cm}^0(0) \) and \( Q_{cr}^0(0) \) are the amount of annual carbon, for manufacturing and remanufacturing units, are not exceeded for a production without penalty.

In this study, we proposed an optimal production planning problem formulation. Our objective is to establish an economical production plans for the manufacturing and remanufacturing units filling the randomly demand under the fixed service level and taking into account the carbon tax to pay.

![Fig. 1. Problem description](image)

III. PROBLEM FORMULATION

A. Production, inventory and Carbone problem model

To determine an economical production plans for manufacturing and remanufacturing units, we state a stochastic model that minimizes the total costs over a finite horizon. The objective of the mathematical model is to determine the greatest combination of production, inventory and Carbone quantity.

Formally, the production problem is defined as follows:
Under constraints:

\[ W_m(k+1) = W_m(k) + U_m(k) + (1-b_k)*U_r(k) + b_k*(1-y)*U_r(k) - d(k) \quad k = \{1, 2, ..., H-1\} \quad (2) \]

\[ W_r(k+1) = W_r(k) - U_r(k) + (1-\beta)\times R(k) \quad k = \{1, 2, ..., H-1\} \quad (3) \]

\[ R(k) = \begin{cases} \alpha \times d(k - \frac{\tau}{\Delta t}) & \text{if } k \geq \tau \\ 0 & \text{otherwise} \end{cases} \quad (4) \]

\[ \text{Prob} \left[ W_m(k+1) > 0 \right] > \theta \quad (5) \]

\[ U_{m_{\text{min}}} \leq U_{m}(k) \leq U_{m_{\text{max}}} \quad (6) \]

\[ U_{r_{\text{min}}} \leq U_{r}(k) \leq U_{r_{\text{max}}} \quad (7) \]

\[ Q_m(k) = Q^{cm}_{m}(0) - \sum_{j=1}^{k} U_{m}(j) \times \Delta t \times X \quad (8) \]

\[ Q_r(k) = Q^{cr}_{r}(0) - \sum_{j=1}^{k} \left[ (1-b_j) \times U_{r}(j) + b_j \times (1-y) \times U_{r}(j) \right] \times \Delta t \times X \quad (9) \]

\[ b_k = \begin{cases} 1 & \text{if } k = \left\lfloor h \times x \right\rfloor \\ 0 & \text{if no} \end{cases} \quad h = \{1, 2, ..., \left\lfloor \frac{H}{x} \right\rfloor \} \quad (10) \]

With \( F \) represents the total cost function of production, inventory and Carbone. The inventory balance level of the principal stock of manufacturing unit \( W_m \) at the period \((k+1)\) is presented by the equation (2), is equals to the inventory level of \( W_m \) at period \( k \) plus the products produced by the manufacturing unit during period \( k \), plus the products produced by the remanufacturing unit during period \( k \) if not subject to a preventive maintenance action or a part of production rate of remanufacturing units if subject to maintenance preventive, minus the customer demand at the period \( k \). Constraint (3) defines the inventory balance level of the remanufacturing unit stock \( W_r \) at period \((k+1)\) is equals to the inventory level of \( W_r \) at period \( k \) minus the products produced by the remanufacturing unit during period \( k \) and plus a part of product returned by customer. Constraint (4) presented the product quantity returned by the market that is a part of the demand returned by the customer after a specific deadline \( \tau \). The service level constraint that is characterized by a probabilistic constraint is defined by the relation (5). Constraints (6) and (7) defines the upper and lower bound on the production rates, during each period \( k \), for manufacturing and remanufacturing units. The remaining amount of carbon at each period \( k \) for the manufacturing and remanufacturing units are given by the equations (8) and (9). Constraint (10) defines a binary variable \( b_k \) is equal to 1 if a maintenance action is applied on period \( k \).
B. Analytical study

To solve this stochastic problem, it must to transform the formulation of the production, inventory and Carbone problem to a deterministic equivalent problem.

