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Reliability Modeling for Rotor Systems with Imbalance Based on Vibration Analysis

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Abstract

In Rotor systems, the imbalance fault generates high dynamic forces. These high dynamic forces are catastrophic in nature at especially at high rotational speeds. This hypothesis is known to engineers, but how it actually affects the reliability is not established. In this research work a vibration based residual generation experimental method is proposed to relate imbalance in terms of vibration amplitude and reliability. A stress-strength interference approach together with a simulation-based methodology is used for modeling and analysis of this complex relationship. The paper also proposes a method for establishing the safe and critical limits of imbalance and rotational speed for achieving the specified reliability targets. Thus the concepts can be effectively used in industrial machinery for controlling the level of imbalance of a rotor designed to operate at a specified rotational speed.

Keywords:
Imbalance; Rotor systems; Reliability; Vibration analysis; Stress-Strength interference.

1. Introduction

During the operational stage of industrial rotor systems, the dynamic forces generated due to machinery faults affect the system performance and reliability. The rotating machineries are vulnerable to varieties of defects and faults such as mass imbalance, shaft misalignment, improper surface finish etc., which may lead to catastrophic accidents [1]. Rotor imbalance is one of the most common faults and it generates unwanted motion of the rotors, such as whirling and vibrations. A detailed literature survey on imbalance is presented in [2].

Many research works are available in the literature for identification and detection of faults in rotor systems. A methodology for fault identification and diagnosis of machine faults based on the parameter estimation, calculation of physical coefficients and symptom-fault tree is presented in [3, 4]. Model based fault identification in the rotor systems is described by [5, 6]. The model based identification of faults for the occurrence of two simultaneous faults (crack and imbalance) in the rotor system is presented by [7]. A model based fault diagnosis of a rotor–bearing system is discussed in [8]. The authors identified faults such as misalignment and imbalance using a test setup by comparing the experimental residual forces with theoretical residual forces from fault models. Model based fault identification for the two imbalance faults in two-spin rotor system is presented in [9]. An equivalent load minimization and vibration minimization method is applied for identification of the imbalance [10]. The aforementioned literature shows that the vibration data of the system is utilized by model based diagnostics to identify the commonly occurring faults such as imbalance, misalignment, crack etc.

Many failure models for evaluating the reliability of the mechanical systems (*viz* rotor systems) are discussed in [11]. Furthermore, this paper identifies that the Stress–Strength Interference (SSI) model is suitable for the failure analysis of mechanical systems and components where strength and stress are often treated as the random variables. A brief literature survey on SSI models and their applications is presented in [2, 12].

The traditional approach for reliability assessment of the rotor systems is based on the factor of safety or safety margin. This concept relies on the empirical methods and experience rather than statistical approach [13]. In actual practice, it can be seen that the stress on a component and its strength are both random variables. For example, the applied load on the rotor shaft is not a constant deterministic quantity. It depends on several factors such as the variations in load, speed, friction and the additional forces due to imbalance, misalignment, etc. Similarly, the strength of rotor shaft is also a function of several stochastic variables such as variations due to manufacturing process, material strength, metal removal due to wear, corrosion, and other factors such as fatigue. As we know, failure occurs when the stress on a component increases beyond its strength. Therefore, the SSI models are found suitable for evaluating the reliability of the rotor systems subjected to the mass imbalance.

A critical review of the literature indicated that some research work is available for reliability modeling of rotor systems considering dynamic forces due to faults and defects. A theoretical model to relate reliability with mass imbalance is presented in [2, 12]. However, experimental validation of such models is a very important step for confirmation and acceptance of the models for practical applications in industry. This paper has presented a frame work for carrying out such studies. The focus is on the most common problem of mass imbalance in rotor systems and its effect on reliability. It is well known that the mass imbalance in rotor systems produces additional centrifugal force and high vibration amplitude at rotational frequency. Therefore, an experimental vibration analysis using a machine fault simulator and SSI approach has been used for developing a reliability model for the rotor system with imbalance. Even though the current focus is only on the problem of imbalance, the proposed frame work can be extended to include forces in rotor systems due to other faults such as misalignment, resonance, oil whirl, etc.

The rest of this paper is organized as follows: Section 2 along with its sub sections present the various steps followed for developing the proposed model. Section 3 illustrates the proposed model for the shaft of a rotor system and the section 4 demonstrates proposed model for the shaft in an experimental setup with data collection, analysis and interpretation of results. Section 5 presents the conclusions with scope of future work. The paper ends with references.

2. Proposed Reliability Model

The force due to imbalance in a rotor system depends on the amount of imbalance mass, its distance from the axis of rotation, and the rotational speed [14]. The force produced due to imbalance is given by,

$$F_{im} = mr\omega^2 \tag{1}$$

From the equation (1), it can be found that a small amount of imbalance mass will generate huge force resulting in substantial amount of stress on the system.

In Industrial rotor systems, it is difficult to estimate the amount of imbalance mass and its distance from the axis of rotation, and hence the equation (1) will be only of theoretical importance. Hence, the residual force generation technique of model based diagnosis which is discussed in [5, 6, 8, 9, 10] is used to estimate the amount of force due to imbalance mass in the practical rotor systems. The model presented here uses the information of a dynamic model of rotor system, and it uses the difference between the vibration response of the rotor system with fault and without fault conditions.

