

# The APT with Lagged, Value-at-Risk and Asset Allocations by Using Econometric Approach

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**Abstract**— In this paper analyzed the APT with lagged, Value-at-Risk and asset allocations by using economic approach. It is assumed that stocks were analyzed following the model of Arbitrage Pricing Theory (APT) with lagged. Where the factor risk premium in the past affects the present changes in stock return. The return of factor is assumed to have non constant volatility and there is effect of long memory. Long memory effects are estimated using the rescaled range method (R/S) or Geweke and Porter-Hudak (GPH) method. The mean and non constant volatility is estimated using ARFIMA-GARCH models. The portfolio risk level is measured by the Value-at-Risk (VaR). Asset allocation problem solving carried out using the Lagrangian multiplier technique and the Kuhn-Tucker method. The purpose of this research forms the efficient portfolio and determines optimal portfolio weights. As a numerical illustration, we analyze some stocks traded on the capital market in Indonesia.

**Keywords**— APT, ARFIMA, GARCH, Value-at-Risk, Asset Allocation.

## I. INTRODUCTION

Formulation of Arbitrage Pricing Theory (APT) has important implications in determining stock prices [3]. It's stated that the return of a stock (or portfolio) will be affected by one or several explanatory variables (factor index). However, APT does not mention (explicitly) what variables affect the stock return. To determine the factors that influence the degree of sensitivity and magnitude of asset returns to each factor, is to set a number of factors that allegedly had an influence on stock return [7; 11]. These factors include the industrial variables (egg market index, alternative products, etc.) and economic variables (egg inflation, interest rates, etc.) [9; 17].

APT applies the law of one price, in equilibrium, the relationship between risk and stock return occurs in one area (if there is more than one factor). This situation can be achieved through a process of arbitrage. Arbitrators will cause all the portfolio is located in one and the same area [19; 11]. The location of each portfolio will be determined by the proportion (weight) of funds invested in the establishment of a portfolio [9; 3]. Determination of the proportion (weight) portfolio is a problem that should be sought the solution. To determine the proportion (weight) can be conducted using a portfolio optimization [10; 13]. The

composition obtained proportions will affect the return expectations and risk portfolio [17; 15]. Highly popular portfolio risk is measured using a Value-at-Risk (VaR) [1; 4; 5].

In this paper, the formulation of APT as a means of determining the stock price will be expanded by considering the factors of lagged. Where the factors of the last period is assumed influence the formulation of APT. The return of factors in APT is assumed to have non constant volatility and there is an effect of long memory. Non constant volatility and long memory effect will be analyzed using ARFIMA-GARCH models [22]. The mean and variance of returns of stocks estimated based on the APT with the lagged, which has non constant volatility and long memory effects. Using the mean estimator and the variance will be arranged the problem of Asset allocation. Asset allocation is based on the mean-VaR approach [18]. The settlement of asset allocation problem is based on Lagrangian multiplier technique and the Kuhn-Tucker methods [17]. Thus the analysis could be performed, because many stocks have characteristics such as the discussion here. The aim is to establish an efficient portfolio and determine the proportion (weight) portfolio optimally. The empirical research carried out on a few stocks that are traded at the capital markets in Indonesia.

## II. METHODOLOGIES

**Determining stocks return.** Let  $P_{it}$  and  $r_{it}$  denote the prices and the returns of stock  $i$  ( $i = 1, \dots, N$  and  $N$  is the number of stocks that are analyzed), respectively, at the time  $t$  ( $t = 1, \dots, T$ ,  $T$  denotes the period of data observation). Stocks return  $r_{it}$  is calculated using the formula  $r_{it} = \ln(P_{it} / P_{it-1})$ . Let  $F_{jt}$  and  $\hat{r}_{jt}$  respectively denote the price and the return of factor index  $j$  ( $j = 1, \dots, M$  and  $M$  is the number of factors index in the APT), at the time  $t$ ,  $t = 1, \dots, T$ . In the same way to calculate  $r_{it}$ , the factors index return  $\hat{r}_{jt}$  are calculated by  $\hat{r}_{jt} = \ln(F_{jt} / F_{jt-1})$  [22; 8].

### A. Mean Modeling

In the next stage we identify the existence of long memory effect in the data return of factor index using the rescale range method (R/S) or Geweke and Porter-Hudak (GPH) method. The parameter estimation of fractional difference index  $d_j$ ,  $j = 1, \dots, M$ , is performed using the maximum likelihood method [22; 21]. The confidence interval  $(1 - c)100\%$  for  $d_j$  is  $\hat{d}_j - z_{c/2} \cdot \sigma_{d_j} < d_j < \hat{d}_j + z_{c/2} \cdot \sigma_{d_j}$  where  $\hat{d}_j$  denotes estimator of  $d_j$ , and  $z_c$  denotes the percentile of standard normal distribution at the significance level  $c$ . Let  $\mu_{d_j}$  and  $\sigma_{d_j}$  respectively denote the mean and standard deviation of  $d_j$ . We can test the null hypothesis  $H_0 : \hat{d}_j = 0$  against  $H_1 : \hat{d}_j \neq 0$  using statistic  $z_{d_j} = (d_j - \mu_{d_j}) / \sigma_{d_j}$ . We reject  $H_0$  if the value  $z_{d_j} < z_{c/2}$  or  $z_{d_j} > z_{1-c/2}$  [14; 16].

