Superstructure Optimization of Multiple Gas-Solid Cyclone Arrangements

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Abstract — Cyclones are the most widely used industrial dust collecting devices for removing particulate matter from air streams. In this work, a new optimization model of multiple cyclone arrangement is presented using General Algebraic Modeling System (GAMS) software in obtaining the optimal number and dimensions of the cyclone, and the best cyclone arrangement for a certain condition with respect to the minimum total cost, including the operating cost and the capital cost. The proposed model of nonlinear programming (NLP) and mixed integer nonlinear programming (MINLP) has been successfully applied to different case studies. The NLP model is applied to an NPK (Nitrogen, Phosphorus, and Potassium) fertilizer plant to find the optimal number and dimensions of the 1D3D, 2D2D, and 1D2D cyclones arranged either in parallel or series. In another case study, the best cyclone arrangement of parallel-series for three different combinations of the 1D3D and 2D2D cyclone is obtained through the use of MINLP modeling. The results show that each type of cyclone (i.e., 1D3D, 2D2D, and 1D2D) has an alternative that can be arranged either in parallel or in series configuration with different optimal numbers of cyclones. Furthermore, in the parallel-series arrangement, the cyclone of 2D2D+2D2D is found to be more economical and efficient compared to the others.

Keywords — Cyclone, Optimization, Arrangement, GAMS software, Modeling

I. INTRODUCTION

The gas-solid cyclone is one of the most widely used of all types of industrial gas-cleaning devices for removing particulate matter from air streams. Designing optimum cyclone arrangements became more essential with the growing concern of the environmental effects of particulate pollution. In general, cyclones are made in a variety of configurations. The most common geometry of a gas-solid cyclone is determined by the following dimensions as shown in Fig. 1:

Where:
1) a = the inlet height (m)
2) b = the inlet width (m)
3) B = the dust outlet diameter (m)
4) D = the diameter of the cyclone (m)
5) De = the gas outlet or vortex finder diameter (m)
6) S = the gas outlet length (m)
7) h = the cylinder height of the cyclone (m)
8) H = the overall height of the cyclone (m)

Fig.1 Schematic diagram of a reverse-flow cyclone

The configurations of cyclones that are considered in this work are 1D3D [1], 2D2D [2], and 1D2D [3]. The letter D in the 2D2D designation refer to the diameter of the cyclone. A 2D2D cyclone means a cyclone with cylinder height and cone height of two times of cyclones diameter. All the mentioned configurations are listed in Table 1. It should be noted that if the values of both H₀ and h₀ are set for any configuration of the cyclone, the rest of the ratios will be known for that specific configuration.
The design of using a single cyclone connected to each particulate matter source device is common in many industrial applications. In spite of the fact that each cyclone has been designed with excellent performance to handle separation of particles, there are many situations wherein a single cyclone is inadequate for the particle separation task. In such situations, it is often feasible to use multiple units either in series or in parallel or parallel-series. Mathematical programming (i.e., linear or nonlinear programming and mixed integer programming) can be used to determine the optimum cyclone arrangement in order to minimize particulate emissions. Development of these mathematical models can be challenging when the operation cost and the capital cost of the cyclone arrangement are taken into account. The capital cost is proportional to diameter of the cyclone and the number of cyclone. Meanwhile, the operating cost is proportional to inlet flow rate to the cyclone and the cyclone pressure drop. Installing the cyclones in parallel, would lead to higher capital cost. On the other hand, the cyclones in series arrangement would bring to higher operating cost instead. The challenge of this work is therefore, the models must have a capability to optimize the number of cyclones and dimensions when determining the optimum cyclone arrangement (in series and/or in parallel) with the minimum total cost.

In this paper, the mathematical modelling will be developed to determine the optimum cyclone arrangement for two cases; nonlinear programming optimization of 1D3D, 2D2D, and 1D2D cyclones arrangement in parallel and in series in NPK (Nitrogen, Phosphorus, and Potassium) fertilizer plant and MINLP optimization of 1D3D and 2D2D cyclones arrangement in parallel-series in paper mill plant. The best cyclone configurations and the optimum dimension and number of cyclones are optimized with respect to the minimum total cost, including the operating cost and the capital cost. All mathematical models are implemented in the General Algebraic Modeling System (GAMS) software [4]. The optimization models which are developed can be implemented to control the emissions of particulate matter in plants that operate cyclones as their dedusting system.