2.1 Residual Force Generation

The vibrations of an undamaged rotor system represented by the displacement vector $\mathbf{x}_o(t)$ at N degrees of freedom due to the operating load $\mathbf{F}_o(t)$ during normal operating condition is described by the linear equation of motion (2),

$$\mathbf{M}\ddot{\mathbf{x}}_0(t) + \mathbf{C}\dot{\mathbf{x}}_o(t) + \mathbf{K}\mathbf{x}_o(t) = \mathbf{F}_o(t) \tag{2}$$

The occurrence of a fault in the undamaged system induces the change in the dynamic behavior of the system. The change in dynamic behavior depends on the type of fault, severity of the fault and location of its occurrence. The introduction of the fault in the undamaged system adds the additional load and influences the change in the vibration response. The equation of motion for the faulty system is represented by,

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{F}_o(t) + \Delta\mathbf{F}(t) \tag{3}$$

The difference between the faultless system response and the faulty system response will give the residual displacements, velocities and accelerations. These are represented as,

$$\Delta\mathbf{x}(t) = \mathbf{x}(t) - \mathbf{x}_o(t) \tag{4}$$

$$\Delta\dot{\mathbf{x}}(t) = \dot{\mathbf{x}}(t) - \dot{\mathbf{x}}_o(t) \tag{5}$$

$$\Delta\ddot{\mathbf{x}}(t) = \ddot{\mathbf{x}}(t) - \ddot{\mathbf{x}}_o(t) \tag{6}$$

The subtraction of equation of motion of undamaged system (2) from the equation of motion of the faulty system (3) and substituting (4, 5 and 6) yields the following equation,

$$\mathbf{M}\Delta\ddot{\mathbf{x}}(t) + \mathbf{C}\Delta\dot{\mathbf{x}}(t) + \mathbf{K}\Delta\mathbf{x}(t) = \Delta\mathbf{F}(t) \tag{7}$$

In this model it is assumed that the rotor system is linear and hence the system matrices (**M**, **C**, and **K**) remain unchanged. The equivalent load represented as $\Delta\mathbf{F}(t)$ induces the change in the dynamic behavior of the undamaged linear rotor system and this equivalent load will fetch the magnitude of the fault and the location of its occurrence. This additional force ($\Delta\mathbf{F}(t)$) due to fault is also termed as residual force.
To calculate the residual vibration response due to the fault in the rotor system, the measured vibration response data for both undamaged and faulty rotor system should be available for the same operating and measurement conditions [5, 7, 8, 15].

2.2 Modal expansion

To compute the residual force vector ($\Delta\mathbf{F}(t)$) at all the nodes, the residual vibration response at all the degrees of freedom (N) of the system should be available in the form of residual displacement, residual velocity and residual acceleration. But due to the practical limitations, the vibration response is measured only in few degrees of freedom (n) and hence $n \ll N$.
The vibration response for the non-measurable degrees of freedom has to be estimated from the measured vibration response. To estimate the residual vibration at all degrees of freedom from the measured residual vibration response $\Delta\mathbf{x}_{\mathrm{n}}(t)$, the modal expansion technique is used. This modal expansion technique is rooted on the approximation of the residual vibration by a linear combination of few eigenvectors ($\widehat{\boldsymbol{\Phi}}$). The full residual vector $\Delta\mathbf{x}(t)$ can be approximated by a reduced modal matrix $\widehat{\boldsymbol{\Phi}}$ which contain a set of mode shapes $\widehat{\mathbf{X}}_{\mathrm{k}}$

$$\widehat{\boldsymbol{\Phi}} = [\widehat{\mathbf{X}}_1, \widehat{\mathbf{X}}_2, \ldots \widehat{\mathbf{X}}_{\mathrm{k}}] \tag{9}$$

The number of mode shapes used in the reduced modal matrix $\widehat{\boldsymbol{\Phi}}$, logically may not exceed the number of independently measured vibration response contained in the $\Delta\mathbf{x}_{\mathrm{n}}(t)$, i.e. $K \leq n$.
Now, the measured residual vibration response $\Delta\mathbf{x}_{\mathrm{n}}(t)$ is related to the full residual vector $\Delta\mathbf{x}(t)$ by the transformation matrix $\widetilde{\mathbf{T}}$ **,** which uses the modal matrix $\widehat{\boldsymbol{\Phi}}$, and given by

$$\Delta\mathbf{x}_{\mathrm{n}}(t) = \widetilde{\mathbf{T}}\,\Delta\mathbf{x}(t) \tag{8}$$

In this research paper, for the expansion of the available measured data $\Delta\mathbf{x}_{\mathrm{n}}(t)$ the System Equivalent Reduction Expansion Process (SEREP) has been used. The SEREP expansion technique utilizes the SEREP transformation matrix to expand the measured degrees of freedom in all the degrees of the system [16]. The SEREP transformation matrix is given by

$$\widetilde{\mathbf{T}} = [\phi][\hat{\phi}]^g \tag{9}$$

where

$$[\hat{\phi}]^g = \left[[\hat{\phi}]^T \quad [\hat{\phi}]\right]^{-1} [\hat{\phi}]^T \tag{10}$$

2.3 Residual Force

The residual force $\Delta\mathbf{F}(t)$ which characterizes the fault is calculated by substituting the system matrices which describes the system mass, damping and stiffness, and the residual vibration response of the full vibration state in the form of residual displacement, residual velocity and residual acceleration, in the equation (7). And the corresponding substitution of equation (8) finally yields the equation (11),

$$\mathbf{M}\widetilde{\mathbf{T}}\Delta\ddot{\mathbf{x}}_{\mathrm{n}}(t) + \mathbf{C}\widetilde{\mathbf{T}}\Delta\dot{\mathbf{x}}_{\mathrm{n}}(t) + \mathbf{K}\widetilde{\mathbf{T}}\Delta\mathbf{x}_{\mathrm{n}}(t) = \Delta\mathbf{F}(t) \tag{11}$$

The equation (11) can be used to estimate the equivalent loads at all the degrees of freedom from the few measured vibration responses.