Fractional difference process is defined as:

$$(1 - B)^{d_j} \hat{r}_{jt} = a_{jt}, -0.5 < d_j < 0.5; \quad (1)$$

where  $\{a_{jt}\}$  is the error component which is the white noise process, and  $B$  denotes the backshift operator? If the sequence of fractional difference  $(1 - B)^{d_j} \hat{r}_{jt}$  is following the model of ARMA( $p, q$ ), then we call  $\hat{r}_{jt}$  *autoregressive fractionally integrated moving average degree  $p$ ,  $d$  and  $q$  process*, or ARFIMA( $p, d, q$ ) [22; 16]. The ARMA( $p, q$ ) follows the following form

$$r_{jt} = \psi_{j0} + \sum_{g=1}^p \psi_{jg} r_{jt-g} + a_{jt} + \sum_{h=1}^q \theta_{jh} a_{jt-h}, \quad (2)$$

with  $\psi_{j0}$  constant and  $\psi_{jg}$  ( $g = 1, \dots, p$ ) and  $\theta_{jh}$  ( $h = 1, \dots, q$ ) the parameter coefficients of mean model of factors index return  $j$ ,  $j = 1, \dots, M$ . We assume that  $\{a_{jt}\}$  is the error sequence of white noise process with mean zero and variance  $\sigma_{a_j}^2$  [20; 22].

Stages of mean modeling process include: (i) Identification of the model, (ii) parameters estimation, (iii) diagnostic tests, and (iv) Prediction [22].

### B. Non Constant Volatility Modeling

The non constant volatility of the returns of factor index is modeled using *generalized autoregressive conditional heteroscedastic* (GARCH) models. Suppose  $\mu_{jt}$  and  $\sigma_{jt}^2$  respectively denote the mean and non constant volatility of return of factor index  $j$  ( $j = 1, \dots, M$  and  $M$  denotes the number of factors index in the APT), at the time  $t$  ( $t = 1, \dots, T$  and  $T$  is the period of data observation). The error  $a_{jt}$  can be calculated as  $a_{jt} = r_{jt} - \mu_{jt}$  [22; 23]. The non constant volatility  $\sigma_{jt}^2$  will follow the GARCH model of degree  $m$  and  $n$  or GARCH( $m, n$ ), if

$$a_{jt} = \sigma_{jt} \varepsilon_{jt}, \sigma_{jt}^2 = \alpha_{j0} + \sum_{k=1}^m \alpha_{jk} a_{jt-k}^2 + \sum_{l=1}^n \beta_{jl} \sigma_{jt-l}^2 + \varepsilon_{jt} \quad (3)$$

where  $\alpha_{j0}$  is a constant and  $\alpha_{jk}$  ( $k = 1, \dots, m$ ) and  $\beta_{jl}$  ( $l = 1, \dots, n$ ) denote the parameter coefficients of non constant volatility model of factor index return  $j$  ( $j = 1, \dots, M$ ). Here we assume  $\{\varepsilon_{jt}\}$  is the sequence independent and identically distribution (iid) random variable with mean zero and variance 1,  $\alpha_{j0} > 0$ ,  $\alpha_{jk} \geq 0$ ,  $\beta_{jl} \geq 0$ , and  $\sum_{k=1}^{\max(m,n)} (\alpha_{jk} + \beta_{jk}) < 1$  [20; 22].

The stages of non constant volatility modeling include: (i) The estimation of mean model, (ii) Testing the effect of ARCH, (iii) Model identification, (iv) Non constant volatility model estimation, (v) Diagnostic test, and (vi) Prediction.

We further use the mean model (2) and the non constant volatility model (3), to calculate  $\hat{\mu}_{jt} = \hat{r}_{jT}$  (1)  $\hat{\sigma}_{jt}^2 = \hat{\sigma}_{jT}^2$  (1), i.e. the 1-step ahead prediction after time period  $T$  of the mean and the variance [22].

### C. Modeling of Stock Return under APT with Lagged

In this section expand the APT to APT model with lagged. It is known that  $r_{it}$  the return of stock  $i$  at the time  $t$ , and  $\hat{r}_{jt}$  returns the index factor  $j$  at the time  $t$ . Suppose  $\tilde{r}_t$  is the risk free asset return at the time  $t$  ( $t = 1, \dots, T$  and  $T$  the period of data observation). APT regression model with lagged expressed as equation

$$\begin{aligned} r_{it} - \tilde{r}_t = & \omega_{i0} + \omega_{i10} (\hat{r}_{1t} - \tilde{r}_t) + \omega_{i11} (\hat{r}_{1t-1} - \tilde{r}_{t-1}) + \omega_{i12} (\hat{r}_{1t-2} - \tilde{r}_{t-2}) + \dots + \omega_{i1L} (\hat{r}_{1t-L} - \tilde{r}_{t-L}) + \dots \\ & + \omega_{i20} (\hat{r}_{2t} - \tilde{r}_t) + \omega_{i21} (\hat{r}_{2t-1} - \tilde{r}_{t-1}) + \omega_{i22} (\hat{r}_{2t-2} - \tilde{r}_{t-2}) + \dots + \omega_{i2L} (\hat{r}_{2t-L} - \tilde{r}_{t-L}) + \dots \\ & + \omega_{iM0} (\hat{r}_{Mt} - \tilde{r}_t) + \omega_{iM1} (\hat{r}_{Mt-1} - \tilde{r}_{t-1}) + \omega_{iM2} (\hat{r}_{Mt-2} - \tilde{r}_{t-2}) + \dots + \omega_{iML} (\hat{r}_{Mt-L} - \tilde{r}_{t-L}) + u_{it}, \end{aligned}$$

or it can be written into

$$r_{it} - \tilde{r}_t = \omega_{i0} + \sum_{j=1}^M \sum_{\zeta=0}^L \omega_{ij\zeta} (\hat{r}_{jt-\zeta} - \tilde{r}_{t-\zeta}) + u_{it} \quad (4)$$