II. NONLINEAR PROGRAMMING OPTIMIZATION OF 1D3D, 2D2D, AND 1D2D CYCLONE ARRANGEMENT IN PARALLEL AND IN SERIES

A. Problem Statement

Cyclone is used as a first stage to control the emissions of particulate matter that are released from the two types of dryers, the cooler, and the vibrating screen in Nitrogen, Phosphorus, and Potassium (NPK) granulation fertilizer plant. These particulate matters are actually the dust of raw materials that can be recycled to the granulator. In this work, it is desired to observe a feasibility to use multiple units of cyclone in order to reduce the emissions, especially when the NPK fertilizer plant must be operated at its maximum capacity. The input feed as the parameters for both cyclone arrangements (parallel and series) to be processed is described in Table 2. The flow rate (Q) of input feed to the cyclones in both arrangements is roughly the sum of all particulates’ flow rate that is generated from the two dryers, the cooler, and the screen of the proposed NPK plant.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>13.97 (m³/s)</td>
</tr>
<tr>
<td>ρ_p</td>
<td>1042 (kg/m³)</td>
</tr>
<tr>
<td>ρ</td>
<td>1.33 (kg/m³)</td>
</tr>
<tr>
<td>μ</td>
<td>19.34 x 10⁻⁶ (N.s/m²)</td>
</tr>
<tr>
<td>d_p</td>
<td>21.63 x 10⁻⁶ (m)</td>
</tr>
</tbody>
</table>

B. Mathematical Models of Parallel Cyclone Arrangement

For parallel cyclone arrangement (Fig. 2), the diameter of the cyclone (D_p, m), the inlet velocity (v_{ip}, m/s), and the pressure drop (ΔP_p, N/m²) are variables that will be optimized. The flow rate of each cyclone in parallel arrangement (Q_p, m³/s), the inlet velocity (v_{ip}, m/s), and the pressure drop (ΔP_p, N/m²) can be calculated from (1) – (4) [5].

TABLE II. SPECIFICATION OF INPUT FEED TO THE CYCLONE

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Cyclone 1D3D</th>
<th>Cyclone 2D2D</th>
<th>Cyclone 1D2D</th>
</tr>
</thead>
<tbody>
<tr>
<td>a_0 = a/D</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>b_0 = b/D</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>c_0 = c/D</td>
<td>0.125</td>
<td>0.125</td>
<td>0.625</td>
</tr>
<tr>
<td>D_a = D_a/D</td>
<td>0.5</td>
<td>0.5</td>
<td>0.625</td>
</tr>
<tr>
<td>H_a = H_a/D</td>
<td>4</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>h_a = h_a/D</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>B_a = B_a/D</td>
<td>0.25</td>
<td>0.25</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Where $Q$ is the total inlet volumetric flow rate of the dust leaving the NPK plant, $N$ is number of cyclones, and $N_H$ is the number of inlet velocity heads of the gas.

The equation of the diameter of cyclone in parallel ($D_p$, m) is obtained by substituting (1) and (2) into (5), yields (6).

$$d_p = \left[ \frac{9 \mu b}{\pi N_i (\rho_p - \rho) v_i} \right]^\frac{1}{2}$$

$$D_p = \left[ \frac{d_p^2 \pi N_i (\rho_p - \rho) Q}{9 \mu b_0^2 a_0 N} \right]^\frac{1}{3}$$

$$N_i = \frac{1}{a} \left[ h + \frac{(H-h)}{2} \right]$$

Where $d_p$ is the cut-size diameter of the cyclone as in [6]. $N_i$ is the number of spiral turns of particle inside the cyclone. $\mu$ is the viscosity of gas. $\rho_p$ is the density of particle. $\rho$ is the density of gas.

The objective function is total cost of parallel cyclone arrangement ($c_{tot,p}$, $$/second) minimization and calculated from the following expression as indicated in (8).

$$MIN c_{tot,p} = Q A P_p c_e + \frac{F e}{Y t_w} N D_p^j$$

Where $c_e$ is the cost of utilities ($= 1.5 \times 10^{-8}$/ J [7]). $F$ is the investment factor ($= 4.4$ [8]). $Y$ is the number of years over which depreciation occurs ($= 5$). $t_w$ is time worked per year ($= 2.16 \times 10^7$ second/year). $e$ is constant ($= 4924.61$/m [8]). $j$ is constant ($= 1.2$ [8]).

