Proceedings of the 2016 International Conference on Industrial Engineering and Operations Management
Detroit, USA, September 23-25, 2016

Analysis Of Accuracy Multivariate Control Chart T2 Hotelling Free Distribution With Outlier Removal

(Case Study: Production Process Ink at PT. EPSON Batam)

Yuyun Hidayat
Department of Statistics, Faculty of Mathematics and Natural Sciences
Padjadjaran University
Bandung, Indonesia
yuyun.hidayat@unpad.ac.id

Titi Purwandari
Department of Statistics, Faculty of Mathematics and Natural Sciences
Padjadjaran University
Bandung, Indonesia
titipurwandari@yahoo.com

Alvionita
Department of Statistics, Faculty of Mathematics and Natural Sciences
Padjadjaran University
Bandung, Indonesia
Alvionita110293@gmail.com

Sukono
Department of Mathematics, Faculty of Mathematics and Natural Sciences
Padjadjaran University
Bandung, Indonesia
sukono@unpad.ac.id

Abstract

The quality of products are competitive advantage for PT EPSON Batam-Indonesia. One of their products is ink, its quality are controlled by collecting data during 1st April 2014 till 7th July 2014 and use these data to set up a control chart on five variables, namely, Viscosity / viscosity (DPAs), pH, Surface Tension , Wavelength (Å), and Particle Size (m). We noted that 14 points plot above the upper control limit, so the process is not in control.These points must be investigated to see whether an assignable cause can be determined. However, analysis of the data (chamber process) does not produce any reasonable or logical assignable cause for these, and we decide to retain the point and conclude that the process is in control. This indicates that we have false alarm, so clearly this is an important issue to consider in control chart implementation. The ink quality is determined by five variables which are multivariate in nature, while PT EPSON Batam is applying univariate control charts to each individual variable. Control chart is basically used on data having normal distribution, but in fact the data we have is not normal distribution so Chebyshev theorem for free T2Hotelling control chart distribution can be applied. In addition, to address the presence of outliers, the control chart is constructed without outliers (outlier data removal). The Accuracy of control chart is evaluated by G-index measurement accuracy. The proposed control chart produced 4 out of control observations and after decomposition analysis are known that the cause of out of control occurrence are the variable of the surface tension and viscosity. In addition, the accuracy of the free distribution T2Hotelling control chart, 97.94% shows that the control chart can be used for future samples.

Keywords: False alarms, multivariate Chebyshev theorem, free distribution T2 Hotelling control charts, the level of accuracy of the control chart, the G-index, Outliers

I. INTRODUCTION

As a manufacturing company, which is directly supervised under the EPSON Company in Japan, PT EPSON Batam always made quality and service of their product as a top priority. To monitor the quality of goods, PT EPSON Batam is formed a QA(Quality Assurance) Departmentwhich responsible for the quality of products, in every production unit. To assure the quality of ink, PT EPSON Batam gives more attention to the ink's characteristic. To control the ink quality PT EPSON Batam use $\bar{x}$ control chart.

Based on results of observations made during1st April 2014 till 7th July 2014 it is known that there are 14 out of control observations. Then a chamber process is done to those fourteen observations.Based on the result of the chamber process, those fourteen observations are still includedinto the good quality products. The chamber process indicates there are incorrect out-of-control signal or falsealarm generated by the control chart.

Because the ink quality is influenced by several characteristics, such as viscosity, pH, surface tension, wavelength, and the size of the particle, then to reduce false alarm we suggested the implementation of multivariate control chart instead of univariate control chart as used by PT EPSON Batam.

The control chart analysis is applied to a normal distributed data but in fact the data is not normally distributed. This practice will cause the false alarm too. To tackle this problem, Chebyshev theory is implemented so that the data on the control chart is stated as distribution free. After forming a control chart, we evaluate the accuracy of control chart using G-index by forming a 2x2 contingency table between the chart result and the real condition based on observation.The purpose of this research is to obtain inaccurate control chart to control the ink's characteristics.