Assumed that  $\{u_{it}\}$  is the white noise of regressions residual [7; 11; 19]. Where  $\omega_{i0}$  and  $\omega_{ij\zeta}$  ( $i = 1, \dots, N$ ;  $j = 1, \dots, M$ ;  $\zeta = 0, \dots, L$  and  $L$  is length of lagged), respectively declare constants and parameter coefficients of regression for the APT with lagged of stock return  $i$  at the time  $t$ . To estimate the constant  $\omega_{i0}$  and parameter coefficients  $\omega_{ij\zeta}$  regression of equation (4) can be performed using the least squares method. Length of lagged estimated based on the Ad-Hoc method, namely by looking at the consistency changes of parameter coefficients sign, positive (+) continue or the negative (-) continued, when lagged extended. Referring Blume (1971), the ability of the index factors explain changes in individual stock returns ranging between 25%-51% rate coefficient of determination [9].

As previously described,  $\hat{\mu}_{jt}$  and  $\hat{\sigma}_{jt}^2$  successively states the mean and variance of the index return factor  $j$  at the time  $t$ . Suppose that  $\tilde{\mu}_t$  and  $\tilde{\sigma}_t^2$ , in succession states mean and variance of return risk-free asset. Based on the equation (4), the mean stock return  $i$  at the time  $t$ , which  $\mu_{it}$  can be estimated using the following equation:

$$\mu_{it} = E(r_{it}) = \tilde{\mu}_t + \omega_{i0} + \sum_{j=1}^M \sum_{\zeta=0}^L \omega_{ij\zeta} (\tilde{\mu}_{jt-\zeta} - \tilde{\mu}_{t-\zeta}). \quad (5)$$

It is assumed that  $E[(\tilde{r}_{jt-\zeta} - \tilde{r}_{t-\zeta}), (\tilde{r}_{j't-\zeta'} - \tilde{r}_{t-\zeta'})] = 0$ , where  $j, j' = 1, \dots, M$ ,  $j \neq j'$  and  $\zeta, \zeta' = 0, \dots, L$ ,  $\zeta \neq \zeta'$ . Stock return variance  $i$  at the time  $t$ , that  $\sigma_{it}^2$  can be estimated using the equation

$$\sigma_{it}^2 = Var(r_{it}) = \tilde{\sigma}_t^2 + \sum_{j=1}^M \sum_{\zeta=0}^L \omega_{ij\zeta}^2 (\hat{\sigma}_{jt-\zeta}^2 - \tilde{\sigma}_{t-\zeta}^2) + \sigma_{u_{it}}^2. \quad (6)$$

Where  $\sigma_{u_{it}}^2 = Var(u_{it})$  the regressions residual variance of APT with is lagged of stock return  $i$  at the time  $t$ . Based on the assumptions in equation (6), covariance between stock  $i$  with stock  $i'$ , which are stated to  $\sigma_{ii'}$  be estimated with equation

$$\sigma_{ii'} = Cov(r_{it}, r_{i't}) = \sum_{j=1}^M \sum_{\zeta=0}^L \omega_{ij\zeta} \omega_{i'j\zeta} (\hat{\sigma}_{jt-\zeta}^2 - \tilde{\sigma}_{t-\zeta}^2); i \neq i'. \quad (7)$$

Estimator mean, variance and covariance of stock return  $i$  ( $i = 1, \dots, N$  and  $N$  the number of stock that were analyzed), at the time  $t$  ( $t = 1, \dots, T$  and  $T$  the period of data observation), then used for the following portfolio formation.

#### D. Asset Allocation Based on Mean-VaR

Let  $\ddot{r}_t$  denote the return of portfolio at the time  $t$ , and  $w_i$  ( $i = 1, \dots, N$ ) weight of stock  $i$ . Return of portfolio  $\ddot{r}_t$  can be determined using the equation [9; 17]:

$$\ddot{r}_t = \sum_{i=1}^N w_i r_{it}; \text{ Terms } \sum_{i=1}^N w_i = 1 \text{ and } 0 < w_i < 1 \text{ (} i = 1, \dots, N \text{)}. \quad (9)$$

Suppose  $\boldsymbol{\mu}^T = (\mu_{1t} \dots \mu_{Nt})$ ,  $i = 1, \dots, N$  is the mean vector, and  $\mathbf{w}^T = (w_1 \dots w_N)$  the weight vector of portfolio. From equation (9), the weight vector  $\mathbf{w}^T$  follows the property  $\mathbf{e}^T \mathbf{w} = 1$ , where  $\mathbf{e} = (1 \dots 1)^T$ . The mean of portfolio return  $\ddot{\mu}_t$  can be estimated using the equation:

$$\ddot{\mu}_t = \sum_{i=1}^N w_i \mu_{it} = \boldsymbol{\mu}^T \mathbf{w}. \quad (10)$$

The variance of portfolio return  $\ddot{\sigma}_t^2$  can be estimated using the equation:

$$\ddot{\sigma}_t^2 = \sum_{i=1}^N w_i^2 \sigma_{it}^2 + \sum_{i=1}^N \sum_{i'=1}^N w_i w_{i'} \sigma_{ii'} = \mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w}; i \neq i'. \quad (11)$$

where  $\sigma_{ii'} = Cov(r_{it}, r_{i't})$  denotes the covariance between stock  $i$  and stock  $i'$  [17].