**C. Mathematical Models of Series Cyclone Arrangement**

For series cyclone arrangement (Fig. 3), the cyclones are connected in series and set to have up to 3 cyclones with similar diameters. Equation (9) is used to calculate the diameter of the first cyclone or primary cyclone ($D_{s1}$, m).

$$D_{s1} = \left[ \frac{d_{s1}^2 \pi N_i (\rho_p - \rho) Q}{9 \mu b_0^2 a_0 N} \right]^\frac{1}{3}$$
The overflow mass of the particles \(m_p\) from the first cyclone will be charged to the next (second and third) cyclone with certain amount that is affected by the efficiency \(\eta\) of the previous cyclone and calculated by using (10) and (11).

\[
\begin{align*}
    m_{p2} &= (1 - \eta_1) m_p \quad (10) \\
    m_{p3} &= (1 - \eta_2) m_{p2} = (1 - \eta_1)(1 - \eta_2) m_p \quad (11)
\end{align*}
\]

Based on the definition of mass particle \(m_p\) by Clift et al. [9], equations (10) and (11) can be altered into (12) and (13).

\[
\begin{align*}
    m_{p2} &= (1 - \eta_1) \left( \frac{\pi d_{p2}^3}{6} \right) (\rho_p - \rho) \quad (12) \\
    m_{p3} &= (1 - \eta_2) \left( 1 - \eta_1 \right) \left( \frac{\pi d_{p3}^3}{6} \right) (\rho_p - \rho) \quad (13)
\end{align*}
\]

The equation for calculating the diameter of the second \(D_{S2}, m\) and third \(D_{S3}, m\) cyclone can be obtained by setting the two expressions for terminal velocity \(v_t\) (i.e., terminal velocity of a particle settling in the earth’s gravitational field under steady-state conditions [10] and terminal velocity in the time-flight model proposed by Rosin et al. [6] equal to each other and rearranging (14) with (15), yields (16) and (17).

\[
\begin{align*}
    v_t &= \frac{2 m_p v_i^2}{3 \pi \mu d_p} \quad (14) \\
    v_t &= \frac{b v_i}{\pi D \mu} \quad (15)
\end{align*}
\]

\[
\begin{align*}
    D_{S2} &= \left[ \frac{2 m_{p2} \mu_i Q}{3 d_{p2} \mu b_2^3 a_0} \right]^{\frac{1}{3}} \quad (16) \\
    D_{S3} &= \left[ \frac{2 m_{p3} \mu_i Q}{3 d_{p3} \mu b_3^3 a_0} \right]^{\frac{1}{3}} \quad (17)
\end{align*}
\]

Substituting (12) into (16) and (13) into (17) yields (18) and (19), respectively.

\[
\begin{align*}
    D_{S2} &= \left[ \frac{a_{p2}^2 \pi \mu_i (1 - \eta_1)(\rho_p - \rho) Q}{9 \mu b_2^3 a_0} \right]^{\frac{1}{3}} \quad (18) \\
    D_{S3} &= \left[ \frac{a_{p3}^2 \pi \mu_i (1 - \eta_1)(1 - \eta_2)(\rho_p - \rho) Q}{9 \mu b_3^3 a_0} \right]^{\frac{1}{3}} \quad (19)
\end{align*}
\]

The flow rate for each cyclone in series arrangement would be the same, the inlet velocity \(v_{is}, \text{m/s}\) and the pressure drop \(\Delta P_i, \text{N/m}^2\) are calculated from (20) and (21).

\[
\begin{align*}
    v_{is} &= \frac{Q}{ab} = \frac{Q}{a_0 b_0 b_i^3} \quad (20) \\
    \Delta P_i &= \frac{1}{2} \rho \frac{v_{is}^2}{N_h} = \frac{1}{2} \rho \left( \frac{Q}{a_0 b_0 b_i^3} \right)^2 N_h \quad (21)
\end{align*}
\]
In the series cyclone arrangement, the overall efficiency ($\eta_{ov}$) of the cyclone is maximized while the total cost of two series cyclones or three series cyclones is minimized. The overall efficiency for two cyclones in series and three cyclones in series can be calculated as given in (22) and (23), respectively.

$$\eta_{ov} = 1 - \left[ (1 - \eta_1)(1 - \eta_2) \right]$$  \hspace{1cm} (22)

$$\eta_{ov} = 1 - \left[ (1 - \eta_1)(1 - \eta_2)(1 - \eta_3) \right]$$  \hspace{1cm} (23)

The total cost of series cyclone arrangement is minimized by following expression:

$$MIN \ c_{tot,s} = Q \ c_v \ \sum_{s=1}^{N_s} \Delta p_s + \frac{\rho e}{\gamma \ Y_w} N \ \sum_{s=1}^{N_s} D_s^l$$  \hspace{1cm} (24)

Where $N_s$ is number of stage in the arrangement.