This template, modified in MS Word 2007 and saved as a "Word 97-2003Document" for the PC, provides authors with most of the formatting specifications needed for preparing electronic versions of their papers. All standard paper components have been specified for three reasons: (1) ease of use when formatting individual papers, (2) automatic compliance to electronic requirements that facilitate the concurrent or later production of electronic products, and (3) conformity of style throughout a conference proceedings. Margins, column widths, line spacing, and type styles are built-in; examples of the type styles are provided throughout this document and are identified in italic type, within parentheses, following the example. Some components, such as multi-leveled equations, graphics, and tables are not prescribed, although the various table text styles are provided. The formatter will need to create these components, incorporating the applicable criteria that follow.

II. LITERATURE

A. Multivariate Analysis

According to Johnson [2], univariate analysis is performed o analyze avariableoreachvariable tothe observations result. Univariate analysis serves to summarize the results of the measurement data so the data can be transformed into useful information. While the multivariate analysis is related to more than two variable analyzed simultaneously based on a sample of multivariate observations.

TABLE I. MULTIVARIATE DATA STRUCTURES

Observation-i	Variable (X_1)	Variable 2 (X_2)	Variable 3 (X_3)	…	Variable p (X_p)
1	x_{11}	x_{21}	x_{31}	…	x_{p1}
2	x_{12}	x_{22}	x_{32}	…	x_{p2}
…	…	…	…	…	…
m	x_{1m}	x_{2m}	x_{3m}	…	x_{pm}

B. Correlation Test among Variables

Variables $\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_p$ are independent if the correlation matrix between variables is an identity matrix. To test the independence between these variables we can conduct a Bartlett Sphericity test with the following hypothesis formulation [6]:

H0:R=I (variable ink characteristics mutually independent)

H1:R≠I (variable ink characteristics mutually dependent)

With the statistics test used is

$$\chi^2_{\text{hitung}} = -\left\{\text{m} - 1 - \frac{2\text{p}+5}{6}\right\}\ln|\mathbf{R}| \quad (1)$$

Where R is the correlation matrix of x with the following formula,

$$\mathbf{R} = \begin{bmatrix} 1 & r_{12} & \cdots & r_{1p} \\ r_{12} & 1 & \cdots & r_{2p} \\ \vdots & \cdots & \ddots & \vdots \\ r_{1p} & r_{2p} & \cdots & 1 \end{bmatrix} \quad (2)$$

$$r_{jk} = \frac{1}{m-1}\sum_{i=1}^{m}\left(\frac{x_{ij}-\bar{x}_j}{\sqrt{s_{jj}}}\right)\left(\frac{x_{ik}-\bar{x}_k}{\sqrt{s_{kk}}}\right) \quad (3)$$

Where,
r_{jk}: the correlation between the variables j and k

x_{ij}: the value of the i-th observation on variable j

$\bar{x}_j$: the average value of observations on the variable j

s_{jj}: standard deviation of the value of the variable j observations

x_{ik}:observation value to the variable i in the k

$\bar{x}_k$: the average value of observations on the variable k

s_{kk}: standard deviation of observed values of variables k

i: 1,2,3, ..., m

j: 1,2,3, ..., p

k: 1,2,3, ..., p

r: 1,2,3, ..., p

p: number of variables

m: number of observations

by using the statistics at equation(1) then reject the null hypothesis if the $\chi^2_{hitung} \geq \chi^2_{\frac{1}{2}p(p-1)}$or p-value$\leq \alpha = 0.05$ so it can be stated that the variable in k characteristics dependent, so the multivariate analysis fit to use.