Value-at-Risk (VaR) of an investment portfolio based on standard normal distribution approach is calculated using the equation [18; 4]:

$$VaR_t = -W_0 (\ddot{\mu}_t + z_c \ddot{\sigma}_t) = -W_0 \{ \mathbf{w}^T \boldsymbol{\mu} + z_c (\mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w})^{1/2} \}. \quad (12)$$

where  $W_0$  the number of fund is allocated in the portfolio and  $z_c$  is the percentile of standard normal distribution at the significance level  $c$ . When it is assumed  $W_0 = 1$  unit, the equation (12) becomes:

$$VaR_t = -(\ddot{\mu}_t + z_c \ddot{\sigma}_t) = -\{\mathbf{w}^T \boldsymbol{\mu} + z_c (\mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w})^{1/2}\}. \quad (13)$$

A portfolio  $\mathbf{w}^*$  is called (mean-VaR) efficient if there is no other portfolio  $\mathbf{w}$  with  $\ddot{\mu}_t \geq \ddot{\mu}_t^*$  and  $VaR_t < VaR_t^*$  [17]. To obtain the efficient portfolio, we used the objective function, to maximize  $\{2\tau \ddot{\mu}_t - VaR_t\}$ ,  $\tau \geq 0$  where  $\tau$  denotes the investor risk tolerance factor. For the investor with the risk tolerance  $\tau \geq 0$  therefore we must solve an optimization problem [17; 12]:

$$\begin{aligned} &\text{Maximize } \{2\tau \boldsymbol{\mu}^T \mathbf{w} + \boldsymbol{\mu}^T \mathbf{w} + z_c (\mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w})^{1/2}\} \\ &\text{such that } \mathbf{e}^T \mathbf{w} = 1 \end{aligned} \quad (14)$$

Equation (14) is a quadratic concave optimization problem. Its Lagrangian function can be written as  $L(\mathbf{w}, \lambda) = (2\tau + 1)\boldsymbol{\mu}^T \mathbf{w} + z_c (\mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w})^{1/2} + \lambda(\mathbf{e}^T \mathbf{w} - 1)$ . Using the Kuhn-Tucker theorem, the optimal solution can be obtained using the first derivatives, as follows [18; 6]:

$$\partial L / \partial \mathbf{w} = (2\tau + 1)\boldsymbol{\mu} + z_c \boldsymbol{\Sigma} \mathbf{w} / (\mathbf{w}^T \boldsymbol{\Sigma} \mathbf{w})^{1/2} + \lambda \mathbf{e} = 0 \quad \text{and} \quad \partial L / \partial \lambda = \mathbf{e}^T \mathbf{w} - 1 = 0. \quad (15)$$

Solving the equation (15) as the function of  $\lambda$ , we obtain a quadratic function  $A\lambda^2 + B\lambda + C = 0$ . where  $A = \mathbf{e}^T \boldsymbol{\Sigma}^{-1} \mathbf{e}$ ,  $B = (2\tau + 1)(\boldsymbol{\mu}^T \boldsymbol{\Sigma}^{-1} \mathbf{e} + \mathbf{e}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu})$  and  $C = (2\tau + 1)^2 (\boldsymbol{\mu}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}) - z_c^2$ . The solution can be obtained directly from the solution of the quadratic function [18]:

$$\lambda = \{-B + (B^2 - 4AC)^{1/2}\} / 2A; \lambda \geq 0. \quad (16)$$

For  $\tau \geq 0$ , we obtain the weight vector  $\mathbf{w}$  as

$$\mathbf{w} = \frac{(2\tau + 1)\boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} + \lambda \boldsymbol{\Sigma}^{-1} \mathbf{e}}{(2\tau + 1)\mathbf{e}^T \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} + \lambda \mathbf{e}^T \boldsymbol{\Sigma}^{-1} \mathbf{e}}. \quad (17)$$

By substituting the vector  $\mathbf{w}$  into the equation (10) we obtain the mean value of portfolio return. When vectors  $\mathbf{w}$  are substituted into the equation (13), we obtain the value of the investment portfolio risk level  $VaR_t$ . The sets of point pairs  $(\ddot{\mu}_t, VaR_t)$  form a graph of efficient frontier. Among the efficient frontier, there are optimum portfolios, which have the largest ratio  $\ddot{\mu}_t / VaR_t$  [18].

### III. RESULTS AND ANALYSIS

#### A. The Data

For empirical study, we analyze the following stocks: ASII, TRUB, BBKA, BBRI, HMSP, and TLKM, and are denoted as  $S_1$  until  $S_5$ . As the factors index, we use Composite Stock Price Index (IHSG), the rate of inflation, exchange rate of the rupiah against the euro, the rupiah against the U.S. dollar, and the rupiah against the yen, and are denoted as  $F_1$  until  $F_5$ . For the risk-free asset data, we use a government bond price. The data are obtained from <http://www.finance.go.id/>. The period of observation is January 2, 2011 until December 30, 2015. The empirical analysis is done using the software's of: MS Excel 2007, Eviews 6, Maple 9.5 and R.

