Optimal Ordering Policy for non-instantaneous deteriorating items with partially backlogged and stock dependent demand under two-warehouse storage facility

Chandra K. Jaggi
Department of Operational Research,
Faculty of Mathematical Sciences,
New Academic Block,
University of Delhi, Delhi 110007, India
ckjaggi@yahoo.com

Abstract

This paper develops a two warehouse inventory model for non-instantaneous deteriorating items with partial backlogging and stock-dependent demand. In the model, shortages are allowed and the backlogging rate is variable and dependent on the waiting time for the next replenishment. The necessary and sufficient conditions of the existence and uniqueness of the optimal solution are shown. Numerical example is presented to demonstrate the developed model, followed by the sensitivity analysis of the optimal solution with respect to key parameters of the inventory system. The purpose of this study is to determine the optimal replenishment policy by minimizing the total optimal cost. The proposed model can assist a decision maker in making important replenishment decisions.

Keywords
Inventory; non-instantaneous deterioration; two-warehouse; partial backlogging; Stock-dependent demand

1. Introduction and Literature Review

Generally, deterioration is defined as damage, spoilage, decay, obsolescence, evaporation, pilferage, etc., that result in decreasing the usefulness of the original one. For items such as steel, hardware, glassware, and toys, the rate of deterioration is low, so there is little need for considering deterioration in the determination of the economic lot size. However, some items such as food items, pharmaceuticals, chemicals, blood, alcohol, gasoline, and radioactive chemicals, etc. deteriorate rapidly over time. Thus, the loss from deterioration should not be ignored. Ghare and Schrader [1] presented an EOQ model for deteriorating items assuming exponential decay. Covert and Phillip [2] extended Ghare and Schrader's [1] model with the assumption of Weibull distribution deterioration. Two surveys on trends in modeling of continuously deteriorating inventory are presented by Raafat et al [3], Goyal & Giri [4] and Bakker et al. [5]. In current scenario, two warehouse inventory models are followed to avoid frequent transportation inconvenience, to avail the advantage of price concession, trade credit and to guard against the scarcity of the commodity. Thus when large quantity of goods cannot be stored in own warehouse (OW), with limited capacity, rented warehouse (RW) is hired. In recent years good number of two warehouse inventory models has been discussed by several researchers. Chung and Huang [6], Liang and Zhou [7], Bhunia et al. [8] developed two warehouse inventory models for deteriorating items considering permissible delay in payments. Continuing the research ahead, a number of research papers in this interesting area have been published by several researches over the last few decades. One may refer to the recent works of Zhou and Yang [9], Lee [10], Yang [11, 12, 13], Hsieh et al. [14], Niu & Xie, [15], Lee & Hsu [16], Liao et al. [17], Jaggi et al. [18, 19, 20] and Tiwari et al. [21].
Incorporating the above facts, this paper develops a two warehouse inventory model for non-instantaneous deteriorating items with partial backlogging and stock-dependent demand. In the model, shortages are allowed and the backlogging rate is variable and dependent on the waiting time for the next replenishment. The necessary and sufficient conditions of the existence and uniqueness of the optimal solution are shown. Numerical example is presented to demonstrate the developed model, followed by the sensitivity analysis of the optimal solution with respect to key parameters of the inventory system. The purpose of this study is to determine the optimal

replenishment policy by minimizing the total optimal cost. The proposed model can assist a decision maker in making important replenishment decisions.

2. Assumption and Notations

The mathematical models of the two-warehouse inventory problems are based on the following assumptions:

1. The own warehouse (OW) has a fixed capacity of W units; the rented warehouse (RW) has unlimited capacity.
2. The inventory costs (including holding cost and deterioration cost) in the RW are higher than those in OW.
3. Replenishment rate is instantaneous.
4. Lead-time is negligible.
5. Demand rate $D(t)$ is a function of stock level *I(t)* at time *t* which is given by

   $D(t) = \begin{cases} a + bI(t) & ; I(t) > 0 \\ a & ; I(t) < 0 \end{cases}$, where *a, b* are positive constants.
6. The planning horizon of the inventory system is infinite.
7. Unsatisfied demand/shortages are allowed. Unsatisfied demand is partially backlogged and the fraction of shortages backlogged is a differentiable and decreasing function of time t, denoted by $g(t)$, where t is the waiting time up to the next replenishment. We have defined the partial backlogging rate $g(t) = e^{-\delta t}$, where δ is a positive constant.

In addition, the following notations are used throughout this paper.