C. Multivariate Normal Distribution

According to Montgomery [5] variables $\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_p$is stated as multivariate normal distribution with parameters $\boldsymbol{\mu}$and $\Sigma$where -∞ <**x**<∞ and j = 1,2,3, ..., p with notation $N(\boldsymbol{\mu}, \Sigma)$if it has a density probability function:

$$f(\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots, \boldsymbol{x}_p) = \frac{1}{2\pi^{p/2}|\boldsymbol{\Sigma}|^{1/2}} e^{\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})} \quad (4)$$

For a number of variable, p and m observations then the variable will be compiled into a vector $\boldsymbol{x}' = [x_1, x_2, \ldots, x_p]$where the average vector formed is $\boldsymbol{\mu}' = [\mu_1, \mu_2, \ldots, \mu_p]$ variance covariance matrix Σ is matrix of **x** the size p x p the main diagonal is the variance of the **x** value obtained from equation

$$s_j^2 = \frac{1}{m-1}\sum_{i=1}^{m}(x_{ij} - \bar{x}_j)^2 \quad (5)$$

and other elements of the covariance between variables obtained from the equation

$$s_{jk}^2 = \frac{1}{m-1}\sum_{i=1}^{m}(x_{ij} - \bar{x}_j)(x_{ik} - \bar{x}_k) \quad (6)$$

where,

s_{jk}^2: covariance of the observation value of the variable j and k variables

D. Testing Multivariate Normal Distribution

Testing multivariate normal distribution can be donevisuallyby making a Q-Q plot of Mahalanobis distance value approach.

$$d_i^2 = (\boldsymbol{x_i} - \overline{\boldsymbol{x}})'\mathbf{S^{-1}}(\boldsymbol{x_i} - \overline{\boldsymbol{x}}), i = 1,2, \dots, m \quad (7)$$

Establish distance value then sort from smallest to largest value.

$$d_{(1)}^2 \leq d_{(2)}^2 \leq d_{(3)}^2 \leq \cdots \leq d_{(m)}^2 \quad (8)$$

Sequencing results are then plotted against each qi value so that the linear plot results tend to show that the data follow multivariate normal distribution.

$$q_i = \chi_p^2\left(\frac{i-\frac{1}{2}}{m}\right) \quad (9)$$

In addition to visual examination using a QQ plot the multivariate normal test of hypothesis can be run through the value of skewness and kurtosis. Skewness and kurtosis of multivariate normal is defined as follows [3]:

$$\beta_{1,p} = E[(\boldsymbol{x} - \boldsymbol{\mu})'\,\boldsymbol{\Sigma^{-1}}(\boldsymbol{x} - \boldsymbol{\mu})]^3 \quad (10)$$

$$\beta_{2,p} = E[(\boldsymbol{x} - \boldsymbol{\mu})'\,\boldsymbol{\Sigma^{-1}}(\boldsymbol{x} - \boldsymbol{\mu})]^2 \quad (11)$$

When x ~ N (μ, Σ) and the value of the third-order central moments of the multivariate normal distribution is zero, $\beta_{1,p} = 0$ and

$\beta_{2,p} = p(p+2)$. The value for $\beta_{1,p}$and $\beta_{2,p}$are obtained from the following equation,

$$b_{1,p} = \frac{1}{m^2}\sum_{i=1}^{m} g_i^3 \quad (12)$$

$$b_{2,p} = \frac{1}{m}\sum_{i=1}^{m} g_i^2 \quad (13)$$

With a g_i value is Mahalanobis distance values obtained from equation(7) skewness value for testing the hypothesis of multivariate normal is performed as follows,

H_0: $\beta_{1,p} = 0$ (data has normally distributed)
H_1: $\beta_{1,p} \neq 0$ (data has not normally distributed)

With the test statistics used is

$$z_{skew} = \frac{(p+1)(m+1)(m+3)}{6[(m+1)(p+1)-6]} b_{1,p} \quad (14)$$

Decision criteria are reject the null hypothesis if $z_{skew} \geq \chi^2_{\frac{1}{6}p(p+1)(p+2)}$or reject the null hypothesisif thep-value≤α=0.05.