#### B. Empirical Results

In this study, the factors index used are  $F_1$  until  $F_5$ , as described above. We first calculate the returns of each factor index, then identify the existence of the effects of long memory in the returns, and finally estimated the mean and volatility models of the returns.

**Identification of long memory effects.** To identify the effect of long memory, we estimate the parameters of fractional difference  $\hat{d}_j$  ( $j = 1, \dots, 5$ ) as in equation (1). The estimation is performed using the rescale range method (R/S) or Geweke and Porter-Hudak (GPH) method. The results are summarized in Table-1.

TABLE I. THE IDENTIFICATION OF THE LONG MEMORY EFFECTS

Stock	$\hat{d}_j$	$\sigma_{d_j}$	Intervals Confidence	$z_j$	Effect of Long Memory
$F_1$	0.361	0.1462	$0.075 < d_1 < 0.648$	5.86	Significant
$F_2$	0.263	0.1323	$0.004 < d_2 < 0.522$	2.57	Significant
$F_3$	0.635	0.5853	$-0.512 < d_3 < 1.782$	1.12	Not Significant
$F_4$	0.098	0.1731	$-0.241 < d_4 < 0.437$	2.62	Significant
$F_5$	0.518	0.6312	$-0.719 < d_5 < 1.755$	1.51	Not Significant

To ensure the existence of long memory patterns, we test the hypothesis  $H_0 : d_j = 0$  against,  $H_1 : d_j \neq 0$ ,  $j = 1, \dots, 5$ . The values of statistic  $z_j$  is given in table above, while for the level of significance  $c = 0.95$ , from the standard normal distribution table values, we obtain  $Z_{0.95/2} = 1.96$ . Because of the values  $z_1$ ,  $z_2$  and  $z_4$  are larger than the value  $Z_{0.95/2}$ , it is concluded that the test results are significant, the returns of factor index data,  $F_1$ ,  $F_2$  and  $F_4$  have long memory effects. However, in  $F_3$  and  $F_5$  there are no long memory effects.

In the next step, we identify and estimate the best mean and volatility models to difference fractional  $\hat{d}_j$  of the returns data  $F_1$ ,  $F_2$  and  $F_4$ , where for  $F_3$  and  $F_5$ , the analysis is applied directly to the returns data.

**Identification and estimation of mean models.** Identification of the mean models is done using the sample autocorrelation function (ACF) and partial autocorrelation function (PACF). Based on the patterns of ACF and PACF of each factor index returns (or the fractional differenced data), we obtain the best models for  $F_1$  until  $F_5$ , which also passed the standard diagnostic check. The results are summarized in Table 2.

**Identification and estimation of volatility models.** We further identify and estimate of volatility model using generalized autoregressive conditional heteroscedasticity (GARCH) models. Based on the correlogram of quadratic residual  $a_{jt}^2$ , we select the plausible volatility model for the data. Estimation of volatility models of each factors index return is done simultaneously with the mean models. The results, obtained for the best model which is also passed the diagnostic checks, are given in Tabel-2.

TABLE II. THE ESTIMATION RESULTS OF MEAN AND VOLATILITY MODEL OF FACTOR INDEX RETURNS

Factor ( $F_j$ )	Model	Mean and Volatility Equations
$F_1$	ARFIMA(1, $\hat{d}_1$ , 0)- GARCH(1,1)	$r_{1t} = 0.111341 r_{1t-1} + a_{1t}$ $\sigma_{1t}^2 = 0.00000866 + 0.137021 a_{1t-1}^2 + 0.834528 \sigma_{1t-1}^2 + \varepsilon_{1t}$
$F_2$	ARFIMA(1, $\hat{d}_2$ , 1)- GARCH(1,1)	$r_{2t} = 0.993306 r_{2t-1} - 0.990698 a_{2t-1} + a_{2t}$ $\sigma_{2t}^2 = 0.016328 + 0.447513 a_{2t-1}^2 + 0.043462 \sigma_{2t-1}^2 + \varepsilon_{2t}$
$F_3$	AR(1)- GARCH(1,2)	$r_{3t} = -0.070772 r_{3t-1} + a_{3t}$ $\sigma_{3t}^2 = 0.000000853 + 0.140811 a_{1t-1}^2 + 0.300641 \sigma_{3t-1}^2 + 0.563666 \sigma_{3t-2}^2 + \varepsilon_{2t}$
$F_4$	ARFIMA(1, $\hat{d}_4$ , 0)- GARCH(2,1)	$r_{4t} = -0.078681 r_{4t-1} + a_{4t}$ $\sigma_{4t}^2 = 0.00000837 + 0.38691 a_{4t-1}^2 + 0.478577 a_{4t-2}^2 + 0.372516 \sigma_{4t-1}^2 + \varepsilon_{4t}$

$$F_5 \quad \text{AR(1)-TGARCH(2,1)} \quad \begin{aligned} r_{5t} &= -0.09410 \, r_{5t-1} + a_{5t} \\ \sigma_{5t}^2 &= 0.000000467 + 0.280911 \, a_{5t-1}^2 + 0.234836 \, a_{4t-2}^2 + 0.951865 \, \sigma_{4t-1}^2 + \varepsilon_{5t} \end{aligned}$$

**Estimated regression model of APT with lagged.** In this section estimation of APT model with lagged, conducted by estimating regression models of each of the five stock return data, against the data of return of the five-factor index. Estimation made refers to the equation (4), helped by Eviews 4 software. The return of risk-free asset data (bond) is relatively constant, therefore, taken the mean size  $\tilde{\mu}_t = 0.026462$  and variance  $\tilde{\sigma}_t^2 = 0$ . To simplify the writing, for example  $\varsigma_{it} = r_{it} - 0.026462$ ,  $i = 1, \dots, 5$  and the risk premium of factor index with the lagged  $I_{jt-\zeta} = \hat{r}_{jt-\zeta} - 0.026462$  ( $j = 1, \dots, 5$  and  $\zeta = 0, 1, \dots, L$ , where  $L$  length of is lagged). The Estimation results written covering the regression equation with  $t$ -statistic values, written in parentheses below each parameter coefficients. Similarly, given coefficient of determination of  $R_i^2$ , and Durbin-Watson statistic (D-W) from each of the regression. The regression equation of APT with lagged those given in Table-3.

