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An Efficient Approach For Traveling Salesman Problem Solution With Branch-and-Bound

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Abstract—Travelling salesman problem is a known problem.In this paper will be used method of branch-and-Bound efficient to solve the problem Traveling salesman. Tour starts from a given starting vertex and all vertices to be passed, then returned to the initial vertex. Costs given a different path. Furthermore, will be found to the tour cost is minimal between the city (node) connected.

Keywords— *Travelling salesman, path, Branch and Bound.*

I. Introduction

The Traveling Salesman Problem, TSP for short, deals with creating the ideal path that a salesman would take while traveling between cities. The solution to any given TSP would be the cheapest way to visit a finite number of cities,visiting each city only once, and then returning to the starting point [3]. We alsomust assume that if there are two cities, city A and city B for example, it costs the same amount of money to travel from A to B as it does from B to A [2].

For the most part, the solving of a TSP is no longer executed for the intention its name indicates. Instead, it is a foundation for studying general methods that are applied to a wide range of optimization problems [1].

A. Applictions

The TSP naturally arises as a sub problem in many transportation and logistics applications.

Scheduling of a machine to drill holes in a circuit board or other object.In this case the holes to be drilled are the cities, and the cost of travel is the time it takes to move the drill head from one hole to the next.

II. Optimal Solution for TSP using Branch and Bound

A branch-and-bound algorithm consists of a systematic enumeration of all candidatesolutions, where large subsets of fruitless candidate s are discarded,byusing upper and lower estimated bounds of the quantity being optimized.

The Branch and Bound strategy divides a problem to be solved into a numberof sub-problems. It is a system for solving a sequence of sub problems each of which may have multiple possible solutions and where the solution chosen for one sub-problem may affect the possible solutions of later sub-problems.

Suppose it is required to minimize an objective function. Suppose that wehave a method for getting a lower bound on the cost of any solution among those in the set of solutions represented by some subset. If the best solution found so far costs less than the lower bound for this subset, we need not explore this subset at all [3].

Let S be some subset of solutions.

L(S)=a lower bound on the cost of any solution belonging to S

Let C=cost of the best solution found so far

If $C \leq L(S)$,there is no need to explore S because it does not contain any bettersolution.

If $C > L(S)$,then we need to explore S because it may contain a better solution.[2]

The input to the method is the cost matrix, which is prepared using the convention:

$$C = \begin{cases} \infty, \text{If there is no direct path from A to B} \\ W, \text{If there is no direct path from A to B} \end{cases}$$

While solving the problem, we first prepare the state space tree, which represents all possible solution.

III. Solution to the Problem Using Branch and Bound Method

Here in this problem |V|=5. Which is the number of total nodes on the graph or the cities in the map. The following matrix is the Cost Matrix which shows the distance between the two cities [2].

$$\text{Cos matrix}= \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} \infty & 10 & 8 & 9 & 7 \\ 10 & \infty & 10 & 5 & 6 \\ 8 & 10 & \infty & 8 & 9 \\ 9 & 5 & 8 & \infty & 6 \\ 7 & 6 & 9 & 6 & \infty \end{pmatrix} \end{array}$$

We know that the sum of row minimum gives us the lower bound. Now we have to find the reduced matrix by subtracting the minimum element from every row. So, row minimum will be 31.

$$\text{Reduced Matrix} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} \infty & 3 & 1 & 2 & 0 \\ 5 & \infty & 5 & 0 & 1 \\ 0 & 2 & \infty & 0 & 1 \\ 4 & 0 & 3 & \infty & 1 \\ 1 & 0 & 3 & 0 & \infty \end{pmatrix} \end{array}$$

In the above reduced matrix there should be a Zero in every row and every column. Now apply the column minimum principle which means for a particular city I will have come into that city from other city. So, now subtracting the column minimum , we get the lower bound 31+1 = 32.

$$\text{Reduced Matrix} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} \infty & 3 & 0^{(2)} & 2 & 0^{(1)} \\ 5 & \infty & 4 & 0^{(1)} & 1 \\ 0^{(1)} & 2 & \infty & 0^{(0)} & 1 \\ 4 & 0^{(1)} & 2 & \infty & 1 \\ 1 & 0^{(0)} & 2 & 0^{(0)} & \infty \end{pmatrix} \end{array}$$

Now we have to make the assignments Zero's in the above matrix. If we can not make an assignment at that particular Zero we have to go for the next highest element in that row. If there is same value, we have to calculate the sum of row penalty and column penalty. If the sum of penalties is more then we have to make that assignment because we incur an additional cost if we don't make that assignment.