Parameters	Descriptions
A	replenishment cost per order
c	purchasing cost per unit
W	capacity of the owned warehouse
Q	order quantity per cycle
Z	maximum inventory level per cycle.
H	holding cost per unit per unit time in OW.
F	holding cost per unit per unit time in RW, where $F > H$
s	the backlogging cost per unit per unit time, if shortage is backlogged.
c_1	unit opportunity cost due to lost sale, if the shortage is lost.
α	deterioration rate in OW, where $0 \le \alpha < 1$
β	deterioration rate in RW, where $0 \le \beta < 1$; $\beta < \alpha$.
t_d	time period during which no deterioration occurs.
t_r	time at which the inventory level reaches zero in RW.
t_w	time at which the inventory level reaches zero in OW.
T	the length of the replenishment cycle in year.
$I_0(t)$	inventory level in the OW at any time *t* where $0 \le t \le T$
$I_r(t)$	inventory level in the RW at any time *t* where $0 \le t \le T$
TC_i	present value of total relevant cost per unit time for case i=1, 2
$B(t)$	backlogged level at any time *t* where $t_w \le t \le T$
$L(t)$	number of lost sales at any time *t* where $t_w \le t \le T$

3. Model Formulation

This section formulates the replenishment problem of a two warehousing inventory model for a single non-instantaneous deteriorating item with complete backlogging. Initially, a lot size of *Q* units enters in the system. After meeting the backorders, *Z* units left in the inventory system, out of which W units are kept in OW and the remaining (*Z-W*) units are kept in the RW. As the deterioration of item is non-instantaneous, so initially, the units do not deteriorate for the time period (t_d) and after that the deterioration begins. Broadly, there can be two cases: when t_d (time during which no deterioration occurs) is less than t_r (time during which inventory in RW becomes zero) and when t_d is greater than t_r.

Case 1: When $t_d < t_r$

During the time interval $[0,\ t_d]$, there is no deterioration, the inventory in RW is depleted only due to demand whereas in OW inventory level remains the same. In the time interval $[t_d,\ t_r]$ the inventory level in RW is dropping to zero due to the combined effect of demand and deterioration and in OW inventory depleted due to deterioration only. Further, during the time interval $[t_r,\ t_w]$ depletion of inventory occurs in OW due to the combined effect of demand and deterioration and it reaches to zero at time t_w. Moreover, during the interval $[t_w,\ T]$, the demand is backlogged. The behavior of the model over the whole cycle $[0,\ T]$ has been graphically represented below in Figure 1.

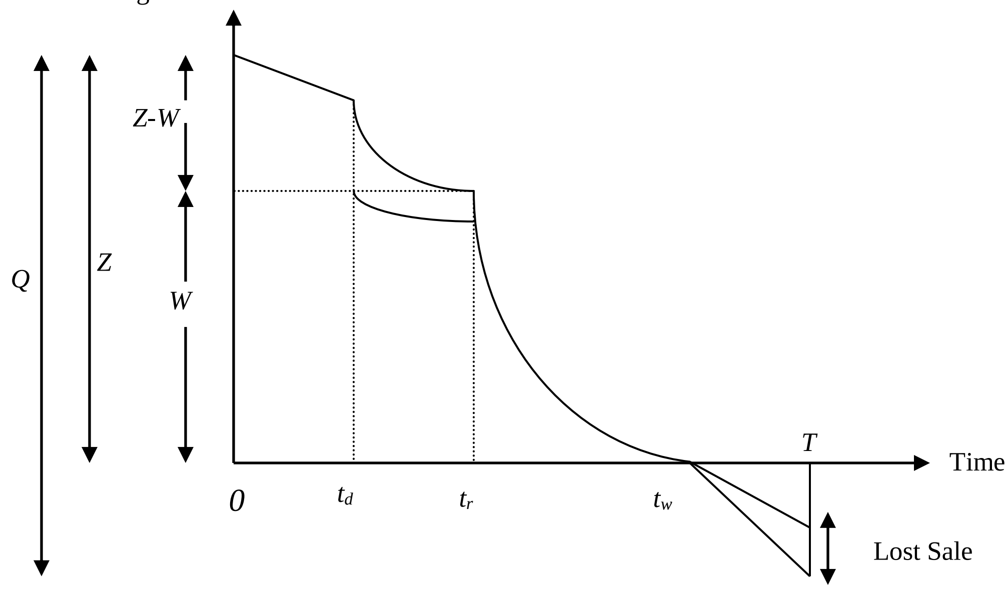


Figure 1: Two-warehouse inventory system when $t_d < t_r$

Therefore, the differential equations that describe the inventory level in the RW and OW at any time *t* over the period (*0*, *T*) are given by:

$$\frac{dI_r(t)}{dt} = -\left(a + bI_r(t)\right), \qquad 0 \le t \le t_d \tag{1}$$

