Various Techniques to Design Controllers for Single Link Flexible Arm

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Abstract— Robotic systems are playing a very essential role in the field of small as well as large industrial applications. In this paper, the design of first and second order sliding mode controllers are proposed to perform the desired tasks. To illustrate this technique a single link flexible robotic arm is used. One of the most powerful algorithm for designing controllers is the sliding mode controller (SMC) with the drawback of chattering which degrades its performance. So to avoid it and to track our desired trajectory twisted algorithms is used. Lyapunov stability criteria was applied for the verification of controllers' stability. Finally all the two algorithms were compared using simulations.

Keywords— Controller, Sliding Mode, Robotics, Chattering, Twisted Mode Controller, Flexible joint, single link.

I. INTRODUCTION

At the present time progress and development in the industrial sector is mostly due to the field of robotics, and main task is to control them according to our desires. Two significant objectives in the control system regarding the robotics is stability and robustness [1]. The systems may be stabilized utilizing linear and non-linear controllers, where linear one having the ease of implementation. One of the most challenging task is to control and stabilize second order systems including non-linearities.

Flexible system are known to be extremely integrated and vastly nonlinear system [1], and it's difficult to obtain a precise mathematical model for them. So by using conventional techniques, it's very difficult to control them. In past, the neural network controllers have attracted abundant of consideration due to its amended control of the manipulator and advantage of having not more knowledge about the system dynamics. Sliding mode control started in Soviet Union during mid-90's [2]. It is a certain sort of Variable Structure System (VSS) with the advantage that the dynamical performance [3] of the system can be directly personalized to the optimal values of switching function, due to which the desired performance can be measured. In a specific class of system uncertainty i.e. matched uncertainty, the closed-loop response turn into completely unaffected [2]. In order control unreliable, nonlinear, and blaring systems, some investigators had applied fuzzy theory on controllers called fuzzy logic controller (FLC) [4] and a self-tuning fuzzy sliding mode control was designed for second order systems later. At high speeds the nonlinear behaviour of the structure not only degrades end-point accuracy, but also complicates controller development.

Recently sliding mode controller (SMC) is using in many of the broadly spread areas i.e. in robotics, in power converters, aerospace applications and process control. Though, SMC has two central weaknesses: the first one is chattering problem, it can be directed to the oscillated output at high-frequencies and the second is its sensitive problem as at reachability condition when the input signal is near to desired point or zero, the controller become very sensitive to the noise. Same problems are also discussed in many other papers too [4]. A specific sliding surface is selected to solve the existence problem for the system. In [5] the tracking based sliding mode control and adaptive control law was applied on a non-linear system. In [8] the rotating single link manipulator's dynamical analysis were done, also the modelling and dynamical performance of a prismatic joint with translating flexible link and rotational motion is also discussed in literature [5, 6].

An advance robotic system is single link flexible robot arm. Various techniques have been used for the control of a single link flexible arm robot. An amalgamated method for the manipulator control was proposed in late 90's and develop an adaptive controller which estimated the gain of the manipulator through transforming the dynamic equations. Also the vibration control of a flexible link were discussed in before. In 2009 a generalized predictive control for a single link flexible joint robot [7, 8] to track the desired trajectory has been applied. A flexible link has various advances as compared to the conventional arm, including low power consumption, low manufacturing cost and can be transported easily [PCFF]. Nuclear maintenance [5] and space applications [9] are done by single link and its control is considered to be an inspiring research area.



Fig. 1. Single Link Flexible Arm

This paper includes two different techniques to design a stable controller for a complex non-linear system i.e. single link flexible robotic arm. Both first and second order controller were applied, sliding mode and twisted controllers algorithm to get a stabilized controller. The stability was verified by Lyapunov stability criteria. Then finally the controllers are also simulated and results are discussed.

II. DYNAMIC MODEL AND CONTROL APPROACH FOR A SINGLE LINK FLEXIBLE ARM

The dynamical behavior of a single link flexible arm [10] can be described by the differential equations as [11]:

$$I\ddot{q_1} + MgLsin(q_1) + k(q_1 - q_2) = 0$$
(1)
$$J\ddot{q_2} - k(q_1 - q_2) = u$$

where q_1 is the link angle, q_2 is the motor angle, *I* is the inertia of the link, *J* is the inertia of the motor, *k* is the stiffness of the spring, *L* is the length of link, *M* is the mass of the link, *g* is the acceleration due to gravity and *u* is the input torque, respectively.