TABLE III. REGRESSION MODEL ESTIMATION RESULTS OF APT WITH LAGGED

Stocks ( $S_i$ )	Regressions Model	$R_i^2$	D-W Stat.
$S_1$	$\begin{aligned} \varsigma_{1t} &= 0.0251 + 1.4723 I_{1t} + 0.2994 I_{1t-1} + 0.1205 I_{1t-2} - 0.0005 I_{2t} - 0.5251 I_{3t} - 0.1840 I_{3t-1} \\ &\quad + 0.6843 I_{4t} + 0.1151 I_{4t-1} - 0.0535 I_{5t} - 0.0171 I_{5t-1} + u_{1t} \end{aligned}$ <p>(3.74) (17.91) (3.74) (1.65) (-2.43) (-3.42) (-2.19) (2.67) (2.45) (-2.28) (-2.09)</p>	58.75%	1.944
$S_2$	$\begin{aligned} \varsigma_{2t} &= 0.0040 + 0.9429 I_{1t} - 0.0009 I_{2t} - 0.0006 I_{2t-1} - 0.0003 I_{2t-2} + 0.0442 I_{3t} + 0.1466 I_{3t-1} \\ &\quad + 0.0239 I_{3t-2} + 0.0432 I_{3t-3} + 0.1236 I_{4t} - 0.2557 I_{5t} - 0.0110 I_{5t-1} + u_{2t} \end{aligned}$ <p>(1.66) (14.11) (-1.83) (-2.53) (-2.31) (2.35) (2.25) (3.25) (2.46) (2.58) (-1.66) (-2.12)</p>	47.79%	2.081
$S_3$	$\begin{aligned} \varsigma_{3t} &= 0.0091 + 1.4491 I_{1t} + 0.1313 I_{1t-1} - 0.0011 I_{2t} - 0.0012 I_{2t-1} - 0.2223 I_{3t} - 0.0515 I_{4t} \\ &\quad - 0.0072 I_{4t-1} - 0.0190 I_{5t} + u_{3t} \end{aligned}$ <p>(2.52) (19.10) (1.83) (-1.90) (-1.98) (-2.55) (-2.21) (-2.06) (-2.11)</p>	60.56%	1.963
$S_4$	$\begin{aligned} \varsigma_{4t} &= 0.0022 + 0.3894 I_{1t} + 0.0978 I_{1t-1} + 0.0822 I_{1t-2} - 0.0014 I_{2t} - 0.1061 I_{3t} - 0.2743 I_{3t-1} \\ &\quad + 0.1369 I_{4t} + 0.0966 I_{4t-1} + 0.2395 I_{4t-2} + 0.1977 I_{5t} + 0.2174 I_{5t-1} + u_{4t} \end{aligned}$ <p>(2.26) (4.52) (1.65) (2.03) (-2.22) (-1.66) (-1.70) (2.51) (2.36) (1.91) (2.00) (2.09)</p>	48.50%	2.553
$S_5$	$\begin{aligned} \varsigma_{5t} &= -0.0007 + 0.9196 I_{1t} - 0.0007 I_{2t} - 0.2695 I_{3t} + 0.0075 I_{4t} + 0.0840 I_{5t} \\ &\quad + 0.007 I_{5t-1} + 0.1067 I_{5t-2} + 0.0997 I_{5t-3} + u_{5t} \end{aligned}$ <p>(-2.14) (14.92) (-1.80) (-2.31) (2.04) (2.59) (2.10) (1.66) (2.54)</p>	46.66%	1.905

As presented by Blume (1971), the ability of the index factors to explain changes in individual stock returns ranging between 25% -51% rate coefficient of determination. Looking at the results in Table-3, it appears that the coefficient of determination  $R_i^2$  of regression models of each stock worth nearly 51%. Means the return of five shares  $S_1$  until  $S_5$  a relatively strong correlation with the return of five risk premium of the index factor  $I_{jt-\zeta}$  ( $j = 1, \dots, 5$  and  $\zeta = 0, 1, \dots, L$ , where  $L$  length of is lagged). Based D-W statistic whose value is also relatively small, showing the five regression models in Table-3 is significant. It can also be shown the residuals  $u_{it}$  of each regression model are normally distributed, with zero mean and variance  $\sigma_{u_{it}}^2$ . Estimator value  $\sigma_{u_{it}}^2$  of each regression equations are given in Table-4. Regression model is then used to estimate the mean and variance of stock return.

**The estimated mean and variance values of stocks return.** The values of constants, coefficient of parameters and regression of residual variance  $\sigma_{u_{it}}^2$  in Table-3, and the mean value estimator  $\hat{\mu}_{jT} = \hat{r}_{jT}(1)$  and variance  $\hat{\sigma}_{jT}^2 = \hat{\sigma}_{jT}^2(1)$ , then used to estimate the mean and variance values of stock return  $S_1$  until  $S_5$ . The mean value estimated using equation (6), while the variance value is estimated based on equation (7). The estimation results are given in Table-4.