$$\frac{dI_r(t)}{dt} + \beta I_r(t) = -\left(a + bI_r(t)\right), \qquad t_d < t \le t_r \tag{2}$$

$$\frac{dI_0(t)}{dt} + \alpha I_0(t) = 0, \qquad t_d < t \le t_r \tag{3}$$

$$\frac{dI_0(t)}{dt} + \alpha I_0(t) = -\left(a + bI_0(t)\right), \qquad t_r < t \le t_w \tag{4}$$

$$\frac{dB(t)}{dt} = ae^{-\delta(T-t)}, \qquad t_w < t \le T \tag{5}$$

The solutions of the above five differential equations (1), (2), (3), (4) and (5) with boundary conditions $I_r(0) = Z - W,\ I_r(t_r) = 0,\ I_0(t_d) = W,\ I_0(t_w) = 0\ \&\ B(t_w) = 0$ respectively are

$$I_r(t) = (Z - W)e^{-bt} + \frac{a}{b}\left(e^{-bt} - 1\right), \qquad 0 \le t \le t_d \tag{6}$$

$$I_r(t) = \frac{a}{\beta + b}\left(e^{(\beta+b)(t_r - t)} - 1\right), \qquad t_d < t \le t_r \tag{7}$$

$$I_0(t) = W e^{\alpha(t_d - t)}, \qquad t_d < t \le t_r \tag{8}$$

$$I_0(t) = \frac{a}{\alpha + b}\left(e^{(\alpha+b)(t_w - t)} - 1\right), \qquad t_r < t \le t_w \tag{9}$$

$$B(t) = \frac{a}{\delta}\left\{e^{-\delta(T-t)} - e^{-\delta(T-t_w)}\right\}, \qquad t_w < t \le T \tag{10}$$

The number of lost sales at time *t* is

$$L(t) = \int_{t_w}^{t} a\left\{1 - e^{-\delta(T-t)}\right\} dt; \qquad t_w < t \le T$$

$$= a\left[(t - t_w) - \frac{1}{\delta}\left\{e^{-\delta(T-t)} - e^{-\delta(T-t_w)}\right\}\right] \tag{11}$$

Considering continuity of $I_r(t)$ at $t = t_d$, it follows from Equations (6) and (7) that

$$(Z - W)e^{-bt_d} + \frac{a}{b}\left(e^{-bt_d} - 1\right) = \frac{a}{\beta + b}\left(e^{(\beta+b)(t_r - t_d)} - 1\right) \tag{12}$$

which implies that the maximum inventory level per cycle is

$$Z = W + \frac{a}{\beta + b}\left(e^{(\beta+b)(t_r - t_d)} - 1\right)e^{bt_d} - \frac{a}{b}\left(1 - e^{bt_d}\right) \tag{13}$$

Considering continuity of $I_0(t)$ at $t = t_r$ it follows from Equations (8) and (9) that

$$W e^{\alpha(t_d - t_r)} = \frac{a}{\alpha + b}\left(e^{(\alpha+b)(t_w - t_r)} - 1\right)$$

$$t_w = t_r + \frac{1}{\alpha + b}\ln\left(\frac{a + (\alpha + b)W e^{\alpha(t_d - t_r)}}{a}\right) \tag{14}$$

Putting $t = T$ in Equation (10), the maximum amount of demand backlogged per cycle is

$$B(T) = \frac{a}{\delta}\left(1 - e^{-\delta(T - t_w)}\right) \tag{15}$$

Therefore, the order quantity over the replenishment cycle can be determined as

$Q = Z + B(T)$ (Using Equations (13) and (15))

$$= W + \frac{a}{\beta + b}\left(e^{(\beta+b)(t_r - t_d)} - 1\right)e^{bt_d} - \frac{a}{b}\left(1 - e^{bt_d}\right) + \frac{a}{\delta}\left(1 - e^{-\delta(T - t_w)}\right) \tag{16}$$

The total cost per cycle consists of the following elements:

1. The replenishment cost is A
2. The inventory holding cost in RW

$$= F\left(\int_0^{t_d} I_r(t)\,dt + \int_{t_d}^{t_r} I_r(t)\,dt\right)$$

$$= F\left\{\frac{(Z - W)}{b}\left(1 - e^{-bt_d}\right) + \frac{a}{b^2}\left(1 - e^{-bt_d} - bt_d\right) + \frac{a}{(\beta + b)}\left\{\frac{e^{(\beta+b)(t_r - t_d)} - 1}{(\beta + b)} + (t_d - t_r)\right\}\right\}$$