### III. MIXED INTEGER NONLINEAR PROGRAMMING OPTIMIZATION OF 1D3D AND 2D2D CYCLONES ARRANGEMENT

#### A. Problem Statement

The problem of multiple cyclone arrangement can be expressed as follows: for a given input feed, it is desired to determine the best cyclone arrangement of $n$ parallel cyclone lines ($n = 1, 2, ..., N_P$) with $s$ ($s = 1, 2, ..., N_S$) series cyclones in each line from various $k$ levels ($k = 1, 2, ..., N_K$). On each level, a number of similar dimensions of cyclones are connected in parallel-series as can be seen in Fig. 4. Cyclones in the parallel lines are actually the duplication units and the number of cyclone in series is considered as a stage of separation. The number of stages to be considered in this optimization is two stages ($N_S = 2$) based on the feasible results from experiments conducted by Whitelock and Buser [11]. The experiment shows that the use of three or four identical cyclones in series increases the efficiency only slightly along with a significant increase in the pressure drop across all cyclones. Moreover, there are four levels (Fig. 4) that are considered in this optimization where the composition of each level is given in Table 3.

<table>
<thead>
<tr>
<th>TABLE III. COMPOSITION OF EACH LEVEL</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Level (k)</strong></td>
</tr>
<tr>
<td>-------------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

The input feed data provided by Ravi et al. [12] is used in this study and as shown in Table 4. A total flow rate ($Q$) of 165 m$^3$/s represents a stream to be processed in a paper mill [12, 13].
### TABLE IV. SPECIFICATION OF INPUT FEED TO THE CYCLONE SYSTEM

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q</td>
<td>165 (m³/s)</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>1600 (kg/m³)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.7895 (kg/m³)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>24.8 x 10⁻⁶ (N.s/m²)</td>
</tr>
<tr>
<td>MMD</td>
<td>10 x 10⁻⁶ (m)</td>
</tr>
<tr>
<td>GSD</td>
<td>2.5</td>
</tr>
</tbody>
</table>

#### B. Mathematical Models of Parallel – series Cyclone Arrangement

The total flow rate through the arrangement, $Q_t$, is the summation of the flow rate of each level $k$, $Q_k$, as expressed as follows:

$$Q_t = \sum_{k=1}^{N_k} Q_k$$  \hspace{1cm} (25)

$$Q_k = Q_t \times k$$  \hspace{1cm} (26)

Where $N_k$ is number of level of the cyclone arrangement and $x_k$ is the fraction of total flow through level $k$.

It is assumed that the mass fraction has the same value with the volume fraction. Hence, the mass particle that enter the first and second cyclone of each level can be calculated by using (27) and (28), respectively.

$$m_{p1k} = m_p \times k$$  \hspace{1cm} (27)

$$m_{p2k} = (1 - \eta_{1k}) m_{p1k} = (1 - \eta_{1k}) m_p \times k$$  \hspace{1cm} (28)

Where $\eta_{1k}$ is the efficiency of the first cyclone on level $k$.

There are two types of mass balance involved in calculating the mass of the emission of the cyclone system ($m_{\text{out}}$): First, by summing each mass of particle emitted from the last stage. Second, by assuming that the cyclone system as the one big cyclone and it has an overall cyclone efficiency. From those mass balance definitions, the overall efficiency of the cyclone system ($\eta_{\text{out}}$) is calculated by using following equations:

$$\eta_{\text{out}} = 1 - \left[ \sum_{k=1}^{N_k} (1 - \eta_{1k}) x_k \right]$$  \hspace{1cm} (29)

$$\eta_{\text{out}} = 1 - \left[ (1 - \eta_{1k}) (1 - \eta_{2k}) \right]$$  \hspace{1cm} (30)

Where $\eta_{1k}$ is the overall efficiency of the parallel – series cyclone arrangement on level $k$.

According to Wang et al. [14], the overall efficiency of each cyclone is a function of the particle size distribution (PSD). Therefore, if it is assumed that the inlet particle size distribution is a lognormal distribution with mass median diameter (MMD) and geometric standard deviation (GSD), the log-normal distribution function can be used to calculate the theoretical model of cyclone overall efficiency as shown below:

$$F(d_p) = \int_{d_p}^{\infty} \frac{1}{\sqrt{2\pi} d_p \ln (GSD)} \exp \left[ -\frac{1}{2} \left( \ln(d_p) - \ln(\text{MMD}) \right)^2 \right] dd_p$$  \hspace{1cm} (31)

It should be noted that the value of the cut-size diameter in (31) shall be corrected by introducing a cut-size diameter correction factor ($K$). Hence, the equation of the cut-size diameter of cyclone becomes:

$$d_p = K \left[ \frac{9 \mu b}{\pi N_i (\rho_p - \rho)} \right]^{1/2} = \frac{1}{K^2} \left( \frac{\mu}{3} \right)^{1/2} \left( \frac{2 m_p N_i v_i}{3 \mu b} \right)^{1/2}$$  \hspace{1cm} (32)

Where $K$ for 1D3D cyclone and 2D2D cyclone are shown in (33) and (34), respectively.