2.4 Effective stress

The force due to imbalance (F_{im}) has to be found from the vector of equivalent load ($\Delta\mathbf{F}(t)$). The force due to imbalance is accountable for the additional stress in the system, apart from the design stress. The imbalance force is used to compute equivalent additional stress. This additional stress is to be incorporated in to the design equation of the system, to find the effective stress. The effective stress is used to find the reliability of the system. This procedure of finding effective stress has been shown in Figure 2. Following sub-section explains about the reliability computation using the effective stress in the component and the ultimate strength of the component.

2.5 Reliability modeling for imbalance

To calculate the reliability of the rotor system the stress-strength interference (SSI) failure model is used [11, 16]. For this SSI failure model, the following reliability expression (12) has been used. Equation (12) represents the reliability of the component or system for all the possible values of strength (S_{TH}) of the component and stress (S_{ts}) on the component.

$$R = \int_{-\infty}^{\infty} f_{S_{TH}}(S_{TH}) \left[\int_{-\infty}^{S_{TH}} f_{S_{ts}}(S_{ts})\, \mathrm{d}S_{ts} \right] \mathrm{d}S_{TH} \tag{12}$$

The proposed model has been shown in the form of flowchart in figure 1 and it is illustrated by a numerical example in the following section.

3. Illustration for the Proposed Reliability Model for a Shaft

The shaft element of rotor system has been considered here for the illustration. The design equation of the shaft is used to find the effective stress in the shaft. The Guest's and Rankine's shaft design equations (13, 14) are used to estimate the design stress [18]. It is suggested that design stress may be obtained by using both the theories (13, 14) and the larger of the two values is adopted for designing the shaft [18].

$$\tau_{max} = \frac{16\sqrt{M_b^2 + T^2}}{\pi d^3} \tag{13}$$

$$\sigma_{max} = \frac{32\left(\frac{M_b + \sqrt{M_b^2 + T^2}}{2}\right)}{\pi d^3} \tag{14}$$

The addition of bending moment (M_{im}) due to imbalance force (F_{im}) in the stress equations (13, 14) will lead to the effective stress equation. The effective stress equations for shaft based on the maximum shear stress and maximum normal stress are given in equations (15, 16).

$$\tau_{eff} = \frac{16\sqrt{(M_{im} + M_b)^2 + T^2}}{\pi d^3} \tag{15}$$

$$\sigma_{eff} = \frac{32\left(\frac{(M_{im} + M_b)^2 + \sqrt{(M_{im} + M_b)^2 + T^2}}{2}\right)}{\pi d^3} \tag{16}$$

The effective stress equations (15, 16) will estimate the total stress in the shaft, including the stress produced due to imbalance force.

The effective stress equations (15, 16) of any mechanical component or system will have variables which are probabilistic in nature, rather than deterministic. This randomness makes the reliability calculations difficult and complex. To calculate the reliability of the component with such randomness, we require large amount of field test data to compute the stresses accurately. This is not practically feasible. Therefore, the simulation technique is used to generate and compute the stresses. For example, in the equations (15, 16), variables such as, radius of shaft, length of the shaft, ultimate tensile stress and ultimate normal stresses are probabilistic in nature. The arbitrariness in the aforementioned variables are due to many reasons such as, manufacturing variations while metal working or casting etc., heat treatment variations, dimensional errors due to human or machine inaccuracy, power fluctuations, environmental effects, etc.

The random variables, such as the radius of the shaft, ultimate tensile stress, and ultimate normal stress will exhibit arbitrariness. To theoretically calculate the reliability for a shaft, the generation of random variables representing the arbitrariness of the radius of the shaft, ultimate tensile and normal stresses according to their probabilistic characteristics is necessary. For this, random numbers (distributed between 0 and 1) are generated using a computer program. These random numbers are used to generate the values for random variables to represent the probabilistic nature of the variables. The generation of random numbers and generation of continuous and discrete random variables using random numbers are discussed in [19].

