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An integrated MOEA and MCDM for multi-objective optimization (Case study: control chart design)

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Abstract

This paper proposes to design an X-bar control chart through an integrated three-stage multi-objective optimization process. The multi-objective formulation reflects the needs of X-bar control chart designing process's multiple objectives, e.g., the expected time that the process remains in statistical control status, the type I error, and the detection power. The optimization process starts with many-objective NSGA-II (NSGA-III), which is a multi-objective evolutionary algorithm (MOEA), to search the Pareto frontiers. Then, Data Envelopment Analysis model (DEA) is applied to find the efficient optimal solutions. After obtaining a manageable size of efficient solutions, a popular Multiple Criteria Decision Making (MCDM) technique, known as VIKOR, is applied to rank the optimal solutions. The proposed multi-stage multi-objective optimization process is applied in a case study and the outcomes are compared with the results in the literature. The comparison results reveal that the proposed multi-stage optimization process introduces more ranked practical efficient solutions.

Keywords

Multi-objective evolutionary algorithm (MOEAs), Multi-criteria decision making (MCDM), Control chart design, Data Envelopment Analysis (DEA), NSGA-III, VIKOR

1. Introduction

The engineering implementation of control charts involves a number of technical and behavioral decisions, which the important one is the design of control chart. Different methods of designing control charts have been proposed in the literature including using a simple rule suggested by Shewhart, a statistical criterion, an economic criterion, or a joint economic statistical criterion. Each method has some advantages and disadvantages relating to ease of use, statistical properties and cost effectiveness which are mentioned by Saniga (1989).

Generally, the design of a control chart requires the specification of three parameters, namely, sample size (n), the time interval (h) between successive samples, and the number of standard deviations (k) away from the in-control process mean that constructs the control limits width. The first economic model to monitor the mean of a normal process under a single assignable cause and to determine the design parameters with minimum total costs was developed by Duncan (1956). As a ratio between the expected cost during a cycle and the expected cycle time length, the expected hourly cost in (Duncan, 1956) is defined as

$$E_{HC} = \frac{a_1 + a_2 n}{h} + \frac{\lambda(a_3 + a_4 A + a_5 B)}{1 + \lambda B}, \qquad (1)$$

where

- a_1 is the fixed cost of sampling an inspection unit,
- a_2 is the variable cost of sampling an inspection unit,
- a_3 is the average cost to detect an assignable cause,
- a_4 is the cost of verifying a false alarm,
- a_5 is the hourly loss due to poor quality of units,
- A is the average number of false alarms per cycle, i.e. $A = \alpha/(e^{\lambda h} - 1)$,
- $\alpha = 2\int_{-\infty}^{-k} \phi(z)dz$ is the probability of false alarm, where $\phi(z)$ is the standard normal probability density function (pdf),
- B is the average time of the process being in out-of-control state, i.e. $B = h/P - \tau + gn + D$,
- $P = \int_{-\infty}^{-k-\delta\sqrt{n}} \phi(z)dz + \int_{k-\delta\sqrt{n}}^{\infty} \phi(z)dz$ is the detection power, where δ is the magnitude of shift occurred,
- $\tau = 1 - (1 + \lambda h)e^{-\lambda h}/(\lambda - \lambda e^{-\lambda h})$ is the average time of occurrence of an assignable cause between samples,
- g is the time required to sample, inspect and interpret the results,
- D is the time to discover and repair the assignable cause.

Since then, this research has motivated the most of subsequent studies in this area. There have been attempts in the literature to optimize the multi-objective design of control chart using different multi-objective optimization tools. Chen and Liao (2004) considered all possible combinations of design parameters as Decision Making Units (DMUs) and then applied DEA to compare DMUs base on relative efficiency factor. Faraz and Saniga (2013) applied Genetic Algorithm (GA) to a 2-objective economic statistical design of control charts with an application to x-bar and S-squared charts. Safaei et al. (2012)0 incorporated the Taguchi lost function and the intangible external costs in the economic design of X-bar control chart and used NSGA-II evolutionary algorithm to obtain Pareto optimal solution. Yang et al. (2011) applied multi-objective Particle Swarm Optimization (PSO) algorithm to optimize the design of X-bar and S control charts. Mobin et al. (2015) applied NSGA-II algorithms to generate Pareto optimal frontier and then DEA to reduce the Pareto optimal solutions to a workable size of efficient solutions for implementation. For attribute control charts, Amiri and Jafarian-Namin (2015) implemented a procedure using DEA for the design of different C charts. Moreover, Jafarian-Namin and Hasanzadeh (2016) applied a heuristic algorithm to solve nonlinear multi-objective problems by nonlinear lexicography goal programming (NLGP) for the design of fraction defective control chart.