Now if we don't have path from 1 to 3 i.e,

V(13) = 0 (1)

we have an additional cost of 2 and the lower bound becomes 32+2 = 34.

If we have path from 1 to 3 i.e,

V(13) = 1 (2)

we eliminate the respective row and column.

$$\text{Reduced Matrix} = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 4 & 5 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{pmatrix} 5 & \infty & 0 & 1 \\ \infty & 2 & 0 & 1 \\ 4 & 0 & \infty & 1 \\ 1 & 0 & 0 & \infty \end{pmatrix} \end{array}$$

We have to check whether every row and every column has a zero otherwise we have to subtract the minimum element from every respective row or column. So, we get

$$\text{Reduced Matrix} = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 4 & 5 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{pmatrix} 4 & \infty & 0 & 0 \\ \infty & 1 & 0 & 0 \\ 3 & 0 & \infty & 0 \\ 0 & 0 & 0 & \infty \end{pmatrix} \end{array}$$

Here we made a column reduction for two columns (1 & 5) which gives a lower bound of 32+1+1 = 34.

Again we calculate the penalty for the above matrix.

$$\text{Reduced Matrix} = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 4 & 5 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{pmatrix} 4 & \infty & 0^{(0)} & 0^{(0)} \\ \infty & 2 & 0^{(0)} & 0^{(0)} \\ 3 & 0^{(0)} & \infty & 0^{(0)} \\ 0^{(3)} & 0^{(0)} & 0^{(0)} & \infty \end{pmatrix} \end{array}$$

Now if we don't have path from 5 to 1 i.e, V(51) = 0 (3)

we have an additional cost of 3 and the lower bound becomes 34+3 = 37.

If we have path from 5 to 1 i.e, V(51) = 1 (4)

we eliminate the respective row and column.

$$\text{Reduced Matrix} = \begin{array}{cc} & \begin{array}{ccc} 2 & 4 & 5 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 5 \end{array} & \begin{pmatrix} \infty & 0 & 0 \\ 2 & 0 & \infty \\ 0 & \infty & 0 \end{pmatrix} \end{array}$$

Here we a Zero in every row and column. So, the bound remains the same i.e, 34+0 = 34.

If we calculate the penalty for the above matrix,

$$\text{Reduced Matrix} = \begin{array}{cc} & \begin{array}{ccc} 2 & 4 & 5 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 5 \end{array} & \begin{pmatrix} \infty & 0^{(0)} & 0^{(0)} \\ 2 & 0^{(2)} & \infty \\ 0^{(2)} & \infty & 0^{(0)} \end{pmatrix} \end{array}$$

Now if we don't have path from 3 to 4 i.e, V(34) = 0(5)

we have an additional cost of 2 and the lower bound becomes 34+2 = 36.

If we have path from 3 to 4 i.e, V(34) = 1 (6)

we eliminate the respective row and column.

$$\text{Reduced Matrix} = \begin{array}{cc} & \begin{array}{cc} 2 & 5 \end{array} \\ \begin{array}{c} 2 \\ 4 \end{array} & \begin{pmatrix} \infty & 0 \\ 0 & \infty \end{pmatrix} \end{array}$$

Here we a Zero in every row and column. So, the bound remains the same i.e, 34+0 = 34.

Now we have only two possible assignments with Zero penalty cost which would give us a feasible solution.

V(25) = 1 (7)

V(42) = 1 (8)

On adding all the paths we get, V(13) + V(34) + V(42) + V(25) + V(51) = 34 (9)

IV. Conclusion

The proposed method, which is using Branch & Bound, is better because it prepares the matrices in different steps. At each step the cost matrix is calculated. From the initial point we come to know that what can be the minimum cost of the tour. The cost in the initial stages is not exact cost but it gives some idea because it is the approximated cost. At each step it gives us the strong reason that which node we should travel the next and which one not. It gives this fact in terms of the cost of expanding a particular node.

References

[1] http://en.wikipedia.org/wiki/Travelling salesman problem.

[2] http://lcm.csa.iisc.ernet.in/dsa/node187.html.

[3] http://www.youtube.com/watch?v=nN4K8xA8ShM, 2010.