3. The inventory holding cost in OW

$$= H\left(\int_0^{t_d} W\,dt + \int_{t_d}^{t_r} I_0(t)\,dt + \int_{t_r}^{t_w} I_0(t)\,dt\right)$$

$$= H\left\{W t_d + \frac{W}{\alpha}\left(1 - e^{\alpha(t_r - t_d)}\right) + \frac{a}{(\alpha + b)}\left\{\frac{e^{(\alpha+b)(t_w - t_r)} - 1}{(\alpha + b)} - (t_w - t_r)\right\}\right\}$$

4. The backlogging cost is $s\int_{t_w}^{T} B(t)\,dt$

$$= \frac{sa}{\delta}\left\{\frac{1}{\delta}\left(1-e^{-\delta(T-t_w)}\right)-(T-t_w)e^{-\delta(T-t_w)}\right\}$$

5. The opportunity cost due to lost sale is

$$= c_1\int_{t_w}^{T}\left\{1-e^{-\delta(T-t)}\right\}a\,dt$$

$$= c_1 a\left[T-t_w-\frac{1}{\delta}\left\{1-e^{-\delta(T-t_w)}\right\}\right]$$

6. The Deterioration cost is $= c\left[\beta\int_{t_d}^{t_r} I_r(t)dt+\alpha\int_{t_r}^{t_w} I_0(t)dt\right]$

$$= c\left\{\frac{a}{(\beta+b)}\left\{\frac{e^{(\beta+b)(t_r-t_d)}-1}{(\beta+b)}+(t_d-t_r)\right\}+\frac{a}{(\alpha+b)}\left\{\frac{e^{(\alpha+b)(t_w-t_r)}-1}{(\alpha+b)}-(t_w-t_r)\right\}\right\}$$

Hence, using the above elements, the total relevant cost per unit time is given by

$$\begin{aligned} TC_1(t_r,T) = \frac{1}{T}\Bigg[& A+F\left\{\frac{(Z-W)}{b}\left(1-e^{-bt_d}\right)+\frac{a}{b^2}\left(1-e^{-bt_d}-bt_d\right)+\frac{a}{(\beta+b)}\left\{\frac{e^{(\beta+b)(t_r-t_d)}-1}{(\beta+b)}+(t_d-t_r)\right\}\right\} \\ & +H\left\{Wt_d+\frac{W}{\alpha}\left(1-e^{\alpha(t_r-t_d)}\right)+\frac{a}{(\alpha+b)}\left\{\frac{e^{(\alpha+b)(t_w-t_r)}-1}{(\alpha+b)}-(t_w-t_r)\right\}\right\} \\ & +\frac{sa}{\delta}\left\{\frac{1}{\delta}\left(1-e^{-\delta(T-t_w)}\right)-(T-t_w)e^{-\delta(T-t_w)}\right\}+c_1 a\left\{T-t_w-\frac{1}{\delta}\left(1-e^{-\delta(T-t_w)}\right)\right\} \\ & +c\left\{\frac{a}{(\beta+b)}\left\{\frac{e^{(\beta+b)(t_r-t_d)}-1}{(\beta+b)}+(t_d-t_r)\right\}+\frac{a}{(\alpha+b)}\left\{\frac{e^{(\alpha+b)(t_w-t_r)}-1}{(\alpha+b)}-(t_w-t_r)\right\}\right\}\Bigg] \end{aligned} \tag{17}$$

where $Z=W+\frac{a}{\beta+b}\left(e^{(\beta+b)(t_r-t_d)}-e^{bt_d}\right)-\frac{a}{b}\left(1-e^{bt_d}\right)$ and $t_w=t_r+\frac{1}{\alpha+b}\ln\left(\frac{a+(\alpha+b)We^{\alpha(t_d-t_r)}}{a}\right)$

Case2. When $t_d > t_r$

In this case, time during which no deterioration occurs is greater than the time during which inventory in RW becomes zero and the behavior of the model over the whole cycle $[0, T]$ has been graphically represented as in Figure 2