TABLE IV. THE ESTIMATION RESULTS OF MEAN AND VARIANCE OF STOCKS RETURN

Stocks ( $S_i$ )	$\hat{\sigma}_{u_{it}}^2$	$\hat{\mu}_{it}$	$\hat{\sigma}_{it}^2$
$S_1$	0.000788	0.001876	0.001278
$S_2$	0.000536	0.002328	0.000732
$S_3$	0.000705	0.001835	0.001132
$S_4$	0.000852	0.000231	0.000929
$S_5$	0.000456	0.000797	0.000634

In this study assumed that the covariance between stock return  $i$  and  $i'$  at the time  $t$ ,  $\sigma_{ii'} = 0$  ( $i, i' = 1, \dots, 5$  and  $i \neq i'$ ), because its values are also very small close to zero. This means that between stock return  $i$  and  $i'$  at the time  $t$  where the cross correlation does not occur. The values of the mean estimator  $\mu_{it}$  and the variance  $\sigma_{it}^2$  will be used for the following portfolio optimization process.

**Portfolio optimization.** Portfolio optimization in this case is based on the *mean-VaR* model, using the *Maple 9.5. software*. First, the values of the mean estimator in Table-4 column  $\mu_{it}$  is formed into a vector mean of  $\mu^T = (0.001867 \ 0.002328 \ 0.001835 \ 0.000231 \ 0.000797)$ . Second, referring to the number of shares to be analyzed as much as five, mean vector defined identity is  $e^T = (1 \ 1 \ 1 \ 1 \ 1)$ . Third, the values of the variance estimator in Table-4 a column of  $\sigma_{it}^2$ , is formed into the covariance matrix and determined inverse, as follows:

$$\Sigma = \begin{pmatrix} 0.00128 & 0 & 0 & 0 & 0 \\ 0 & 0.00073 & 0 & 0 & 0 \\ 0 & 0 & 0.00113 & 0 & 0 \\ 0 & 0 & 0 & 0.00093 & 0 \\ 0 & 0 & 0 & 0 & 0.000634 \end{pmatrix} \text{ and } \Sigma^{-1} = \begin{pmatrix} 782.47 & 0 & 0 & 0 & 0 \\ 0 & 1366.12 & 0 & 0 & 0 \\ 0 & 0 & 883.39 & 0 & 0 \\ 0 & 0 & 0 & 1076.43 & 0 \\ 0 & 0 & 0 & 0 & 1577.29 \end{pmatrix}$$

Risk tolerance  $\tau$  in this study are determined by simulation. Tolerance values together  $\mu^T$  and  $e^T$  vectors and  $\Sigma^{-1}$  matrix, when substituted into equation (15) will be obtained multiplier value as  $\lambda$ . Where then substituted into equation (16) will be obtained by portfolio weight vector  $w$ . Weight vector  $w$ , then used to calculate the mean value of portfolios using equation (10), and calculate the risk level of VaR using equation (13). For the values of risk tolerance  $0 \leq \tau \leq 7.377$  some calculation results are given in Table-5.

TABLE V. RISK TOLERANCE, WEIGHT, MEAN AND PORTFOLIO RISK

$\tau$	Weight					$\hat{\mu}_t$	$VaR_t$	$\frac{\hat{\mu}_t}{VaR_t}$
	$w_1$	$w_2$	$w_3$	$w_4$	$w_5$			
0.00	0.14083	0.25086	0.15870	0.17945	0.27015	0.001396	0.0204341	0.06832617
0.50	0.14405	0.26148	0.16205	0.16954	0.26287	0.001425	0.0204485	0.06968778
1.00	0.14730	0.27220	0.16542	0.15956	0.25552	0.001454	0.0204922	0.07095751
1.50	0.15059	0.28304	0.16883	0.14945	0.24809	0.001483	0.0205657	0.07213373
2.00	0.15393	0.29406	0.17230	0.13918	0.24053	0.001513	0.0206704	0.07321444
2.50	0.15734	0.30531	0.17584	0.12869	0.23282	0.001544	0.0208077	0.07419726
3.00	0.16084	0.31683	0.17947	0.11795	0.22491	0.001575	0.0209798	0.07507931
3.50	0.16444	0.32871	0.18321	0.10688	0.21677	0.001607	0.0211892	0.07585733
4.00	0.16817	0.34100	0.18707	0.09542	0.20834	0.001641	0.0214394	0.07652736
4.50	0.17204	0.35379	0.19110	0.08349	0.19957	0.001702	0.0234100	0.07271306
5.00	0.17610	0.36718	0.19531	0.07101	0.19039	0.001712	0.0220795	0.07752462
5.50	0.18038	0.38127	0.19975	0.05787	0.18073	0.001750	0.0224812	0.07784036



6.00	0.18491	0.39622	0.20446	0.43936	0.17048	0.001790	0.0229477	0.07802487
<b>6.404</b>	<b>0.18880</b>	<b>0.40903</b>	<b>0.20849</b>	<b>0.03199</b>	<b>0.16169</b>	<b>0.001825</b>	<b>0.0233789</b>	<b>0.07807227</b>
6.50	0.18975	0.41219	0.20948	0.29050	0.15953	0.001834	0.0234893	0.07806955
7.00	0.19497	0.42939	0.21490	0.01301	0.14773	0.001880	0.0241196	0.07796420
7.377	0.19920	0.44335	0.21929	0.000003	0.13816	0.001918	0.0246640	0.07777817

For values of risk tolerance  $\tau > 7.377$  is no longer feasible, because the portfolio weight does not qualify.