Limited work exists in the literature combining the evolutionary algorithm called NSGA-III and DEA in the content of control chart design. There are some papers that used other evolutionary algorithms (e.g. NSGA-II) combined with DEA, but did not ranked the optimal efficient solutions. In other words, the DEA methods only evaluate the relative efficiency of solutions and many solutions will be at the efficient level of 100%. Tavana et al. (2016) used NSGA-III combined with MOPSO to solve the multi-objective design of control chart problem and used TOPSIS to rank the final solutions. In addition to statistical perspective, designing a control chart has several economic consequences as presented before. Thus, taking into account two statistical constraints, the multi-criteria decision making can be formulated as (Tavana et al., 2016):

$$\begin{aligned} &\max_s f_1 = ARL_0(s) \\ &\max_s f_2 = p(s) \\ &\min_s f_3 = E_{HC}(s) \\ &\text{s.t.} \qquad (2) \\ &\quad p(s) \geq p_L \\ &\quad \alpha(s) \leq \alpha_U, \qquad \forall s = (n, h, k) \end{aligned}$$

$$n \in \mathbb{Z}^+, \quad h, k \in \mathbb{R}^+$$

where $s = (n, h, k)$ is a possible set of design parameters, $ARL_0(s) = 1/\alpha$ is average run length when a false alarm occurs; $p(s)$ represents the detection power of the chart (after out-of-control status), and $E_{HC}(s)$ is the expected hourly cost. The constraints include the lower bound of detection power (p_L), and the upper bound (α_U) of type I error ($\alpha(s)$) which is the probability of false alarm. Note that one possible design of the control chart is a combination of n, h, and k, which is denoted by $s = (n, h, k)$ or DMU vector. The main assumptions of the model are summarized as following (Tavana et al., 2016):

1. The quality characteristic follows a Normal distribution,
2. The process is either in-control or out-of-control state only and is initially in the 'in-control' state;
3. When a random assignable cause of magnitude δ occurs, leads the process mean to shift from μ_0 to $\mu_0 + \delta\sigma$,
4. The occurrence of an assignable cause possesses a Poisson distribution with rate λ,
5. The process is allowed to continue during the search and repair.

This paper utilized an integrated NSGA-III, DEA and VIKOR to optimize design of X-bar control chart. After obtaining the optimal solutions in Pareto frontier shapes using NSGA-III, the DEA model is applied to evaluate the solutions obtained by NSGA-III algorithm in terms of relative efficiency. By using the relative efficiency concepts in DEA to eliminate those inefficient Pareto optimal solutions, the number of solutions will not be prohibitive for a decision maker to make choice. Efficient Pareto optimal solutions will be presented to the decision makers, and those designs which are not satisfactory in terms of the relative efficiency could be easily pruned. In addition, a Multiple Criteria Decision Making (MCDM) method called VIKOR is utilized in the last step to rank the optimal efficient solutions. Utilizing VIKOR technique provides a set of high ranked efficient optimal solutions and eases the decision making process for experts.

The remainder of the paper is organized as follows. Section 2 presents three-stage integrated NSGA-III/DEA/VIKOR approach of optimizing the multi-objective design of X-bar control chart. The proposed approach is applied to solve a numerical example borrowed from case study in the literature in Section 3. The final conclusion and future research are discussed in Section 4.

2. Proposed integrated solution methodology

2.1. Evolutionary algorithm

Most real-world problems, including design of control charts, involve simultaneous optimization of several incommensurable and often competing objectives. Often, there is no single optimal solution, but rather a set of alternative solutions. These solutions are optimal in the wider sense that no other solutions in the search space are superior to them when all objectives are considered. They are known as Pareto-optimal solutions. Since developing Evolutionary algorithms (EAs) there has been a growing interest obtaining the Pareto optimal solutions using different EAs. A comprehensive survey of evolutionary-based multi-objective optimization techniques are presented in Coello, C. A. C. (2013).

Specially, Non-dominated Sorting Genetic Algorithm (NSGA) (Yang et al., 2011), is a popular non-domination based GA which uses a non-dominated sorting procedure and applies a ranking method that emphasizes those good solutions and tries to maintain them in the population. Through a sharing method, this algorithm maintains the diversity in the population. The algorithm explores different regions in the Pareto front and is very efficient in obtaining sufficient Pareto optimal sets. However, it has been generally criticized for its computational complexity, lack of elitism and for choosing the optimal parameter value for sharing parameter. A modified version of NSGA, NSGA-II (Deb et al., 2000)0, utilizes a fast non-dominated sorting genetic algorithm. This method is more computationally efficient, non-elitism preventing, and less dependent on sharing parameter for diversity preservation.