The formulation of testing hypothesis, using kurtosis valueare as follow

H_0: $\beta_{2,p} = 0$ (data has normally distributed)
H_1: $\beta_{2,p} \neq 0$ (data has not normally distributed)

With the test statistics used is

$$z_{kurtosis} = \frac{b_{2,p} - p(p+2)}{\sqrt{8p(p+2)/m}} \tag{15}$$

With decision criteria rejectthe null hypothesisif$z_{kurtosis}$value more than the upper limit of 2.5%orreject the null hypothesis if the p-value $\leq \alpha = 0.05$.

E. Control Chart

A major objective of statistical process control is to quickly detect the occurrence ofassignable causes of process shifts so that investigation of the process and corrective actionmay be undertaken before many nonconforming units are manufactured. The control chartis an on-line process-monitoring technique widely used for this purpose.The control chart may also provide information usefulin improving the process. Finally, remember that the eventual goal of statistical process controlis the elimination of variability in the process. It may not be possible to completelyeliminate variability, but the control chart is an effective tool in reducing variability as much as possible [5]. Based on how to obtain the data, then the data can be divided into:

1) Univariate Control Chart

Figure1. presents some guidelines for using univariate control charts to monitor processes with both correlated and uncorrelated data [5].

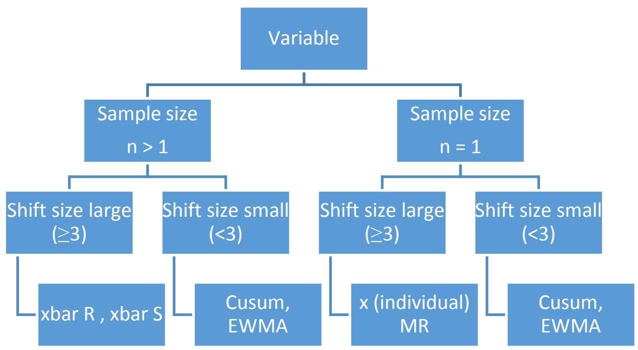

Fig. 1. Some guidelines for univariate control chart selection

2) Multivariate Control Chart

This is a control chart in which the simultaneous monitoring or control of two or morerelated quality characteristics is administered. .Here is the chart of the use of multivariate variable control charts,

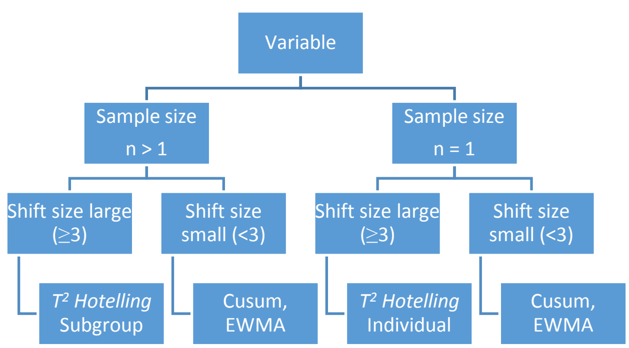

Fig. 2. Some guidelines for multivariate control chart selection

According to Montgomery (2009), T^2 Hotelling control chart is a chart for the control of individual data that can be used for multivariate data in the form of variable data that can be used in large size of the shift is the value $\delta_T \geq 3$and suitable to use in the observation with a large amount or more than 30 observations.

3) T^2 Hotelling Control Chart

According to Montgomery (2009) T^2 Hotelling is the most familiar multivariate process-monitoring and control procedure.T^2 Hotelling control chart using T^2 Hotelling statistics with the equation,

$$T_i^2 = (\boldsymbol{x_i} - \overline{\boldsymbol{x}})'\mathbf{S^{-1}}(\boldsymbol{x_i} - \overline{\boldsymbol{x}}), i = 1,2, \dots, m \tag{16}$$

Where$\bar{x}$ denote the vectorof averageand S is the covariance matrix. As with other control charts, T^2 Hotelling control chart has control limits as follow

$$UCL = \frac{p(m+1)(m-1)}{m^2-mp} F_{\alpha,p,m-p} \tag{17}$$

As for the number of observations (m) is greater than 100 then the control limits become

$$UCL = \frac{p(m-1)}{m-p} F_{\alpha,p,m-p} \tag{18}$$

And for a number of variables (p) is greater or equal to 10 the number of observations (m) is greater than 100 then the control limits become,

$$UCL = \chi^2_{(\alpha,p)} \tag{19}$$

4) T^2HotellingControl Chart Free Distribution

Infact, the dataisnot always meet the assumption of multivariate normal distribution. This led to the analysis of control charts which do not meet the assumption of multivariate normal distribution. In this case wecanuseT2Hotelling control charts with distribution free procedure as said by Mason & Young [4].

