It is at this point that since the successors of (1011) are (0110) and (0111), we choose to add (0111) since (0110) is in the removed set F. We now have

$$C_z = (1101)(1010)(0100)(1000)(0000)(0001)(0010)(0101)(1011)(0111).$$

Continuing this process, we add (1110) and then see that with the addition of (1110) that of the two successors, (1101) is marked and (1100) is in F. Thus we reach the end of the "while" loop with

$$C_z = (1101)(1010)(0100)(1000)(0000)(0001)(0010)(0101)(1011)(0111)(1110):$$

We now enter into the "for" loop and try to find the first node which has an unmarked successor not in F. The first few nodes do not have this property, but we find that the node (0111) has successor (1111) which is unmarked and not in F.

We now use Algorithm 1 on the node (1111) to construct C_y. We will set y = (1111) and then construct C_y while maintaining the same list of marked nodes as before. Since the successors of (1111) are (1111) which is now marked and (1110) which is also marked we see that C_y = (1111) after the "while loop" completes. Now, after completing the construction of C_y, we then go back to our C_z and join it with C_y by inserting C_y after (0111) in C_z. This gives us the cycle

$$C_z = (1101)(1010)(0100)(1000)(0000)(0001)(0010)(0101)(1011)(0111)(1111)(1110).$$

This C_z is a cycle on the 12 remaining nodes in $B(2, 4)$ and is therefore a Hamiltonian cycle on the component containing z = (1101).

Remark 4. It may appear to be coincidence that this algorithm returns a cycle instead of a path, but if we recall that each predecessor of a node has all same successors, then since each of the predecessors of our start node have been visited, then so, too, must all the successors of those predecessors. This means that since the first node in C_z is the only one which has not had an in-going edge and the last node is the only one without an out-going edge, then this last node must be a predecessor of the original node.

V. Concluding Statements

In Section 3 we examined N. G. de Bruijn's original proof in [2] that there are $2^{2^{n-1}-n}$ Hamiltonian cycles in the De Bruijn graph $B(2, n)$.

We then saw in Section 4 that in the case of a single node failure on $B(2, n)$ that we could use the technique of removing a necklace to show that these graphs admit cycles of length greater than or equal to $2^n - n - 1$.

ACKNOWLEDGMENT

Thank you for the program of the academic leadership grant (ALG), Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran (Indonesia), which has been providing facilities to conduct a research and publication.

REFERENCES

[1] Oliver Collins, Sam Dolinar, Robert McEliece, and Fabrizio Pollara. A vlsi decomposition of the De Bruijn graph. J. ACM, 39:931{948, October 1992.

[2] N. G. de Bruijn. A combinatorial problem. Nederl. Akad. Wetensch., Proc., 49:758{764 = Indagationes Math. 8, 461{467 (1946), 1946.

[3] J.-C. Delvenne and R. Carli. Optimal strategies in the average consensus problem. In Decision and Control, 2007 46th IEEE Conference on, pages 2498 {2503, dec. 2007.

[4] Abdol-Hossein Esfahanian and S. Louis Hakimi. Fault-tolerant routing in De Bruijn communication networks. IEEE Trans. Comput., 34(9):777{788, 1985.

[5] W. C. Hu_man and V. Pless. Fundamentals of error-correcting codes. Cambridge Univ. Press, 2003.

[6] Zolt_an K_asa. On arc-disjoint hamiltonian cycles in De Bruijn graphs. CoRR, abs/1003.1520, 2010.

[7] R. Rowley and B. Bose. Edge-disjoint hamiltonian cycles in De Bruijn networks. In Distributed Memory Computing Conference, 1991. Proceedings., The Sixth, pages 707 {709, apr-1 may 1991.

[8] Robert Rowley and Bella Bose. On the number of arc-disjoint Hamiltonian circuits in the De Bruijn graph. Parallel Process. Lett., 3(4):375{380, 1993.

[9] Robert A. Rowley and Bella Bose. Fault-tolerant ring embedding in De Bruijn networks. Computers, IEEE Transactions on, 42(12):1480 {1486, dec 1993.

[10] Vladimir Raphael Rosenfeld. Enumerating De Bruijn sequences. In MATCH Communications in Mathematical and in Computer Chemistry, page 2002.

BIOGRAPHY

Mochamad Suyudi, is a lecturer at the Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran. Bachelor in Mathematics at the Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, and Master in Mathematics at the Faculty of Mathematics and Natural Sciences, Universitas Gajah Mada. Currently pursuing PhD program in the field of Graphs at Universiti Malaysia Terengganu.