The generation of continuous random variable according to their probabilistic nature and the calculation for each realization of random variable in equations (15, 16) will give a probabilistic character for the effective stress. And in the same way the ultimate shear strength and ultimate normal strength for the shaft with a particular material type and shape is generated according to their probabilistic characters. This ultimate shear strength and ultimate normal strength will resist the effective shear stresses and effective normal stresses acting on the shaft. The SSI equation [17] for the maximum shear stress model and maximum normal stress model are given in the following equations (17, 18) respectively,

$$R = \int_{-\infty}^{\infty} f_{\varsigma}(\varsigma)\left[\int_{-\infty}^{\varsigma} f_{\sigma_{eff}}(\sigma_{eff})\,\mathrm{d}\sigma_{eff}\right]\mathrm{d}\varsigma \tag{17}$$

$$R = \int_{-\infty}^{\infty} f_{\mathcal{T}}(\mathcal{T})\left[\int_{-\infty}^{\mathcal{T}} f_{\tau_{eff}}(\tau_{eff})\,\mathrm{d}\tau_{eff}\right]\mathrm{d}\mathcal{T} \tag{18}$$

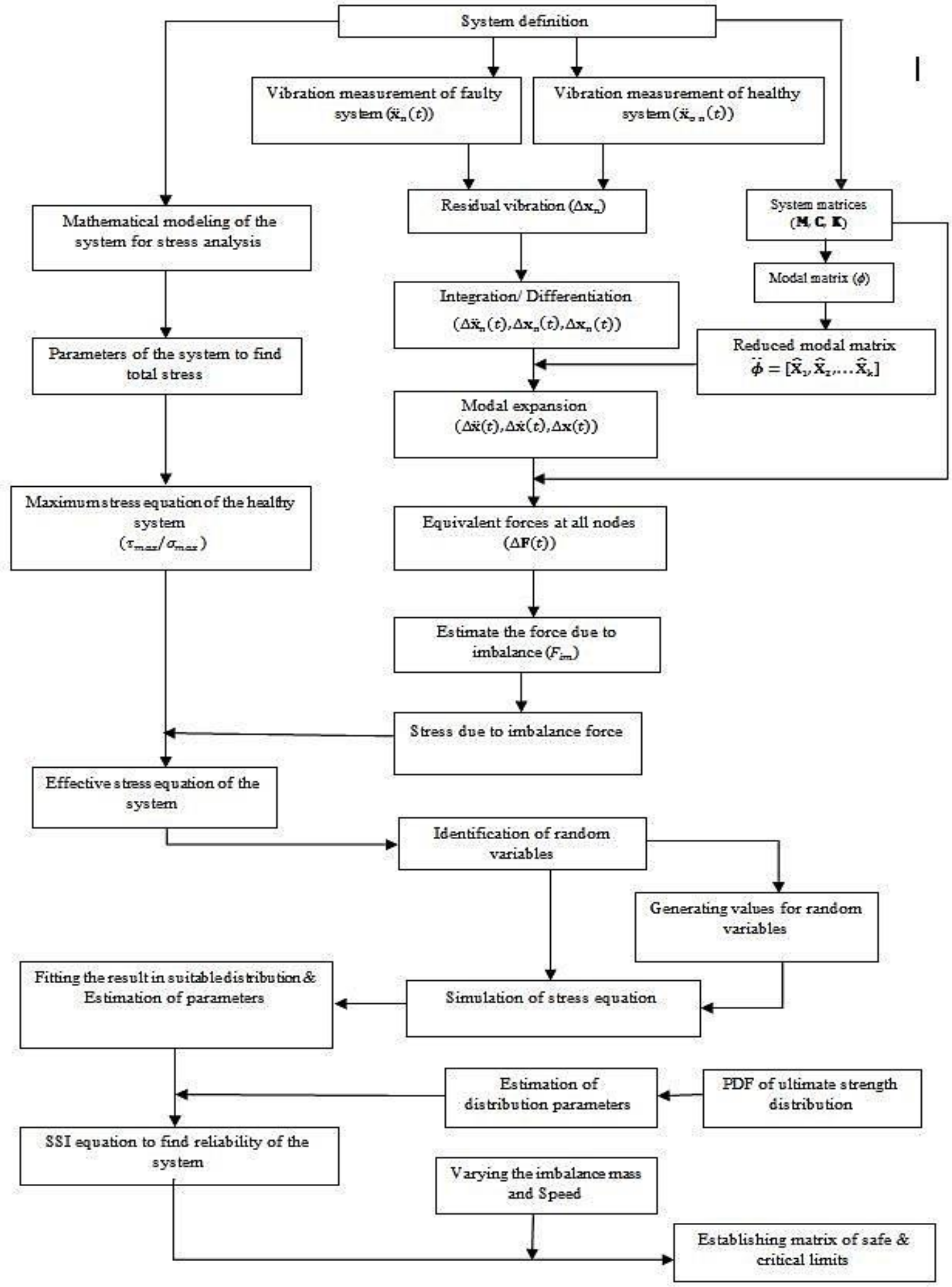


Figure 1: Flow chart of the proposed reliability model for rotor system with imbalance fault

After the simulation cycles (in this illustration 250,000 simulation cycles) the resultant random variables for the τ_{eff} and σ_{eff} are fit into the suitable distribution. A similar procedure is followed for generation of the strength variables. Reliability of the shaft is computed by substituting the density functions and its parameters in the equations (17, 18). An experimental method is used to physically simulate imbalance and to measure the vibration due to this. The stresses are computed based on the measured vibrations.