The analysis of the T2Hotelling control chart with free procedure is same asT2Hotelling control charts except in the determination of the upper control limit(UCL).Where the value of the control limit is determined by the equation,

$$UCL = \bar{T} + ks_T \tag{20}$$

$\overline{T}$ is the average value of T2Hotelling and ST is variance value of T2Hotelling. The α values and equation 21 are important in forming the control limits.

$$k = \sqrt{\frac{1}{\alpha}} \tag{21}$$

The value of the upper control limit (UCL) of the value of x is defined as follows,

$$P(\mu - k\sigma < x < \mu + k\sigma) \geq 1 - \frac{1}{k^2}. \tag{22}$$

5) PlottingControl ChartwithOutlierRemoval

Outliers are observations that do not follow most of the pattern and located away from the center of data [1]. However, outliers can not be thrown away, because sometimes outlier observations provide significant information. Due to the problems and the effects of outliers, then the outlier detection is very important. In detecting outliers or data out of the ordinary in multivariate control charts is to do a gradual formation control chartas follow:

- Form the first control chart with the number of observations(q) as the number of variables plus 2(p +2)
- Determine the value ofT^2forobservationq+1
- Performtestonobservationq+1using the first control chart control limits
- When the q+1 observation result is in control then continue by forming the next control charts
- Whenobservationq+1isout of control then the observation is omitted and recorded in the check sheet. Create the next control chart do test in the next observation. The thing that has to be remembered is the number of observations that for me din the control chart can not be less than (p +2)

6) Decomposition Process for Uncontrolled Point

A very useful approach to diagnosis of an out-of-control signal is to decompose the T2 statistic into components that reflect the contribution of each individual variable.

If T2is the current value of the statistic, and T2(i) is the value of the statistic for all process variables

Except the *i* th one, then Runger, Alt, and Montgomery [5] show that

$$d_j = T^2 - T^2_{(j)} \tag{23}$$

is an indicator of the relative contribution of the ith variable to the overall statistic. When an out-of-control signal is generated, we recommend computing the values of d_i (i = 1, 2, . . . , p)and focusing attention on the variables for which d_i are relatively large. The larger value than the contribution of variable j to the point of uncontrolled is bigger.

7) Control Chart Accuracy through The G Index of Agreement computation

According to Guilford [1] to determine the homogeneity of the observation is by using the G-index through contingency tables as shown in Table II.

TABLE II. CONTINGENCY TABLE 2 X 2

T^2 Hotelling Control Chart	RealData (AfterChamber)		
	Not defective	Defective	Amount
In Control	p_{11}	p_{12}	p_{1+}
Out of control	p_{21}	p_{22}	P_{2+}
Amount	p_{+1}	p_{+2}	p_{++}

thevalue in the table are;

- p11 = m11 / m with the m11 is the number of observations which is in control and is not defective,
- P22 = m22 / m with m22 is the number of observations that are Out of control and defect
- p12 = m12 / m is proportion of in control and defect
- $p_{21} = m_{21}$ / m indicates the proportion of observations that are out of control and not defect
- $p_{1+} = m_{1+}$ / m and $p_{2+} = m_{2+}$ / m
- $p_{+1} = m_{+1}$ / m and $p_{+2} = m_{+2}$ / m is respectively the marginal proportion of not defective and defect on real data

Using data on Table 2, the equation used to calculate the accuracy of the control chart [1],

$$r_g = 2(p_{11} + p_{22}) - 1 \tag{24}$$

8) Significance Test of Control Chart Accuracy

To test the significance of the accuracy value it can be done with the formulation of hypotheses as follows; [1],