$$K_{(1D3D)} = 5.3 + 0.02 \text{ MMD} - 2.4 \text{ GSD}$$  \hspace{1cm} (33)

$$K_{(2D2D)} = 5.5 + 0.02 \text{ MMD} - 2.5 \text{ GSD}$$  \hspace{1cm} (34)

The equations for the inlet velocity of gas through level $k$ ($v_{ik}$) and the pressure drop of each cyclone on level $k$ ($\Delta P_k$) will be the same form as indicated in (2) and (3). Then, these equations are rewritten as shown in (35) and (36).

$$v_{ik} = \frac{Q_k}{a_{0k} b_{0k} d_{ik}^2 N_{ik}}$$  \hspace{1cm} (35)

$$\Delta P_k = \frac{1}{2} \rho v_{ik}^2 N_H = \frac{1}{2} \rho \left( \frac{Q_k}{a_{0k} b_{0k} d_{ik}^2 N_{ik}} \right)^2 N_H$$  \hspace{1cm} (36)
The diameter of 1D3D or 2D2D cyclone as the first cyclone can be calculated by rearranging and substituting (27), (35), and the mass of the particle defined by Clift et al. [9] into (32), yields (37) and (38).

\[
D_{(1D3D)_1} = \left[ \frac{d^2_{(1D3D)_1} \pi N_{(1D3D)_1} (\rho_p - \rho) x_k Q_k}{9 K_{(1D3D)_1} \mu b_{(1D3D)_1} n_{(1D3D)_1} n_p} \right]^{\frac{1}{3}}
\]

\[
(37)
\]

\[
D_{(2D2D)_1} = \left[ \frac{d^2_{(2D2D)_1} \pi N_{(2D2D)_1} (\rho_p - \rho) x_k Q_k}{9 K_{(2D2D)_1} \mu b_{(2D2D)_1} n_{(2D2D)_1} n_p} \right]^{\frac{1}{3}}
\]

\[
(38)
\]

Meanwhile, as the second cyclone, the equation to calculate the diameter of 1D3D or 2D2D cyclone becomes:

\[
D_{(1D3D)_2} = \left[ \frac{d^2_{(1D3D)_2} \pi N_{(1D3D)_2} (\rho_p - \rho) (1 - \eta_{1k}) x_k Q_k}{9 K_{(1D3D)_2} \mu b_{(1D3D)_2} n_{(1D3D)_2} n_p} \right]^{\frac{1}{3}}
\]

\[
(39)
\]

\[
D_{(2D2D)_2} = \left[ \frac{d^2_{(2D2D)_2} \pi N_{(2D2D)_2} (\rho_p - \rho) (1 - \eta_{1k}) x_k Q_k}{9 K_{(2D2D)_2} \mu b_{(2D2D)_2} n_{(2D2D)_2} n_p} \right]^{\frac{1}{3}}
\]

\[
(40)
\]

The objective functions of the optimization are minimizing total cost \( c_{tot} \) and maximizing the efficiency of the cyclone system. Since these objectives are not conflicting with each other, i.e., the minimizing of the total cost will lead to maximize the efficiency of the cyclone \( \eta_{olv} \), the model can obtain the optimal solution simultaneously.

\[
MIN \ c_{tot} = \sum_{k=1}^{N_k} \left[ Q_k c_e (\Delta P_{k1} + \Delta P_{k2}) + \frac{F e}{\rho} N_p \left( D_{k1}^l + D_{k2}^l \right) \right]
\]

\[
(41)
\]

IV. CONSTRAINT

A. For All Cases

According to Buonicore et al. [15] the pressure drop of the cyclone arrangement is normally accepted to be in the range as shown in (42).

\[
500 \leq \Delta P \leq 2500 \ \text{N/m}^2
\]

\[
(42)
\]

For inlet velocity, Shepherd and Lapple [2] proposed the range of practicable cyclone inlet velocity as shown in (43).

\[
15 \leq v_i \leq 30 \ \text{m/s}
\]

\[
(43)
\]

The size range of the diameter of the cyclone of which becomes the lower and upper limit of the diameter of the cyclone in the optimization model can be shown in (44).

\[
0.4 \leq D \leq 3 \ \text{m}
\]

\[
(44)
\]

B. Series Cyclone Arrangement

For the series cyclone arrangement, the constraint for the overall efficiency of the cyclone arrangement is shown in (45). The efficiency of each cyclone based on the experimental result of multiple series cyclones by Whitelock and Buser [11] has lower and upper limit as given in (46) – (48).

\[
0.99 \leq \eta_{olv} \leq 1
\]

\[
(45)
\]

\[
0.9 \leq \eta_1 \leq 0.99
\]

\[
(46)
\]

\[
0.55 \leq \eta_2 \leq 0.99
\]

\[
(47)
\]

\[
0.2 \leq \eta_3 \leq 0.99
\]

\[
(48)
\]