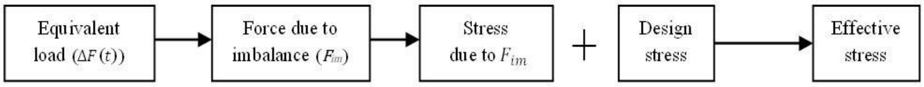


Figure 2: Flow chart for computation of effective stress

4. Experimental work for the illustration

The experimental work to get acquire the vibration response is carried out in a machine fault simulator (MFS) test rig. The fault simulator is having provisions to introduce different types of faults to carry out the experimental work. The finite element modeling [20] of the test setup has been given in the appendix-I. The schematic diagram of the MFS with the essential dimensions, and the photograph has been shown in figure 3. Four ICP accelerometers (0.3 – 10 KHz, 100m V/G) with BNC connectors are used for vibration data acquisition. The OROS software is used to acquire and investigate vibration data. The OROS uses the digital integration to find the displacement and velocity amplitude from the measured acceleration data, and OriginPro is used to plot the data. MATLAB software has been used to write codes for the further analysis of the acquired data.

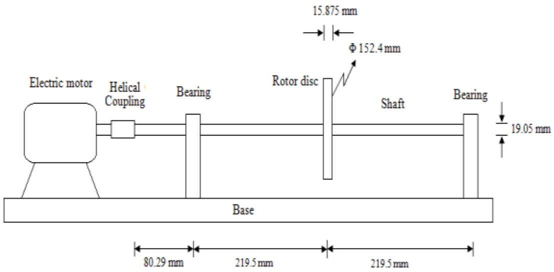


Figure 3:Schematic representation of experimental setup

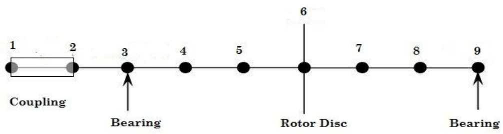


Figure 4: Finite element model of test setup

4.1 Experimental procedure and signal processing

The vibration response was measured without introducing any fault ($\ddot{\mathbf{x}}_0(t)$) and the imbalance is introduced from 0.004 kg to 0.220kg, the vibration responses are measured at the speeds from1500 rpm to 8000 rpm. The rotational speed of the machine has been kept under the critical speed of 9000 rpm. The sample vibration responses plots are given in the figures 5 (for faultless condition) and 6 (with the presence of fault). Figure 6 clearly shows that vibration amplitude of 1x running speed component increases due to presence of imbalance in the system. The 1x and 2x components in figure 5 are due to the residual faults present within the system.

4.2 Imbalance force estimation:

The imbalance force (F_{im}) is found from residual force vector ($\Delta\mathbf{F}(t)$).The force vector ($\Delta\mathbf{F}(t)$) is estimated by substitution of numerical values for residual acceleration, velocity, and displacement from the acquired vibration signal, and substitution of system matrices obtained from the FEM analysis of the test setup. In the FEM analysis, the test setup (figure 4) has been divided into nine nodes and each node is having four degrees of freedom (two translation motion and two rotational motions). The force vector ($\Delta\mathbf{F}(t)$) gives corresponding equivalent load at all DOF. The force due to imbalance (F_{im}) is found by plotting the residual forces at the corresponding nodes. The sample numerical values of F_{im} for different imbalance masses for different rotational speeds are shown in the table 1.

Table 1: Force (F_{im}) due to imbalance mass (N)

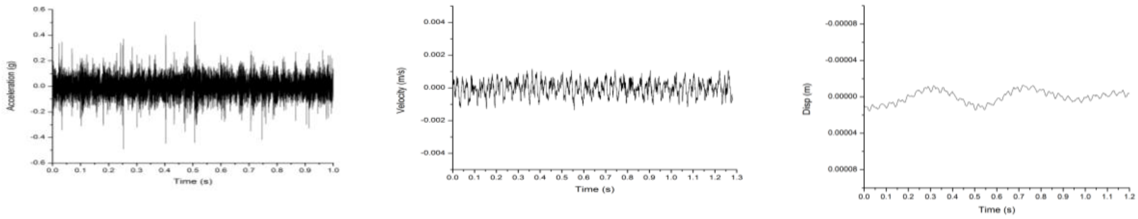
Imbalance mass(kg) / *Speed (rpm)*	**0.004**	**0.01**	**0.02**	**0.04**	**0.08**	**0.12**	**0.2**
1500	6.5820	15.5890	30.8316	66.7234	136.4521	207.7985	345.4312
1800	9.1788	25.4413	48.7849	99.7296	192.9731	297.3096	490.8490
2400	17.0243	39.9079	86.7368	175.3452	355.2653	530.1560	887.5434
2700	21.3257	54.9980	110.2401	220.2149	449.1365	675.5469	1125.4189

4.3 Reliability Estimation:

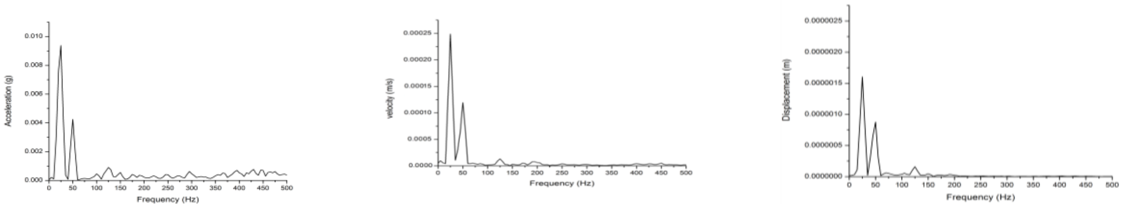
The numerical values of force due to imbalance are used to compute the additional bending moment in the shaft, as explained in sub-section 2.4. The required values of different parameters and additional bending moment are substituted in the equations (15, 16) to get the effective stress values. The shear stress (τ_{eff}) and the ultimate shear strength ($\mathcal{T}$) values obtained after the simulation cycles follow the standard normal distribution. Substitution of the normal distribution density functions of stress and strength in the Eq. (18) the following reliability expression (19) is obtained [17]. By substituting the parameters of the distributions in equation (19) reliability of the shaft is computed.