- Thus the total cost simplified as

Lemma 1:

\[
\text{Min } F = \text{Min } \left\{ \sum_{k=0}^{H} \left[ C_{s_{m}} \times (\hat{W}_{m} (H)^2) + \sum_{k=0}^{H} \left[ C_{s_{m}} \times \left( k \times \sigma_{w}^2 + \hat{W}_{m} (k)^2 \right) + C_{c} \times |Q_{m}(k)|_{[0,k)} \right] \right] + \sum_{k=0}^{H} \left[ C_{s_{r}} \times (\hat{W}_{r} (H)^2) + \sum_{k=0}^{H} \left[ C_{s_{r}} \times \left( k \times \sigma_{w}^2 + \hat{W}_{r} (k)^2 \right) + C_{c} \times |Q_{r}(k)|_{[0,k)} \right] \right] \right\}
\]

Where the mean variables:

\[ E \{ W_{n} \} = \hat{W}_{m} ; \quad E \{ W_{r} \} = \hat{W}_{r} ; \quad E \{ U_{m} \} = U_{m} ; \quad E \{ U_{r} \} = U_{r} ; \quad E \{ Q_{r} \} = Q_{r} ; \quad E \{ Q_{m} \} = Q_{m} \]

And variance variables: \( \nu_{m} = 0 \), \( \nu_{r} = 0 \). (control variables \( U_{m}, U_{r}, Q_{r} \) and \( Q_{m} \) are deterministic).

The inventory balance equations (6) and (7) can be reformulated as:

\[ \hat{W}_{m}(k+1) = \hat{W}_{m}(k) + U_{m}(k) + (1-b_{k}) \times U_{r}(k) + b_{k} \times (1-y) \times U_{r}(k) - \hat{d}(k) \]

\[ \hat{W}_{r}(k+1) = \hat{W}_{r}(k) - U_{r}(k) + (1-\beta) \times \hat{R}(k) \quad k = \{1,2,...,H-1\} \]

\[ \hat{R}(k) = \begin{cases} \alpha * \hat{d}(k - \frac{\tau}{\Delta t}) & \text{if } k \geq \tau \\ 0 & \text{otherwise} \end{cases} \]

- Concerning service level constraint, we transform from probabilistic form into equivalent deterministic inequality by specifying a minimum cumulative production quantity to produce by the manufacturing and remanufacturing units in order to satisfy the customer demand depending on the service level requirements.
Lemma 2

\[
\Pr\{W'_k(k+1) \geq 0\} \geq \theta \Rightarrow \left( U_m(k) + (1-h_1) * U_l(k) + h_2 * (1-h_2) * U_h(k) \geq V_{d,k} * \varphi^{-1}(\theta) - W'_k(k) + \hat{d}(k) \right) \quad k = 0, 1, ..., H-1
\]  

(12)

Where \( \varphi \) is a cumulative Gaussian distribution function with mean \( \hat{d}(k) \) and finite variance \( V_{d,k} \), and \( \varphi^{-1} \) denotes the inverse distribution function.

IV. NUMERICAL EXAMPLE

A numerical example, consider a manufacturing and remanufacturing system that produces one type of products in order to satisfy the random demands below. Using the analytical model, we will determine the optimal production plans for manufacturing and remanufacturing units minimizing the total cost over a finite planning horizon: \( H=12 \) periods with a length \( \Delta t=1 \). We supposed that the standard deviation of demand is the same for all periods \( \sigma_d=0.5 \) and the initial inventory level \( S_0=5 \).

The other data required are:

- Lower and upper boundaries of manufacturing and remanufacturing capacities: \( U_{m\text{min}}=0 \) and \( U_{m\text{max}}=16 \);
- \( U_{r\text{min}}=0 \) and \( U_{r\text{max}}=16 \);
- \( C_{p_m}=3; C_{p_r}=10; C_{s_m}=6; C_{s_r}=6 \);
- \( Q^0_{cm}(0)=1000 \) um; \( Q^0_{cr}(0)=500 \) um;
- \( \alpha =0.3 \);
- \( \tau_r : 1 \);
- Service level \( \theta=90\% \)