A collection of points  $(\ddot{\mu}_t, VaR_t)$  to form the surface efficiently (efficient frontier), as shown in Figure-1. Ratio between  $\ddot{\mu}_t$  toward  $VaR_t$  the values given in last column of Table-5 while the graph is given in Figure-2.

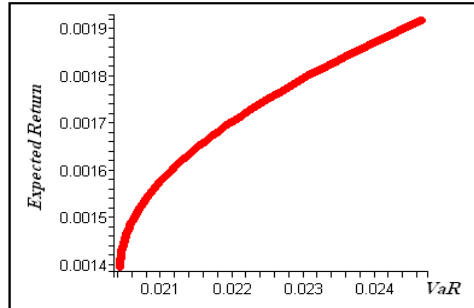


Fig. 1. Efficient Frontier of Portfolio

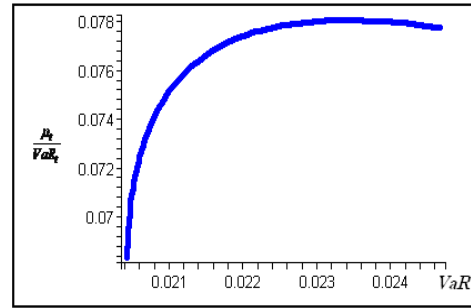


Fig. 2. Mean and VaR Ratio of Portfolio

### C. Discussion

Based on the calculation results given in Table-5, can be seen that with the risk tolerance of 0.00 obtained VaR portfolio composition that produces a minimum, that is equal to 0.020434 with the expected return of portfolio amounted to 0.001396. The amount of risk tolerance can still be improved but with the condition that the resulting values of weighted portfolio of real  $0 < w_i < 1$  ( $i = 1, \dots, 5$ ) and qualify  $\sum_{i=1}^5 w_i = 1$ . In this case the value will be at most risk tolerance as  $\tau = 7.377$ . Where the resulting composition of the portfolio with the highest expected return of portfolio that is equal to 0.0019183 with VaR at 0.024664. Any increase in the value of risk tolerance will cause the increase in expected return portfolio which is also accompanied by an increase in the Value-at-Risk portfolio.

Efficient portfolios lie along the line with risk tolerance of  $0 \leq \tau \leq 7.377$ , as given by the graph of the surface efficiently (efficient frontier) in Figure-1. Along that line of investors eligible to invest in a portfolio consisting of stocks  $S_1$  until  $S_2$ . Of course, each investor should choose one based on preference or risk tolerance level that is believed. For the avoidance of risk investors will usually sets the risk tolerance is small, while for investors challenger will risk taking a big risk tolerance. Large-size specified risk tolerance, of course, will affect the large-size of portfolio expected return obtained.

Having obtained a series of efficient portfolios, the next step is determining the optimum portfolio composition. Every investor wants a portfolio of investment that can yield big returns, but accompanied by a small degree of risk. If it is assumed that investor's preferences based only on expected return and risk of the portfolio. Selection of the optimum portfolio can be determined based on the composition of the efficient portfolios that generate expected return and the Value-at-Risk of portfolio with the largest ratio.

Based on the calculation results given in the last column of Table-5, shows that the ratio of expected return and VaR the largest portfolio is 0.07807227, or obtained when risk tolerance reaches 6404. The ratio of expected return and VaR continues to increase at intervals of risk tolerance  $0 \leq \tau \leq 6.404$  and decreased in the interval  $6.404 \leq \tau \leq 7.377$ . Up-and-downs of these ratios can be seen graph given in Figure-2. Based on the values in Table-5, obtained the results that based on the model of asset allocation mean-VaR, the optimum portfolio composition prepared from the stock  $S_1$  until  $S_5$  the portfolio composition with the weight vector  $\mathbf{w}^T = (0.1888 \ 0.4090 \ 0.2085 \ 0.0320 \ 0.1617)$ . Where is the optimum portfolio composition to generate the expected return of 0.001825 with Value-at-Risk of 0.0233789.

#### IV. CONCLUSION

Mathematical of *Arbitrage Pricing Theory* (APT) model can be expanded into APT with lagged. Based on it the estimators of mean, variance and covariance of stock return are formulated. The empirical research conducted on five stock  $S_1$  until  $S_5$ , and five index factor of  $F_1$  until  $F_5$ . Where return of the index factors in the APT with lagged analyzed using ARFIMA-GARCH model approach. Based on the analysis that the three index return factor, namely  $F_1$ ,  $F_2$  and  $F_4$  there are significant effects of long memory. Parameters estimator of mean, variance and covariance, then used for the analysis of asset allocation problem based on the mean-VaR model. Based on the results of optimization, efficient portfolios are formed for the values of risk tolerance  $0 \leq \tau \leq 7.377$ . An optimum occurs in the value of portfolio risk tolerance  $\tau = 6.404$ . Optimum portfolio has composition allocation weight of  $w^T = (0.1888 \ 0.4090 \ 0.2085 \ 0.0320 \ 0.1617)$ , with a mean portfolio return of 0.001825 and Value-at-Risk of 0.0233789.

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