$$R = 1 - \Phi(z) \tag{19}$$

where $$z = \frac{\left(\mu_{\mathcal{T}} - \mu_{\tau_{eff}}\right)}{\sqrt{(\varrho_{\mathcal{T}})^2 + (\varrho_{\tau_{eff}})^2}}$$

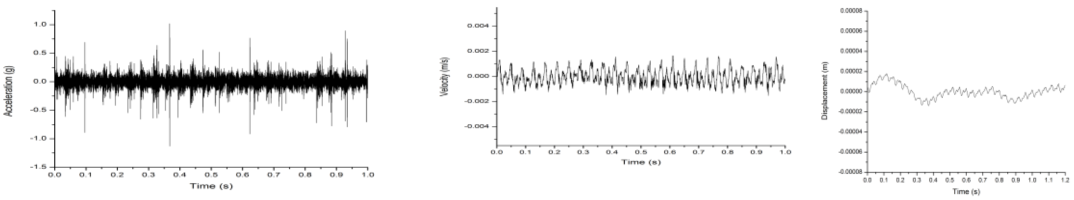


(a)

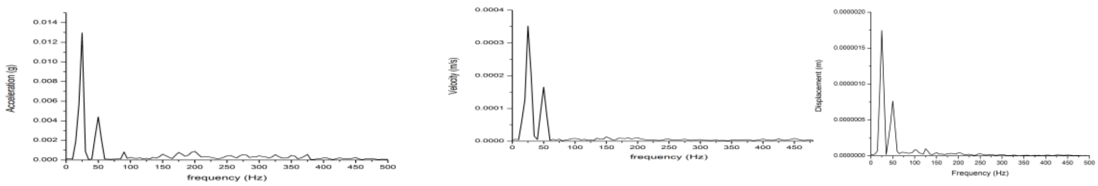


(b)

Figure 5: Vibration response at 25 Hz without imbalance mass (a) Time domain (b) Frequency domain



(a)



(b)

Figure 6: Vibration response at 25 Hz with imbalance mass 0.004kg **(a)** Time domain **(b)** Frequency domain

4.4 Results

The experiment is repeated by increasing the imbalance mass. The results of the numerical illustration has been are presented in the Table 2. Here it is considered that for a given imbalance mass, for shaft reliability of ≥0.99 is safe. If the reliability is less than 0.99 the shaft is considered as critical. The variations of reliability with respect to imbalance mass and the rotational speed are presented in figure 7. The analysis of experimental results for the numerical illustration shows that in the experimental setup, the imbalance mass is having negligible impact on the reliability up to the speed of around 3110 rpm. The reliability decreases considerably if the test setup is operated at higher speeds with the imbalance mass.