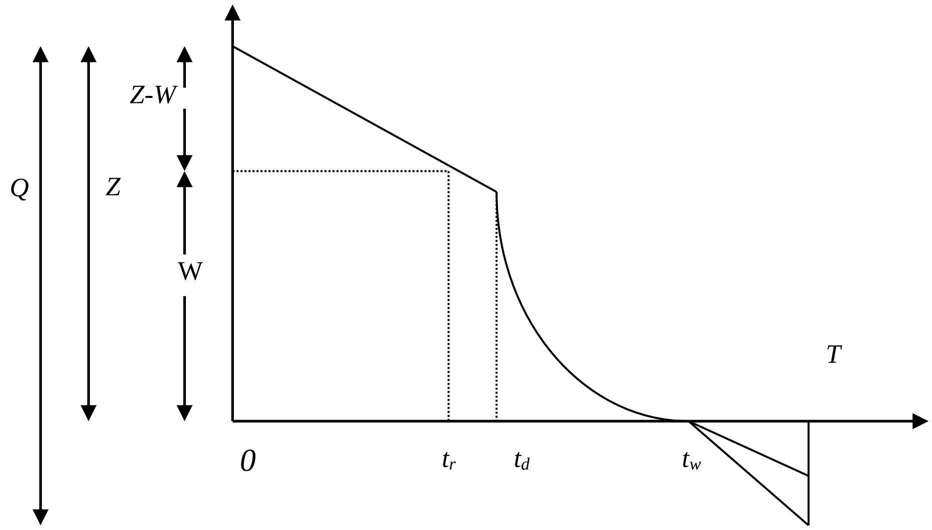


Figure 2: Two-warehouse inventory system when $t_d > t_r$

The differential equations that describe the inventory level in the RW and OW at any time t over the period (0, T) are given by:

$$\frac{dI_r(t)}{dt} = -\left(a + bI_r(t)\right), \qquad 0 \le t \le t_r \tag{18}$$

$$\frac{dI_0(t)}{dt} = -\left(a + bI_0(t)\right), \qquad t_r < t \le t_d \tag{19}$$

$$\frac{dI_0(t)}{dt} + \alpha I_0(t) = -\left(a + bI_0(t)\right), \qquad t_d < t \le t_w \tag{20}$$

$$\frac{dB(t)}{dt} = ae^{-\delta(T-t)}, \qquad t_w < t \le T \tag{21}$$

The solutions of the above four differential equations (18), (19), (20) and (21) with boundary conditions $I_r(t_r) = 0, I_0(t_r) = W, I_0(t_W) = 0, B(t_W) = 0$ respectively are

$$I_r(t) = \frac{a}{b}\left(e^{b(t_r-t)} - 1\right), \qquad 0 \le t \le t_r \tag{22}$$

$$I_0(t) = \left(W + \frac{a}{b}\right)e^{b(t_r-t)} - \frac{a}{b}, \qquad t_r < t \le t_d \tag{23}$$

$$I_0(t) = \frac{a}{\alpha + b}\left(e^{(\alpha+b)(t_w-t)} - 1\right), \qquad t_d < t \le t_w \tag{24}$$

$$B(t) = \frac{a}{\delta}\left\{e^{-\delta(T-t)} - e^{-\delta(T-t_w)}\right\}; \qquad t_w < t \le T \tag{25}$$

The number of lost sales at time t is

$$L(t) = \int_{t_w}^{t} a\left\{1 - e^{-\delta(T-t)}\right\}dt; \qquad t_w < t \le T$$

$$= a\left[(t - t_w) - \frac{1}{\delta}\left\{e^{-\delta(T-t)} - e^{-\delta(T-t_w)}\right\}\right] \tag{26}$$

Considering continuity of $I_0(t)$ at $t = t_d$, it follows from Equations (23) and (24) that

$$\left(W + \frac{a}{b}\right)e^{b(t_r-t_d)} - \frac{a}{b} = \frac{a}{\alpha + b}\left(e^{(\alpha+b)(t_w-t_d)} - 1\right) \tag{27}$$

$$t_w = t_d + \frac{1}{\alpha + b} \ln \left| 1 + \frac{\alpha + b}{a} \left\{ \left(W + \frac{a}{b} \right) e^{b(t_r - t_d)} - \frac{a}{b} \right\} \right| \tag{28}$$

Now, at $t = 0$ when $I_r(t) = Z\text{-}W$ and solving Equation (22) we get the maximum inventory as

$$Z = W + \frac{a}{b}\left(e^{b t_r} - 1\right) \tag{29}$$

Putting $t = T$ in Equation (25), the maximum amount of demand backlogged per cycle is

$$B(T) = \frac{a}{\delta}\left(1 - e^{-\delta(T - t_w)}\right) \tag{30}$$

Therefore, order quantity is $Q = Z + B(T)$

$$= W + \frac{a}{b}\left(e^{b t_r} - 1\right) + \frac{a}{\delta}\left(1 - e^{-\delta(T - t_w)}\right) \qquad \text{(Using Equations (29) and (30))} \tag{31}$$

Again, the total cost per cycle consists of the following elements:

1. The replenishment cost = A

2. The inventory holding cost in RW = $F \int_0^{t_r} I_r(t)\, dt$

$$= \frac{Fa}{b}\left\{ \frac{e^{b t_r} - 1}{b} - t_r \right\}$$

3. The inventory holding cost in OW

$$= H \left(\int_0^{t_r} W\, dt + \int_{t_r}^{t_d} I_0(t)\, dt + \int_{t_d}^{t_w} I_0(t)\, dt \right)$$

$$= H \left\{ W t_r + \left(W + \frac{a}{b} \right)\left(\frac{1 - e^{b(t_r - t_d)}}{b} \right) - \frac{a}{b}\left(t_d - t_r\right) + \frac{a}{(\alpha + b)}\left(\frac{e^{(\alpha + b)t_w}}{(\alpha + b)}\left(e^{-t_d(\alpha + b)} - e^{-t_r(\alpha + b)} \right) - \left(t_r - t_d\right) \right) \right\}$$