$H_0: r_g = 0$ (value of accuracy control chart not significant)
$H_1: r_g \neq 0$(value of accuracy control chart significant)

With the teststatisticss used is

$$z = \frac{(m_{11}+m_{22}) - 0.5m}{0.5\sqrt{m}} \tag{25}$$

or can be simplified to

$$z = r_g\sqrt{m} \tag{26}$$

Where $r_g$is a control chart accuracy value obtained from equation (26) and m is the number of observations, then reject the null hypothesis if the value of $z > z_{\frac{\alpha}{2}}$or p-value $\leq \alpha = 0.05$ so it can be stated that the accuracy of the control chart is significant.

III. Research Methodology

T^2 Hotelling distribution free is used to control the quality of ink for PRODUCT X. The data are 388 productionsunit for the period ranging from 1st April 2014 to 7thJuly 2014, involvingfive quality characteristics of ink, namely,

- X_1: Viscosity / viscosity (DPAs)
- X_2: pH
- X_3: Surface Tension
- X_4: Wavelength (Å)
- X_5: Particle Size (m)

Data processing method

- Testing Independence of Multivariate Data
- Multivariate Normal Distribution Test
- T^2Hotellingfree distribution control chart
- Plotting control chart with outlier removal
- The process of decomposition to the out of control point
- Process Capability Analysis
- Determination of Control chartAccuracy

IV. Results And Discussions

A. TestingIndependence of Multivariate Data

Independence Tests conducted using Bartlett Sphericity test with the following hypothesis formulation,

H_0: R=I (variable ink characteristics mutually independent)
H_1: R≠I (variable ink characteristics mutually dependent)
Where,

$$\mathbf{R} = \begin{bmatrix} 1 & & & & \\ -0.0905 & 1 & & & \\ 0.0275 & 0.0084 & 1 & & \\ 0.1630 & -0.0870 & 0.1053 & 1 & \\ 0.0701 & 0.0648 & -0.0177 & -0.0051 & 1 \end{bmatrix}$$

and | R | = 0.939174,

then by using equation (1) is obtained,

$$\chi^2_{hitung} = -\left\{388 - 1 - \frac{2(5)+5}{6}\right\} 0.939174 = 24.12913$$

so$\chi^2_{hitung} \geq \chi^2_{\frac{1}{2}5(5-1)} = 18.30704$ then rejected H_0, it means the ink characteristic variable mutually dependent and the use of multivariate methods fit for use.

B. Multivariate Normal Distribution Test

To visually test the multivariate normal distribution we can make a Q-Q plot from the distance value using Mahalanobis approach with the equation (7) in order to obtain the results show in Table 3.

TABLE III. DISTANCE VALUE USING MAHALANOBIS APPROACH

I	Observation Results					d_i^2
	v1	v2	v3	v4	v5	
1	3.889	8.63	27.6	0.69	115	7.606
2	3.967	8.57	28	0.67	113	10.11
3	3.920	8.64	27.7	0.68	110	4.381
4	3.826	8.65	27.4	0.67	113	4.482
5	3.889	8.65	27.7	0.68	111	2.655
6	3.967	8.62	27.8	0.68	119	7.730
7	3.861	8.77	27.8	0.67	111	1.679
8	3.830	8.57	27.7	0.68	110	5.376
9	3.854	8.65	27.7	0.68	110	2.959
10	3.808	8.64	27.5	0.68	117	5.892
⋮	⋮	⋮	⋮	⋮	⋮	⋮
384	3.840	8.64	27.6	0.67	114	14.034
385	4.001	8.59	27.5	0.67	120	10.105
386	3.988	8.59	27.6	0.67	111	6.318
387	3.913	8.61	27.5	0.67	119	2.896
388	3.839	8.79	27.8	0.68	112	3.705

Using software R we can see from the graph that the data does not spread around can be stated that the data is not multivariate normal distribution. The Kurtosis calculation also show the same result with Q-Q plot.