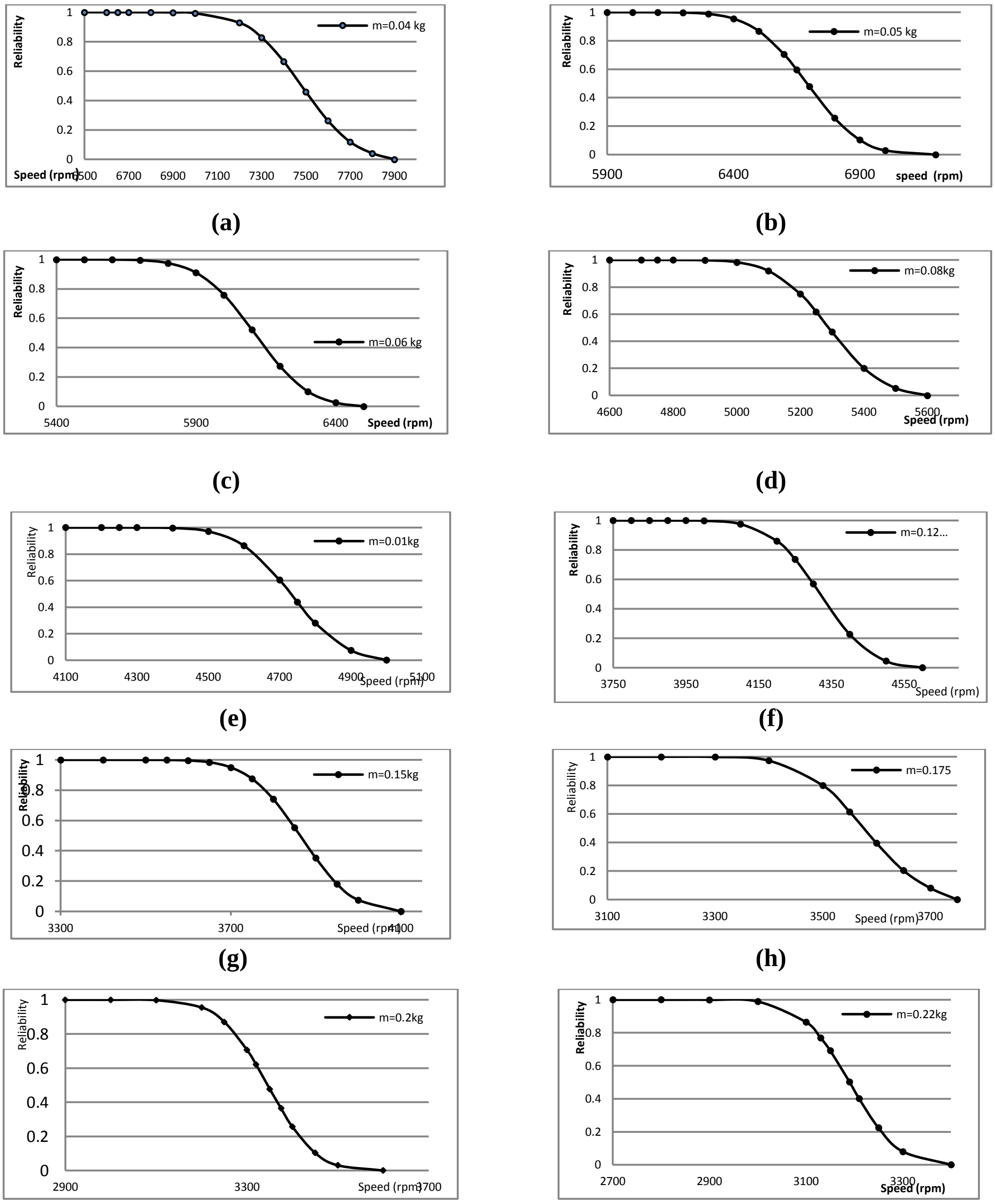


Figure 7: (Contd.) Reliability of the shaft at different speeds with imbalance masses **(a)** 0.04kg **(b)** 0.05kg **(c)** 0.06kg **(d)** 0.08kg **(e)** 0.10 kg **(f)** 0.12 kg **(g)** 0.15 kg **(h)** 0.175 kg **(i)** 0.2 kg **(j)** 0.22 kg

Table 2: Safe and critical RPM for the shaft

Imbalance mass (Kg)	Safe RPM (R ≥ 0.99)	Unsafe RPM (0.95 ≤ R ≤ 0.99)	Critical RPM (R < 0.95)
0.04	≤ 7000	7000 - 7100	>7100
0.05	≤ 6300	6300 - 6450	>6450
0.06	≤ 5750	5750 – 5850	>5850
0.08	≤ 4950	4950 – 5050	>5050
0.10	≤ 4450	4450 – 4550	>4550
0.12	≤ 4030	4030 – 4150	>4150
0.15	≤ 3650	3650 – 3700	>3700
0.175	≤ 3320	3320 - 3420	>3420
0.2	≤ 3110	3110 - 3200	>3200

If the imbalance mass can be limited by regular monitoring and correction, then the system can be highly reliable and safe to operate at higher speeds. Therefore it is suggested that this system needs periodic monitoring and balancing.

5. Conclusions

Imbalance in rotating systems results in huge dynamic forces at higher rotational speeds, leading to failure and catastrophic accidents. In this research work, a reliability model based on the experimental methodology has been presented and illustrated with an example. The model can be effectively used to predict the operational reliability of rotor systems with imbalance, by measuring their vibration amplitude. If the reliability target is specified, safe ranges of operational speed can be established for an existing or new system using this model. The model presented in this research work will have practical application in the areas of safety and reliability in the aviation sector, power generating equipment and could even be applied in surface transport and shipping where huge propellers and gear boxes are used. As future scope, new reliability models may be developed based on experimental methodology for the rotor system faults such as misalignment, and oil whirl.

Notation:

F_{im}	:	Force due to imbalance
m	:	Imbalance mass
r	:	Distance between imbalance mass and the axis of rotation
ω	:	Angular velocity of shaft
$\mathbf{M}$	:	Mass matrix
$\mathbf{C}$	:	Damping matrix
$\mathbf{K}$	:	Stiffness matrix
$\ddot{\mathbf{x}}_0(t)$	:	Acceleration vector of faultless rotor system
$\dot{\mathbf{x}}_o(t)$	:	Velocity vector of faultless rotor system
$\mathbf{x}_o(t)$	:	Displacement vector of faultless rotor system
$\mathbf{F}_o(t)$	:	Force vector of faultless rotor system
$\ddot{\mathbf{x}}(t)$	:	Acceleration vector of faulty rotor system
$\dot{\mathbf{x}}(t)$	:	Velocity vector of faulty rotor system
$\mathbf{x}(t)$	:	Displacement vector of faulty rotor system
$\Delta\mathbf{F}(t)$	:	Equivalent load vector
n	:	Measurable degrees of freedom
N	:	Degrees of freedom
$\Delta\mathbf{x}_n(t)$	:	Measured residual vector
$\tilde{\mathbf{T}}$	:	Transformation matrix
$\Delta\mathbf{x}(t)$	:	Residual displacement vector
$\Delta\dot{\mathbf{x}}(t)$	:	Residual velocity vector
$\Delta\ddot{\mathbf{x}}(t)$	:	Residual acceleration vector
$\hat{\boldsymbol{\Phi}}$	:	Reduced modal matrix
$\hat{\mathbf{X}}_1, \hat{\mathbf{X}}_2, \ldots \hat{\mathbf{X}}_n$	:	Mode shapes
$\Delta\mathbf{q}(t)$	:	Modal coordinate vector
R	:	Reliability
S_{ts}	:	Stress in the shaft
S_{TH}	:	Strength of the shaft
$f_{S_{ts}}(S_{ts})$	:	Probability density function of stress
$f_{S_{TH}}(S_{TH})$	:	Probability density function of strength
τ_{max}	:	Maximum shear stress
M_b		Bending moment
T	:	Torque
d	:	Cross sectional diameter of shaft
σ_{max}	:	Maximum bending stress
τ_{eff}	:	Effective shear stress
M_{im}	:	Bending moment due to imbalance force
σ_{eff}	:	Effective bending stress
$\mathcal{T}$	:	Ultimate shear strength
ς	:	Ultimate normal strength
$f_{\varsigma}(\varsigma)$	:	Density function of ultimate normal strength
$f_{\mathcal{T}}(\mathcal{T})$	:	Density function of ultimate shear strength
$f_{\tau_{eff}}(\tau_{eff})$	:	Density function of effective shear stress