Let $x_1 = q_1$, $x_2 = \dot{q}_1$, $x_3 = q_2$ and $x_4 = \dot{q}_2$. According to the canonical form [12] of a class of under actuated systems, we can transform (1) into the following state space representation

$$\begin{aligned}
\dot{x_1} &= x_2 \\
\dot{x_2} &= f_1 \\
\dot{x_3} &= x_4 \\
\dot{x_4} &= f_2 + bu
\end{aligned}$$
(2)

where $[x] = [x_1 \ x_2 \ x_3 \ x_4]^T$ is the state variable vector, u is the control input, f_1 , f_2 and b are nominal nonlinear functions, expressed as:

$$f_1 = -\frac{k}{I}x_1 + \frac{k}{I}x_3 - \frac{MgLsin(x_1)}{I}$$
$$f_2 = \frac{k}{J}x_1 - \frac{k}{J}x_3$$
$$b = \frac{1}{J}$$

Let in matrices form system can be represented as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -\frac{k}{I} x_1 + \frac{k}{I} x_3 - \frac{MgLsin(x_1)}{I} \\ & x_4 \\ & \frac{k}{J} x_1 - \frac{k}{J} x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J} \end{bmatrix} u, \quad (3)$$

and can be also expressed as:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \end{bmatrix} = \begin{bmatrix} -\frac{k}{I} - \frac{MgLsin(x_1)}{2Ix_1} & 0 & \frac{k}{I} - \frac{MgLsin(x_1)}{2Ix_3} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{I} & 0 & -\frac{k}{I} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{J} \end{bmatrix} u,$$
(4)

Voytsekhovsky et al.[12] proposed a method that can approximate the original system with an input-output linearizable control system in new coordinates. This stabilization method of nonlinear system using sliding mode control is based on coordinate transformation by mapping

 $T: x \rightarrow z$ defined by [13]:

$$z_i = L_{f^{i-1}}g(x); \qquad i = 1, 2, 3, 4$$

$$L_{f^0}g(x) \quad L_{f^1}g(x) \quad L_{f^2}g(x) \quad L_{f^3}g(x)]$$

with $[z] = [z_1 \ z_2 \ z_3 \ z_4]^T \cdot T$ is defined as a local diffeomorphism with T(0) = 0. where, $L_f g(x)$ is the Lie derivative of g(x) along the vector f(x).

Consider the output system function defined by

$$z = x_1$$

With, $z = x_1$. Define the transformation $T: x \to z$ by [12]:

$$T(x) = \begin{bmatrix} L_{f^0}g(x) \\ L_{f^1}g(x) \\ L_{f^2}g(x) \\ L_{f^3}g(x) \end{bmatrix} = \begin{bmatrix} z_1 = T_1(x) \\ z_2 = T_2(x) \\ z_3 = T_3(x) \\ z_4 = T_4(x) \end{bmatrix},$$
(5)

Then,

$$T(x) = \begin{bmatrix} x_1 \\ x_2 \\ -\frac{k}{I}x_1 + \frac{k}{I}x_3 - \frac{MgLsin(x_1)}{I} \\ -\frac{k}{I}x_2 + \frac{k}{I}x_4 - \frac{MgLx_2cos(x_1)}{I} \end{bmatrix},$$
(6)

Differentiating z we obtain

$$\dot{z}_{1} = z_{2} = x_{2},$$

$$\dot{z}_{2} = z_{3} = -\frac{k}{I}x_{1} + \frac{k}{I}x_{3} - \frac{MgLsin(x_{1})}{I}$$

$$\dot{z}_{3} = z_{4} = -\frac{k}{I}x_{2} + \frac{k}{I}x_{4} - \frac{MgLx_{2}cos(x_{1})}{I}$$

$$\dot{z}_{4} = f_{e} + g_{e}u$$
(7)

where,

$$f_e = \frac{k^2}{I^2} x_1 - \frac{k^2}{I^2} x_3 + \frac{MgLksin(x_1)}{I^2} + \frac{MgLkcos(x_1)}{I^2} x_1 - \frac{MgLcos(x_1)}{I} x_3 + \frac{k^2}{IJ} x_1 - \frac{k^2}{IJ} x_3$$
$$g_e = \frac{k}{IJ}$$

We obtain a feedback linearizable nonlinear system in the state z with

2.2 First-order sliding mode controller

Define the surface, $s = \{z \in \Re^4 | s(z) = 0, \text{ for } \lambda > 0, \}$

$$s(z) = \left(\frac{d}{dx} + \lambda\right)^3 (z - z_d),\tag{9}$$

We choose the desired position $z_d = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^T$. The time derivative of *s* along the system trajectory *z* is equal to $\dot{s}(z) = z^{ii} + 3\lambda \ddot{z} + 3\lambda^2 \ddot{z} + \lambda^3 \dot{z}$

The sliding mode control is expressed by:

 $u = u_{eq} + u_{sw}$ where u_{sw} is the switching control, u_{eq} is the equivalent control yielded from $\dot{s}(z) = 0$, and

$$u_{eq} = -\frac{f_e(z) + 3\lambda z + 3\lambda^2 \ddot{z} + \lambda^3 z}{g_e(z)}$$
$$u_{sw} = \eta sign(s)$$

where η is positive constants.

It is notable that for small deviations, we have $g_e(z) < \frac{k}{IJ} < 0$. Choosing the Lyapunov candidate as

$$V = \frac{1}{2}s^2$$

and differentiating V along the trajectories of (14) yields

$$\dot{V} = s\dot{s} = -\eta|s| - ks^2 \le 0$$

Then, the system is stable and the convergence of the sliding mode is guaranteed.