- For the manufacturing system, the fixed preventive maintenance actions every \( h \cdot T_r \) with \( T_r=x \cdot \Delta t \) with \( x=2 \) and \( h:1...H/T_r \) with fixed time duration \( t=y \cdot \Delta t \) with \( y \) characterized by a uniform law with parameters \( \left[ 0,\left( h \cdot x \right) + 1 - h \cdot x \right] \cdot \Delta t \) \( h = \{1,2,...,\left[ H/T_r \right]\} \)

<table>
<thead>
<tr>
<th>( \hat{d}_0 )</th>
<th>( \hat{d}_1 )</th>
<th>( \hat{d}_2 )</th>
<th>( \hat{d}_3 )</th>
<th>( \hat{d}_4 )</th>
<th>( \hat{d}_5 )</th>
<th>( \hat{d}_6 )</th>
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<th>( \hat{d}_8 )</th>
<th>( \hat{d}_9 )</th>
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TABLE I. AVERAGE DEMANDS

Using the numerical procedure method, we realize this optimization, the different results are presented respectively in table II, III, IV, V, VI.

<table>
<thead>
<tr>
<th>( b_0 )</th>
<th>( b_1 )</th>
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<th>( b_4 )</th>
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From the tables, we are involved to find the economical production plan for the manufacturing and remanufacturing units and the cumulative Carbone emission quantity for each periods and for each unit. Table III and IV presented the economical production planning for the manufacturing and remanufacturing unit under a service level equals to 0.9 and with a minimal total cost equals to 18498.1 mu. According to the tables Table V and VI, we can see the cumulative quantity of Carbone emission. We can note that for the manufacturing unit, the payment of Carbone taxes will be start at production period 4 However for the remanufacturing unit the payment of Carbone taxes will be start at production period 10. This can be explained by the fact that the manufacturing unit produced more product in order to respect the service level and in this case will pay more taxes of Carbone comparing by the remanufacturing unit which characterized by a higher cost so it will produce smaller amount and therefore will pay Carbone taxes much less.

V. CONCULSION

In this paper, a new control production policy considering a voluntary emission limit, random demand and service level, which integrates environmental concerns in the production rate control of manufacturing and remanufacturing units, is proposed. An analytical study is presented to minimize the total cost of manufacturing, remanufacturing, inventory and Carbone taxes in order to assure an economical production planning by determining a best combination of production rate and inventory level and showing the influence of product returned, Carbone quantity and service level on the economical production planning for manufacturing and remanufacturing units.

For future research, we will consider a more complex system with considering a maintenance strategy of the manufacturing and remanufacturing units taking into the state of degradation degree for each unit and consequently on the preventive maintenance plan.
REFERENCES


BIOGRAPHY

Bouslikhane Salim is a PHD student in the laboratory of industrial engineering, production and maintenance at the University of Lorraine, Metz. His main areas of research on the optimization of maintenance policies coupled to production and the development of methods and support the design and control tools in the production systems of goods and services.

Hajej Zied is an Associate professor at the University of Lorraine, Metz platform since September 2012. It operates research in the laboratory LGIPM Metz. After obtaining his doctorate at the University of Paul Verlaine - Metz in 2010, he was employed at the University of Metz as contract research engineer until August 2012. His main areas of research on the optimization of maintenance policies coupled to production and the development of methods and support the design and control tools in the production systems of goods and services. He is the author of numerous articles in international community of industrial engineering. Her teaching areas include modeling and organization of manufacturing and logistics systems, the practice of simulation, automation, and quality system production.

Nidhal Rezg is a professor at the University of Lorraine; he is a Doctor of Industrial Automatic from the National Institute of Applied Sciences (INSA) in Lyon in 1996. Accreditation to supervise research at the University of Metz in 2003. he was Professor at the Faculty of Engineering of the University of Moncton, New Brunswick Canada from 1997 to 1999 and Associate professor at the University of Metz until 2004, and currently holds the position of Professor of University. He is director of LGIPM laboratory since October 2006 and scientific responsible of the INRIA CusTom team from 2007 to 2011. His research interest is the optimization of maintenance policies coupled to production, the optimal control SED. He is the author of sixty papers in international journals, directors of 12 theses and 4 Accreditation to supervise research. Keywords researches are modeling, simulation and optimization of stochastic processes, reliability and maintenance and Petri nets.