$f_{\sigma_{eff}}(\sigma_{eff})$	:	Density function of effective normal stress
$\Phi(z)$	:	Standard cumulative normal distribution function
$\mu_{\mathcal{T}}$	:	Mean of ultimate shear strength
$\varrho_{\mathcal{T}}$	:	Standard distribution of ultimate shear strength
$\mu_{\tau_{eff}}$	:	Mean of effective shear strength
$\varrho_{\tau_{eff}}$	:	Standard distribution of effective shear strength
$[\,]^T$	:	Transpose

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Appendix I

Modeling of test setup:

The FEM formulation of Nelson and McVaugh [20] is used to model the rotor system to get the system matrices ($\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$). Here the equation of motion for the interconnected rotor, shaft and bearing has been presented; it is considered here that, four degrees of freedom, two translation motions in horizontal (W) and vertical (V) directions, and two rotational motions about horizontal (θ_W) and vertical (θ_V) axes, respectively at each node. The shaft is divided into nine elements, using the finite element method and using the Lagrangian formulation, the equation of motion for the shaft element presented by Nelson *et al* [20]. The Lagrangian equation of motion of the finite shaft element with the constant rotational speed is given by

$$(\mathbf{M}_T^e + \mathbf{M}_R^e)\ddot{\mathbf{x}}^e - \Omega\mathbf{G}^e\dot{\mathbf{x}}^e + (\mathbf{K}_B^e + \mathbf{K}_A^e)\mathbf{x}^e = \mathbf{F}^e \tag{A.1}$$

where $\{\mathbf{x}\}^T = [W \; V \; \theta_W \; \theta_V]$

The force vector $\{\mathbf{F}^e\}$ include mass imbalance, interconnection forces and other element external effects. In appendix the individual element matrices are listed.

The Lagrangian equation of motion of the rigid disk with four DOF as discussed for the shaft element, with the constant rotational speed (Ω) is given by Nelson *et al* [20]

$$(\mathbf{M}_T^d + \mathbf{M}_R^d)\ddot{\mathbf{x}}^d - \Omega\mathbf{G}^d\dot{\mathbf{x}}^d = \mathbf{F}^d \tag{A.2}$$

The vector $\mathbf{F}^d$ represents the force which includes the mass imbalance, interconnection forces and other external forces on the rotor. In appendix the individual rotor matrices are listed.

In this test set up the anti-friction bearings are used and the general equation of motion for the bearings is given by

$$\mathbf{C}^b\dot{\mathbf{x}}^b + \mathbf{K}^b\mathbf{x}^b = \mathbf{F}^b \tag{A.3}$$

where $\mathbf{x}^b = \begin{Bmatrix} W \\ V \end{Bmatrix}$, $\mathbf{K}^b = \begin{bmatrix} k_{WW}^b & k_{WV}^b \\ k_{VW}^b & k_{VV}^b \end{bmatrix}$, $\mathbf{C}^b = \begin{bmatrix} c_{WW}^b & c_{WV}^b \\ c_{VW}^b & c_{VV}^b \end{bmatrix}$

and $\{\mathbf{F}^b\}$ is the external force vector on the bearings given by [20].

For the isotropic bearings which are used in this present study represented by the following equation,

$$c[\mathbf{I}]\dot{\mathbf{x}}_R^b + k[\mathbf{I}]\mathbf{x}_R^b = \mathbf{F}'^b \tag{A.4}$$

Where c and k are the isotropic bearing damping and stiffness coefficient respectively, and $\mathbf{F}'^b$ and $\mathbf{x}_R^b$ are the external force vector relative to the rotational reference frame [20]. For further detailed study on the modeling of rotor bearing system based on finite elements [20] can be referred.

The final assembled equation of motion of the rotor system of the test rig will be given in the following equation as,

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F} \tag{A.5}$$

Where the $\mathbf{M}$ is the mass matrix and it consist of translational and rotor mass matrices of shaft elements and the rotor, $\mathbf{C}$ includes gyroscopic moments and damping of bearings, and $\mathbf{K}$ is the stiffness matrix and it includes shaft element stiffness and bearing stiffness. The force matrix $\mathbf{F}$ includes the weight of the disk, force due to imbalance at constant rotational speed, bearing force, and other external force if any. For the elemental matrices of the rotor system [8, 10, 20] can be referred.