3 Second-order sliding mode controller

The disadvantage of the first-order SMC is the occurrence of chattering. As a solution to resolve this problem, a higher-order sliding mode (HOS) is proposed in literature. In fact, HOS appears as an effective application to respond the switching control signals and chattering, with higher relative degrees in a finite time[1]. The HOS has been introduced in [12], with the goal to get a finite time on the sliding set of order r defined by $s = \dot{s} = \ddot{s} = \dots = s^{(r-1)} = 0$. s defines the sliding variable with the *rth* order sliding and with its (r-1)th first time derivatives depending only on the state x. The first-order sliding mode tries to keep s = 0. In the case of second-order sliding mode control, which only needs its measurement or evaluation of s, the relation should be verified as

$$s(x) = \dot{s}(x) = 0$$

In the following, a twisting algorithm with a prescribed convergence law is used.

3.1 Twisting controller

3.1.1 Controller approach

Consider the sliding surface

$$s_1 = \left(\frac{d}{dx} + \lambda_1\right)^2 z,\tag{11}$$

Differentiating above equation twice gives

$$\begin{split} \ddot{s_1} &= f_e(z) + g_e(z)u + 2\lambda_1 \ddot{z} + \lambda_1^2 \ddot{z} \\ \ddot{s_1} &= \psi(z) + \varphi(z)u_. \end{split}$$

where $\psi(z) = f_e(z) + 2\lambda_1 \ddot{z} + \lambda_1^2 \ddot{z}$ and $\varphi(z) = g_e(z)$. We assume that functions ψ and φ are bounded such that

$$|\psi| = \psi_d$$
 $0 < \varphi_m \le \varphi \le \varphi_M$
nd φ_d are positive scalars. Then, we have

where $\psi_d, \varphi_m, \varphi_M$ a

$$\left|\frac{\varphi}{\varphi}\right| < \frac{\varphi_d}{\varphi_M}$$

By imposing $\ddot{s}_1 = 0$, the equivalent control can be expressed as $u_{eq} = -\left|\frac{\psi}{\omega}\right|$.

3.1.2 Stability analysis

The dynamic control law using the twisting algorithm is given by [8]

$$u_{sw} = \frac{k}{g_e(z)}(s_1 + \beta sign(\dot{s}))$$

with $\beta > 0, 0 < k \le k_m$ and $k_m > \frac{1}{1-\beta} \cdot \frac{\psi_d}{\varphi_M}$ The total sliding mode control is expressed by

$$u = u_{eq} + u_{st}$$

where u_{sw} is the switching control, u_{eq} is the equivalent control yielded from $\ddot{s}(z) = 0$, and

$$u_{eq} = -\frac{\psi(z)}{\varphi(z)} = -\frac{f_e(z) + 2\lambda_1 \ddot{z} + \lambda_1^2 \dot{z}}{g_e(z)}$$

Choosing the Lyapunov candidate as:

$$V = \frac{1}{2}\lambda_2 s_1^2 + \frac{1}{2}\dot{s_1^2},$$
 (12)

and differentiating V along the trajectories of (14) yields:

$$\dot{V} = \frac{1}{2}\lambda_2 s_1 s_1^2 + \frac{1}{2}s_1 \dot{s_1}^2 = -|\dot{s_1}|[(\lambda_2 - K)|s_1| - K\beta] \le 0$$

Then, the system is stable and the convergence of the sliding mode is guaranteed if $\lambda_2 - K < 0$.

III. SIMULATION RESULTS AND DISCUSSION

Parameters of the single link flexible robotic arm are set as follows:

The initial conditions as: $(q_1, q_2, \dot{q}_1, \dot{q}_2) = (0.1, 0, 0, 0)$. q_1 is the link angle, q_2 the motor angle, $I = 1 kg m^2$ is the inertia of the link, $J = 1.42kg m^2$ is the inertia of the motor, k = 0.55 Nm is the stiffness of the spring, L = 0.93 m is the length of link, M = 1.65 kg is the mass of the link, g = 1.05 N/kg is the acceleration due to gravity and $u = 1 kg m^2 sec^{-2}$ is the input torque, where the desired value is $z_d = 1$ respectively. In simulations both controller have to track the system without oscillation. In figure 2, the sliding mode controller is smooth as compare to the twisted mode. On the other side the fluctuation is less in case of SMC. So the user can select controller regarding to their needs. Also the initial value was 0.1, therefore both controllers start from initial condition as mentioned.







Fig. 3. Shows the TMC for position of link angle (solid line) and motor angle (dashed line)

CONCLUSION

This paper includes two different techniques to design a stable controller for a non-linear system i.e. single link flexible robotic arm. The problem faced in the sliding mode was chattering, so the results are compared with twisted controllers but it takes more time to stabilize. So both controllers have advantage and disadvantage. So it will be correct to say that user can select controller regarding to there need. The results are obtained through simulation.

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BIOGRAPHY

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