C. Parallel – Series Cyclone Arrangement

It is necessary to introduce binary variables (0-1) \( z \) for each level, defining the existence or nonexistence of the level.

\[
\sum_{k=1}^{N_k} z_k \leq 1
\]

\[
(49)
\]

The upper limit of the total flow through level \( k \) is expressed as indicated in (50).

\[
Q_{p_k} N_{p_k} \leq Q_{kU} z_k
\]

\[
(50)
\]

In order to ensure that there is always at least one existing level of the cyclone arrangement and to obtain the volume fractions \( x_k \) at the optimal solution, constraint as shown in (51) will be added.

\[
x_k - z_k \leq 0
\]

\[
(51)
\]
The number of cyclones is restricted by the upper limit of number of parallel lines as indicated in (52).

\[ N_{p,k} - N_{p,k}^U z_k \leq 0 \quad (52) \]

The limit of cyclone overall efficiency (from 0 to 1) has a relationship with the cut-size diameter of cyclone. By substituting (31) with a global surrogate model consisting of a polynomial of degree four, the optimal efficiency of the first cyclone \((\text{MMD}_1 = 10 \times 10^{-6} \text{ m} \text{ and GSD}_1 = 2.5)\) and the second cyclone \((\text{MMD}_2 = 3.7 \times 10^{-6} \text{ m} \text{ and GSD}_2 = 2.5 \text{ [11]}\)) will be obtained.

![Fig. 5 The efficiency vs the cut-size diameter for (a) First cyclone; (b) Second cyclone](image)

**V. RESULTS AND DISCUSSION**

**A. Parallel Cylones Arrangement**

All mathematical models were solved in GAMS (General Algebraic Modeling System, [4]) in a CPU Intel Core i5-4200U, 1.60 GHz. CONOPT 3 solver was used to solve the NLP problem. The diameter of the cyclone has an important role in the resultant value of the optimal number of cyclones, pressure drop, and inlet velocity in this optimization. All of these optimal values were obtained with respect to the minimum total cost. In order to obtain a specific result of the optimal number of cyclones, pressure drop, and inlet velocity, the constraint of the cyclone diameter is divided into five parameters; each parameter has a lower limit of 1.2 m with five different upper limits, i.e., 1.8 m, 2.0 m, 2.5 m, 2.7 m, and 3.0 m. The results of the optimization of 1D3D, 2D2D, and 1D2D cyclones in parallel can be seen in Figs. 6 – 9.

![Fig. 6 Optimal solution of number of the cyclone in parallel](image)

![Fig. 7 Optimal solution of pressure drop of the cyclone in parallel](image)

Fig. 6 confirms that the optimal number of cyclones lies at the upper bound of the diameter. The selection of the upper bound of the diameter as the optimal value of the decision variable can be explained physically wherein by increasing the cyclone diameter, the residence time also increases. The increase in the available time for collection of particles results in an increase in the total collection efficiency. So, the optimal performance of the cyclone can be reached. The 1D3D cyclones connected in parallel have the smallest diameter for the same value of optimal number of cyclones \((N = 3)\) compared with the other two. Using an upper bound diameter up to 3.0 m, the 2D2D cyclone with parallel arrangement model resulted in one cyclone as the optimal number of cyclones.

Fig. 7 presents the optimal values of the cyclone pressure drop in parallel arrangement. It is observed that with the similar size of cyclone diameter (i.e., 1.8 m and 2.0 m), the 1D2D cyclone will have a lower pressure drop compared to 2D2D cyclone. Furthermore, the optimal values of pressure drop of 1D2D cyclone and 2D2D cyclone are obtained by using the upper bound...
of cyclone diameter up to 2.7 m and 3.0 m, respectively. Meanwhile, the optimal values 1D3D cyclone pressure drop is obtained by using the upper bound of cyclone diameter up to 2.5 m. Although the diameter of 1D3D cyclone is smaller than the others, the value of its pressure drop is higher. Higher value of pressured drop will lead to higher operating cost.

![Fig. 8 Optimal solution of inlet velocity of the cyclone in parallel](image)

Fig. 8 presents the optimal value of inlet velocity in the parallel arrangement. The sensitivity of the optimal value of inlet velocity is exactly the same as the pressure drop. However, when the inlet velocity of 1D2D cyclone has the same value with 2D2D cyclone at \( D = 1.8 \) m and 2.0 m, the pressure drop of 1D2D cyclone is lower than 2D2D. So as the pressure drop of cyclone is directly related to the fan power, the 1D2D cyclone will have the lowest operating cost than 2D2D. This result is found to be in accordance with the result from Wang et al. [16] where the 1D2D cyclone has the lowest pressure drop compared to 1D3D and 2D2D cyclones.