4. The backlogging cost = $s \int_{t_w}^{T} B(t)\, dt$

$$= \frac{sa}{\delta}\left\{ \frac{1}{\delta}\left(1 - e^{-\delta(T - t_w)}\right) - \left(T - t_w\right) e^{-\delta(T - t_w)} \right\}$$

5. The opportunity cost due to lost sale $= c_1 \int_{t_w}^{T} \left\{1 - e^{-\delta(T - t)}\right\} D\, dt$

$$= c_1 a \left[T - t_w - \frac{1}{\delta}\left\{1 - e^{-\delta(T - t_w)}\right\} \right]$$

6. The cost for deteriorated items $= c\alpha \int_{t_d}^{t_w} I_0(t)\, dt$

$$= c\, \frac{a}{(\alpha + b)}\left(\frac{e^{(\alpha + b)t_w}}{(\alpha + b)}\left(e^{-t_d(\alpha + b)} - e^{-t_r(\alpha + b)} \right) - \left(t_r - t_d\right) \right)$$

Hence, the total relevant cost per unit time during the cycle $(0, T)$ is given by

$$
\begin{aligned}
TC_2(t_r,T) = \frac{1}{T}\Bigg[& A + \frac{Fa}{b}\left\{\frac{e^{bt_r}-1}{b} - t_r\right\} + H\Bigg\{ W t_r + \left(W + \frac{a}{b}\right)\left(\frac{1-e^{b(t_r-t_d)}}{b}\right) - \frac{a}{b}(t_d - t_r) \\
& + \frac{a}{(\alpha+b)}\left(\frac{e^{(\alpha+b)t_w}}{(\alpha+b)}\left(e^{-t_d(\alpha+b)} - e^{-t_r(\alpha+b)}\right) - (t_r - t_d)\right)\Bigg\} \\
& + \frac{sa}{\delta}\left\{\frac{1}{\delta}\left(1-e^{-\delta(T-t_w)}\right) - (T-t_w)\,e^{-\delta(T-t_w)}\right\} + c_1 a\left\{T - t_w - \frac{1}{\delta}\left\{1-e^{-\delta(T-t_w)}\right\}\right\} \\
& + c\,\frac{a}{(\alpha+b)}\left(\frac{e^{(\alpha+b)t_w}}{(\alpha+b)}\left(e^{-t_d(\alpha+b)} - e^{-t_r(\alpha+b)}\right) - (t_r - t_d)\right)\Bigg]
\end{aligned} \tag{32}
$$

Where $Z = W + \frac{a}{b}\left(e^{bt_r} - 1\right)$ and $t_w = t_d + \frac{1}{\alpha+b}\ln\left|1 + \frac{\alpha+b}{a}\left\{\left(W + \frac{a}{b}\right)e^{b(t_r - t_d)} - \frac{a}{b}\right\}\right|$

Therefore, the present worth of the total relevant cost per unit time during the cycle (*0*, *T*) is given by

$$
TC(t_r, T) = \begin{cases} TC_1(t_r,T) & \text{if} \quad t_d \le t_r \\ TC_2(t_r,T) & \text{if} \quad t_d \ge t_r \end{cases} \tag{33}
$$

which is a function of two continuous variable t_r and T.

Optimality:

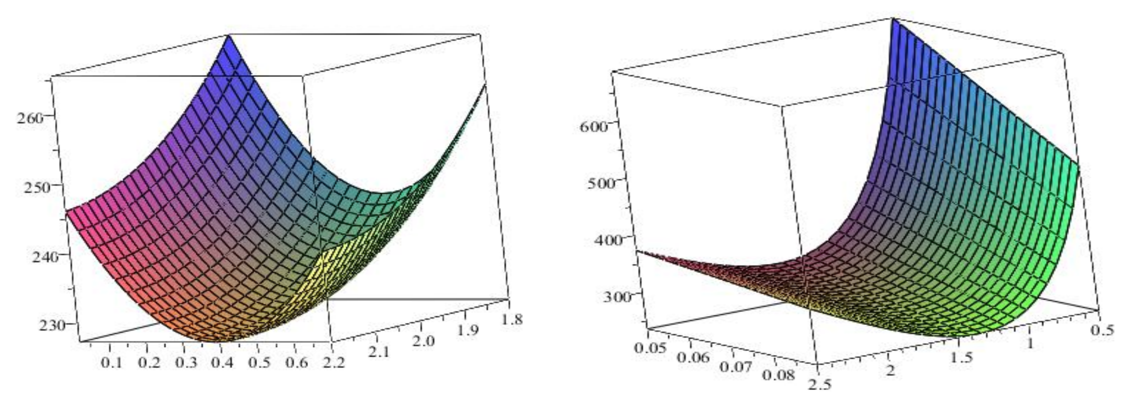
Our problem is to determine the optimum value of t_r and T which minimizes $TC(t_r, T)$. The necessary conditions for minimization of the total cost function given by Equation (33) are