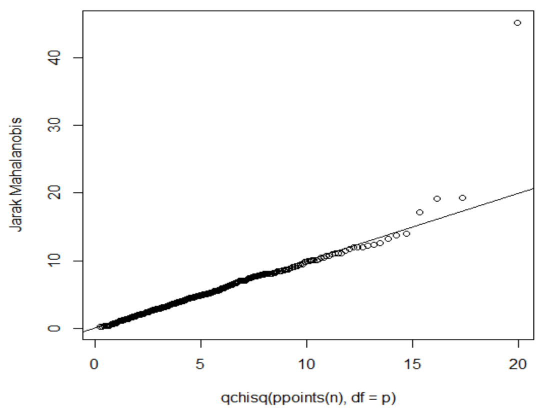

Fig. 3. PlotTestingMultivariateNormal InkQuality

C. Plotting Chart T2 Hotelling control with Outlier Removal Free Distribution

Utilizing 388data and five quality characteristics we compute individual T2Hotelling statistics for every observation point using equation(16). Having obtained the value ofT_1^2foreachobservationthenthe value iscompared with the(UCL) and(LCL).In this casethe LCL is0. UCL values is obtained from equation (20). With the k value obtained from equation (21),the value of k by using α = 0.05is calculated as follows,

$$k = \sqrt{\frac{1}{0.05}} = 4.472136$$

After analyzed388datawe obtain that there are four observations which are beyond control limit namely, theobservation44th, 272th, 327th, and378th.All of these four observations is the out of control observation. Using 388 observation to create the control chart we obtained the control chart in Fig7 .

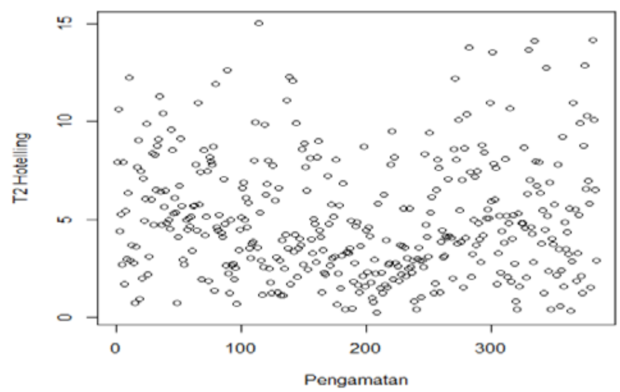

Fig. 4. T^2HotellingControl Chart with Outlier Removal Free Distribution

The control limits are established at 8.43717. Then the test is done to observation 388, with T2Hotelling statistics value of 2.896525.T2Hotelling statistics value is smaller than the control limits then we stated that the observation 388 are in control.

D. The Process Of Decomposition For Out Of Control Points

Based on the earlier stages the results showed that there are four out of control observations which is observation 44, 272, 327, and 378. Further analysis is done using equation (23) to determine which variables influence the out of control observation.

The results are presented in Table 4

TABLE IV. DECOMPOSITION ANALYSIS OBSERVATIONS

Data	T^2	d_1	d_2	d_3	d_4	d_5	Influence
44	11.0692	0.0081	1.7850	6.2915	0.5269	1.8262	variable X_3
272	22.8067	0.3224	0.0567	20.9818	0.0203	1.1486	variable X_3
327	45.3913	0.4773	0.5043	40.6245	0.3440	5.3600	variable X_3
378	19.8065	7.0484	5.7159	0.0019	4.6297	1.0691	Variable X_1

FromTable4 above we obtain some results as follows,

- In observation $44^{th}T^2$ Hotelling statistics value is equal to 11.0692, the variables contributed to the out of control incidence is variable surface tension.
- In observation of all $272^{th}T^2$ Hotelling statistics value is equal to 22.8067 the variables contributed to the out of control incidence is variable surface tension.
- In observation of all $327^{th}T^2$ Hotelling statistics value is equal to 45.3913 the variable contributed to the out of control incidence is variable surface tension.
-
- In observation to $378^{th}T^2$ Hotellingstatistics value is equal to 19.8065 the variable contributed to the out of control incidence is variable viscosity.