As observed in Fig. 9 which presents the minimum total cost of parallel cyclone arrangement, the total cost of all types of cyclones are obtained very close to each other. Furthermore, the number of cyclone has significant effect in calculating the total cost, where higher number of cyclone will lead to higher total cost. Figure 9 also shows that for the same value of total cost and when \( N_p = 3 \), the 1D3D cyclones will have the smallest diameter. Therefore, if the available space (horizontal space) is limited on the field, it is advised to use the 1D3D cyclones connected in parallel. Meanwhile, if there is enough space, the 1D2D cyclones are the best choice to be connected in parallel because it will have a less number of cyclone with a significant lower pressure drop and a lower total cost compared to the others.

### B. Series Cylones Arrangement

The optimization results of two cyclones and three cyclones in series can be observed in Table 5 and Table 6, respectively. From these results, it can be concluded that in order to minimize the total cost while maximizing the overall efficiency of the arrangement, the optimal value of cyclone diameter lies at its upper bound. The selection of upper bound as the optimal value of the diameter of cyclone is in line with the results from Ravi et al. [12] method that selected the \( H_0 \) ratio (H/D) at its upper bound to increase the collection efficiency. By setting the second and third cyclone to have a similar diameter as the first cyclone, the optimum value of the efficiency of the secondary and tertiary cyclone lies at its lower bound while obtaining the maximum value of the overall efficiency of the arrangement (\( \eta_{\text{opt}} \)). This result is found to be in accordance with the fact that since most of large particles are captured in the first cyclone, only small sized materials enter to the next stages. Therefore, a higher efficiency of the second or third stage cyclone can probably be reached when using a smaller diameter or dimensions than the first cyclone.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>1D3D cyclone (Cyclone No.1)</th>
<th>2D2D cyclone (Cyclone No.1)</th>
<th>1D2D cyclone (Cyclone No.1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta ) (%)</td>
<td>97.8 55</td>
<td>97.8 55</td>
<td>97.8 55</td>
</tr>
<tr>
<td>( d_\mu ) (µm)</td>
<td>30.77 206.41</td>
<td>19.86 133.24</td>
<td>22.207 148.97</td>
</tr>
<tr>
<td>( D ) (m)</td>
<td>2.709 2.709</td>
<td>2.709 2.709</td>
<td>2.549 2.549</td>
</tr>
<tr>
<td>( \Delta \rho ) (N/m²)</td>
<td>949.634 949.634</td>
<td>949.636 949.636</td>
<td>882.834 882.779</td>
</tr>
<tr>
<td>( v_1 ) (m/s)</td>
<td>15.232 15.232</td>
<td>15.232 15.232</td>
<td>17.2 17.201</td>
</tr>
<tr>
<td>( c_{\text{tot}} ) ($/s)</td>
<td>0.002</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>( \eta_{\text{opt}} ) (%)</td>
<td>99</td>
<td>99</td>
<td>99</td>
</tr>
</tbody>
</table>
Another result found from this study that there is a significant difference of the pressure drop among all three types of cyclones where the 1D2D cyclone will have the lowest pressure drop with the smallest size of diameter (i.e., 2.5 m). Therefore, the results of this study illustrate that the 1D2D cyclone in a series is more suitable to handle a given input feed. Furthermore, two cyclones in series is more likely to be operated than three cyclones because as can be seen in the results, the total cost and total pressure drop in the system of three cyclones in series are higher than the two cyclones in series.

### C. Parallel – Series Cyclones Arrangement

The MINLP problem in this study was solved using DICOPT (Discrete and Continuous OPTimizer) that runs under GAMS. The MINLP algorithm inside DICOPT solves a series of MILP and NLP sub-problems where the MILP and NLP were solved using CPLEX and CONOPT 3, respectively. The program was run on a CPU Intel Core i5-4200U, 1.60 GHz CPU and 8 Gbyte memory. In this optimization two decision variables, i.e., number of parallel lines ($N_p$) and cyclone diameter ($D$) are used to provide a good boundary to help the optimization process to be convergent, a reasonably wide range value of the upper bound of number of parallel lines ($N_p^U$) should be selected properly for each upper bound of the cyclone diameter ($D_p^U$).