$$
\frac{\partial TC_i(t_r,T)}{\partial t_r} = 0, \text{ and } \frac{\partial TC_i(t_r,T)}{\partial T} = 0 \quad \text{for } i = 1,2 \text{ which gives} \tag{34}
$$

Equation (34) can be solved simultaneously for the optimal values of t_{ri} and T_i (say t_{ri}^* and T_i^*) for $i=1, 2$ provided, it also satisfies the following sufficient conditions.

$$
\frac{\partial^2 TC_i(t_r,T)}{\partial t_r^2} > 0, \frac{\partial^2 TC_i(t_r,T)}{\partial T^2} > 0 \text{ and } \left[\left(\frac{\partial^2 TC_i}{\partial t_r^2} \times \frac{\partial^2 TC_i}{\partial T^2}\right) - \left(\frac{\partial^2 TC_i}{\partial T\,\partial t_r} \times \frac{\partial^2 TC_i}{\partial t_r \partial T}\right)\right] > 0 \text{ for } i = 1,2
$$

Mathematically, it is very difficult to prove the sufficient conditions, so convexities of cost function for both cases are shown graphically in figure 3



Case 1: When $t_d < t_r$ **Case2: When** $t_d > t_r$

Figure 3: Convexity of cost function w.r.t. t_r and T

4. Numerical and Sensitivity analysis

Example: To illustrate the results, let us consider an inventory system with the following data: $A = \$250/order$, $s = \$10/unit/year$, $c = \$20/unit$, $c_1 = \$5/unit$, $H = \$0.5/unit/year$, $F = \$0.7/unit/year$, $W = 200units$, $a = 80$, $b = 10$, $\alpha = 0.05/unit$, $\beta = 0.03/unit$, $R = 0.06$, $\delta = 0.9$, $t_d = 0.1year$.

The optimal solutions for given example is shown in table 1 of our proposed inventory model

Table 1: Solution of Example

Case	t_r	t_w	T	Z^*	B^*	Q^*	$TC^*(t_r, T)$
1	0.1	0.4247	2.5406	331.32	75.65	406.97	469.07
2	0.10	0.4247	2.5597	213.74	76.87	290.61	467.32

5. Conclusion and Future Scope

In this study, a two-warehouse inventory model for non-instantaneous deteriorating items with stock-dependent demand has been developed. Shortages are allowed and partially backlogged. The deterioration rate in OW is assumed to be higher than in RW, and the holding cost in RW is greater than that in OW because of the better storage facilities provided in RW. The items stored have the property of non-instantaneous deterioration. A mathematical model has been developed to determine the optimal ordering policy which minimizes the present worth of total optimal cost. The study concludes with numerical examples. For future research, the present model can be formulated by considering permissible delay in payments, and two-level trade credit policy also can be incorporated. To be more realistic, the model can be extended with different types of variable demand such as stock and price-dependent demand, credit linked demand and advertisement dependent demand, etc.

Acknowledgment

The fauthor acknowledges the support of the University Grants Commission through University of Delhi (Research Grant No. RC/R&D/2014/6820).

References

[1] Ghare, P. M., & Schrader, G. F. (1963). A model for exponentially decaying inventory. *Journal of Industrial Engineering*, *14*(5), 238-243.

[2] Covert, R.P. & Philip, G.C., (1973). An EOQ model for items with Weibull distribution deterioration. *American Institute of Industrial Engineering Transactions*, 5(4), 323–326.

[3] Raafat, F. (1991). Survey of literature on continuously deteriorating inventory models. *Journal of Operational Research Society*, 42(1), 27–37.

[4] Goyal S.K. & Giri, B.C. (2001). Recent trends in modelling of deteriorating inventory. *European Journal of Operational Research*, 134(1), 1–16.

[5] Bakker, M., Riezebos, J., & Teunter, R. H., (2012). Review of inventory systems with deterioration since 2001. *European Journal of Operational Research*, 221(2), 275–284.