E. ProcessCapabilityAnalysis

Process capability analysis is carried out to know if the process is able to produce the products according to the characteristics determined by the company. From testing normality, it is known that the data is not normally distributed so that the process capability analysis is done using equation(24). C_{pc} value established for each variable can be seen in Table 5.

TABLE V. PROCESS CAPABILITY ANALYSIS

Variable	C_{pc} Value	Explanation
X_1	0.14427	$C_{pc} < 1.33$
X_2	0.30578	$C_{pc} < 1.33$
X_3	1.01988	$C_{pc} < 1.33$
X_4	0.30444	$C_{pc} < 1.33$
X_5	2.01572	$C_{pc} > 1.33$

FromTable5, it is obtained that variable viscosity, pH, surface tension and wavelength has C_{pc}smaller than 1.33, it means that for these variables the production process does not comply with the specifications, only for the particle size the production process is capable or has the ability to produce accurately.

F. ControlChartAccuracy

The resulted T2Hotellingcontrolchartare evaluated by constructing data in Table6.

TABLE VI. TABLE ACCURACY CONTROL CHART

T^2 Hotelling Control Chart	RealData(AfterChamber)		
	Not defective	Defective	Amount
In Control	0.9897	0	0.9897
Out of control	0.0103	0	0.0103
Amount	1	0	1

FromTable6 above by using equation(26) it is obtained that the accuracy of the control charts,$r_g = 2(0.9897 + 0) - 1 = 0.9794$

This means that the accuracy value of the control chart is at 0.9794or 97.94%. Having obtained the value of the accuracy of the control chart we can perform significance test of the r_g.

$H_0 : r_g = 0$ (value of accuracy control chart not significant)

$H_1 : r_g \neq 0$(value of accuracy control chart significant)

Testing the significance accuracy of the control chart with equation (28), found that the value of the test statistics is follows,

$$z = 0.9794\sqrt{388} = 19.29194$$

Because the value $z > z_{\frac{0.05}{2}} = 1.96$, then H0 is rejected which indicate that the accuracy value of the control chart is significant.

V. CONCLUSIONS AND RECOMMENDATIONS

A. Conclusion

Based on the research results, it can be concluded that the appropriate control chart for controlling in k characteristics is the T2HotellingControl Charts free distribution with Chebyshev theory. The creation of control chart is done with outlier removal method. Note that there is now only four out-of-control point on the chart. Viscosity and surface tension are variable seem to contribute a lot to the out of control observations. The accuracy of the control chart is 0.9794or97.94%, it means that the control chart is accurate to monitor the quality of the in k.

B. Recomendations

Ink quality data are multivariate in nature, so it is highly recommended that PT. EPSON Batam implement the T2Hotellingcontrol charts free distribution with outlier removal, the chart accuracy to control the ink quality characteristics is at 97.94%.

REFERENCES

[1] Guilford, J.P. 1956. Fundamental Statisticss in Psychology and Education. New York: McGraw-Hill Book.

[2] Johnson, A.R. andWichern, D.W. 2002. Applied MultivariateStatisticsalAnalysis5th.NewJersey:Prentice Hall,Inc.

[3] Mardia, K. V. 1970. Measures of Multivariate Skewness and Kurtosis with Applications.Biometrika. Vol. 50: 519-530

[4] Mason, R. L. and Young, J. C. 2002. Multivariate Statisticsal Process Control with Industrial Application. Philadelphia:ASA-SIAM University City ScienceCentre

[5] Montgomery, D. C. 2009. StatisticsalQualityControl A Modern Introduction Sixth Edition. NewYork:JohnWiley&Sons,inc.

[6] Morrison, D. F. 2005. Multivariate Statisticsal Methods Fourth Edition. The Wharton School University of Pennsylvania.