Figs. 10 – 11 presents the effects of varying the upper bound value of decision variables $D_p^U$ and $N_p^U$ in order to find the optimal value of the cyclone diameter and number of parallel lines. The optimal value of cyclone diameter lies at its upper bound and the optimal solution for the number of parallel lines lies within a certain wide range of values. It should be noted that a certain value of the upper bound of $N_p$ can not be used in obtaining the feasible solutions for all relaxed NLP sub-problems. For instance, the optimization by using $D_p^U = 0.3$ m and $N_p^U = 500$, will only obtain the parallel – series arrangement of 1D3D+2D2D (level 3) as the best arrangement with $N_p = 489$.

![Fig. 10 Optimal solution of number of parallel lines ($D_p = 0.3 – 1$ m)](image)

![Fig. 11 Optimal solution of number of parallel lines ($D_p = 1.1 – 2.5$ m)](image)

Fig. 12 shows the relationship between the overall efficiency and cyclone diameter. The overall efficiency decreased nonlinearly as cyclone diameter increased with different slope of certain type of cyclone arrangement being chosen. Furthermore, higher collection efficiency is accompanied by an increased in value of the inlet velocity and pressure drop across the cyclone, resulting in higher total cost. Similar result of this case was also reported by Gim bun et al. [17]. It is interesting to know that there are only three from four levels available (Table 3) that have been chosen by the model as the best arrangement, where level 2 (2D2D+2D2D) is found to be more dominant than the others. The 2D2D+2D2D arrangement is selected as the best arrangement for a wide range value of the optimum diameter of the cyclone ($D_p = 0.8 – 2.5$ m).

#### TABLE VI. OPTIMAL SOLUTION OF THREE CYCLONES ARRANGED IN SERIES

<table>
<thead>
<tr>
<th>Parameter</th>
<th>1D3D cyclone</th>
<th>2D2D cyclone</th>
<th>1D2D cyclone</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cyclone No.1</td>
<td>Cyclone No.2</td>
<td>Cyclone No.3</td>
<td>Cyclone No.1</td>
</tr>
<tr>
<td>$\eta$ (%)</td>
<td>97.2</td>
<td>55</td>
<td>20</td>
</tr>
<tr>
<td>$d_p$ (µm)</td>
<td>30.77</td>
<td>184.62</td>
<td>275.21</td>
</tr>
<tr>
<td>D (m)</td>
<td>2.709</td>
<td>2.709</td>
<td>2.709</td>
</tr>
<tr>
<td>$\Delta_p$ (N/m²)</td>
<td>949.635</td>
<td>949.635</td>
<td>949.636</td>
</tr>
<tr>
<td>$v_1$ (m/s)</td>
<td>15.232</td>
<td>15.232</td>
<td>15.232</td>
</tr>
<tr>
<td>$e_{tot}$ ($$/s$$)</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td>$\eta_{tot}$ (%)</td>
<td>99</td>
<td>99</td>
<td>99</td>
</tr>
</tbody>
</table>
Fig. 13 indicates that an increased in the optimal value of $D_p$ will result in a decreased in the total cost ($c_{tot}$). Since the operating cost is proportional to $\frac{1}{D^2}$ in the equation, a decreased value of the operating cost tends to be more dominant in affecting the value of total cost.

Another result shows that, at $D_p = 2.1$ m, there are two different types of cyclone arrangements (level 2 and level 3) will have the similar optimum number of parallel lines ($N_p = 13$) and the same total cost (Fig. 13). However, the overall efficiency of 2D2D+2D2D is higher than 1D3D+2D2D (Fig. 12). In addition, the pressure drop of all cyclone arrangements can also be accepted, since the values were obtained in the range of $500 \leq \Delta P \leq 2500$ N/m2.

VI. CONCLUSION

This study proposed the model of mathematical programming for optimization of multiple cyclone arrangement in order to obtain the best cyclone arrangement with the optimal number and dimensions of the cyclone. For this purpose, two mathematical programming techniques are used to optimize different cyclone arrangement i.e., nonlinear programming (NLP) model for 1D3D, 2D2D, and 1D2D cyclones in parallel or in series and mixed integer nonlinear programming (MINLP) model for four combinations of 1D3D and 2D2D cyclones connected in parallel-series. The objective of all these models is to find the optimal number and dimensions of cyclone arrangement with respect to the minimum total cost including the operating cost and the capital cost. The results of NLP optimization show that if the horizontal space is limited on the field, it is advised to use the 1D3D cyclones in parallel because at the same value of minimum total cost and optimal number of cyclone, the 1D3D cyclones will have the smallest dimensions. Meanwhile, for a series arrangement, two 1D2D cyclones in series is more suitable to handle the total dust in the proposed NPK fertilizer plants because it will have the lowest pressure drop with the smallest size of diameter. Moreover, in the MINLP optimization, the cyclone arrangement of 2D2D+2D2D in parallel-series is found as the best arrangement with a higher efficiency and a lower total cost compared to the others.

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REFERENCES


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