[6] Chung, K. J., & Huang, T. S. (2007). The optimal retailer's ordering policies for deteriorating items with limited storage capacity under trade credit financing. *International Journal of Production Economics*, 106(1), 127-145.

[7] Liang, Y., & Zhou, F. (2011). A two-warehouse inventory model for deteriorating items under conditionally permissible delay in payment. *Applied Mathematical Modelling*, 35, 2221–2231.

[8] Bhunia, A. K., Jaggi, C. K., Sharma, A., & Sharma, R. (2014). A two-warehouse inventory model for deteriorating items under permissible delay in payment with partial backlogging. *Applied Mathematics and Computation*, 232, 1125-1137. .

[9] Zhou, Y.W. & Yang S.L. (2005). A two-warehouse inventory model for items with stock- level dependent demand rate. *International Journal of Production Economics*, 95(2), 215-228.

[10] Lee, C. C. (2006). Two-warehouse inventory model with deterioration under FIFO dispatching policy. *European Journal of Operational Research*, 174(2), 861-873.

[11] Yang, H.L., (2004). Two-warehouse inventory models for deteriorating items with shortages under inflation. *European Journal of Operational Research*, 157(1), 344-356.

[12] Yang, H.L. (2006). Two-warehouse partial backlogging inventory models for deteriorating items under inflation. *International Journal of Production Economics*, 103(1), 362-370.

[13] Yang, H.L. (2012). Two-warehouse partial backlogging inventory models with three parameters Weibull distribution deterioration under inflation. *International Journal of Production Economics*, 138(1), 107-116.
[14] Hsieh, T. P., Dye, C. Y., & Ouyang, L. Y. (2008). Determining optimal lot size for a two-warehouse system with deterioration and shortages using net present value. *European Journal of Operational Research*, *191*(1), 182-192.
[15] Niu, B. & Xie, J. (2008). A note on Two-warehouse inventory model with deterioration under FIFO dispatches policy. *European Journal of Operational Research*, 190(2), 571-577.
[16] Lee, C. C., & Hsu, S. L. (2009). A two-warehouse production model for deteriorating inventory items with time-dependent demands. *European Journal of Operational Research*, *194*(3), 700-710.
[17] Liao, J. J., Huang, K. N., & Chung, K. J. (2012). Lot-sizing decisions for deteriorating items with two warehouses under an order-size-dependent trade credit. *International Journal of Production Economics*, *137*(1), 102-115.
[18] Jaggi, C. K., Tiwari, S., & Shafi, A. (2015). Effect of deterioration on two-warehouse inventory model with imperfect quality. *Computers & Industrial Engineering*, *88*, 378-385.
[19] Jaggi, C. K., Tiwari, S., & Goel, S. K. (2016). Credit financing in economic ordering policies for non-instantaneous deteriorating items with price dependent demand and two storage facilities. *Annals of Operations Research*, 1-28.
[20] Jaggi, C. K., Tiwari, S., & Goel, S. K. (2016). Replenishment policy for non-instantaneous deteriorating items in a two storage facilities under inflationary conditions.*International Journal of Industrial Engineering Computations*, *7 (3), 489-506.*
[21] Tiwari, S., Cárdenas-Barrón, L. E., Khanna, A., & Jaggi, C. K. (2016). Impact of trade credit and inflation on retailer's ordering policies for non-instantaneous deteriorating items in a two-warehouse environment. *International Journal of Production Economics*, *176*, 154-169.

Biography

Chandra K Jaggi, Professor, Department of Operational Research, University of Delhi, India. He is Fellow Member of International Science Congress Association since 2012 and in 2009 was awarded Certificate for his Significant Contributions in Operation Management by The Society of Reliability Engineering, Quality and Operations Management, New Delhi. He was awarded Shiksha Rattan Puraskar (for Meritorious Services, Outstanding Performance and Remarkable Role) in 2007 by India International Friendship Society. His research interest lies in the field of Supply Chain and Inventory Management. He has publications papers in various international/national journals including Journal of Operational Research Society, U.K., European Journal of Operational Research, International Journal of Production Economics, International Journal of Systems Sciences, Applied Mathematics and Computation, Applied Mathematical Modelling, Annals of Operations Research, Computers and Industrial Engineering, Applied Mathematics & Information Sciences and Scientia Iranica, Transaction E: Industrial Engineering. etc. He is a reviewer of more than 25 International/National journals. He is Editor-in-Chief of IJICM and Associate Editor of IJSAEM, Springer, Co-Editor / Reviewer-In-Charge of The Gstf JMSOR and on the Editorial Board of the IJSS: Operations & Logistics, IJSOI, AJOR, IJECBS, JASR, AJBAS. He has travelled extensively in India and abroad and delivered Key note and invited talks.