Recently, a reference-point based many-objective NSGA-II algorithm (called NSGA-III) is proposed by Deb and Jain (2014) which shows its performance in dealing with more than two objectives problems. The main difference of NSGA-III and NSGA-II is in the selection mechanism where the former one is based on reference points to maintain diversity of the population, and the original NSGA-II is based on crowding distance. The main steps of the NSGA-III are presented in Fig. 1. The NSGA-III algorithm starts with N_{Pop} initial solutions called P_0 which are randomly generated. Recall that parameters of the problem is $n \in \mathbb{N}^+$ and $h, k \in \mathbb{R}^+$, and each solution in NSGA-III is represented by $s_i = (n_i, h_i, k_i)$ for $i = 1, .., N_{Pop}$.

1. ***Input***:
   P_0(Initial Population),
   N_{Pop} size of population,
   t (iteration) = 0,
   It_{max} (Maximum iteration of NSGA-III).
2. **While** $t < It_{max}$
3. Create Offspring Q_t
4. Mutation on Q_t
5. Set $R_t = P_t \cup Q_t$
6. Apply non-dominated sorting on R_t and find F_1, F_2, …
7. $S_t = \{\}$, $i = 1$;
8. **While** $|S_t| \leq N_{Pop}$
9. $S_t = S_t \cup F_i$
10. $i = i + 1$
11. **End**
12. **IF** $|S_t| = N_{Pop}$
13. $P_{t+1} = S_t$; break
14. **Else**
15. $P_{t+1} = \cup_{i=1}^{l-1} F_i$
16. *Normalize* S_t using min and intercept points of each objective
17. *Associate* each member of S_t to a reference point
18. *Choose* $N_{Pop} - |P_{t+1}|$ members from F_l by niche-preserving operator
19. **End**
20. $t = t + 1$
21. **End**
22. Report P_t

Figure 1: Pseudo-code of NSGA-III (Tavana et al., 2016)

The next step is generating offspring Q_t, which is based on the arithmetic crossover operator. In this operator, two randomly selected individuals, let say s_i and s_r, are selected to generate two offspring q_i and q_r as presented in (3) and (4), respectively.

$$q_i = (\beta)s_i + (1-\beta)s_r \quad (3)$$

$$q_r = (1-\beta)\, s_i + (\beta)\, s_r \quad (4)$$

where s_i and s_r are individuals from the current generation, q_i and q_r are individuals from new generation, and β is uniform random number between 0 and 1. The crossover operator is performed for each gene separately. Then, add the generated offspring to Q_t.

After generating offspring, Gaussian mutation operator is applied on the members of Q_t. This operator provides many mutations at the beginning of the algorithm and few at the end. In this way, it avoids disrupting the process of evaluation deeply modifying characteristics of chromosomes (and therefore of individual) (Coello, C. C., et al., 2007).

Recall n_i of each individual s_i ($i \in Q_t$) should be integer, and if it is not integer it should be rounded. Furthermore, the addressed problem has two constraints that should be satisfied by each individual. As suggested in (Deb and Jain, 2014), constraints of the model are normalized as (5) and (6), respectively.

$$g_1(s_i) = \frac{p(s_i)}{p_L} - 1 \geq 0, \forall s_i \quad (5)$$

$$g_2(s_i) = -\alpha(s_i)/\alpha_U - 1 \geq 0, \forall s_i \quad (6)$$

Then, constraint violation value ($CV(s_i)$) of s_i is calculated as (7), where $\langle x \rangle$ is $-x$ if $x < 0$, 0 otherwise.

$$CV(s_i) = \langle g_1(s_i) \rangle + \langle g_2(s_i) \rangle \quad (7)$$

Next, the parent population P_t and offspring Q_t are merged, $R_t = P_t \cup Q_t$. Note that size of R_t is equal to $2 * N_{Pop}$. After that, the fast non-dominated sorting based on Pareto dominance, suggested in 0, is applied on R_t to classify it into different non-dominance levels F_1, F_2, and so on. If all members of R_t are infeasible, the solution with the smallest constraint violation value will be assigned to F_1, and the solution with the second smallest constraint violation value will be assigned to F_2, and so on. If all members of R_t is feasible, then the usual non-dominating sorting procedure can be applied. If some of members of R_t are feasible and some of them are infeasible, the feasible solutions based on usual non-dominating sorting procedure will be assigned to the first levels, and then the infeasible solutions will be assigned to the higher levels. Having F_1, F_2, …, start from F_1, different level of non-dominance levels are selected to generate next generation S_t until its size reaches to N_{Pop} or its exceeds N_{Pop} for the

first time, let say, at non-dominance level *l*. Solutions which are in levels higher that *l* are rejected, and $S_t \backslash F_l$ are chosen for the next generation P_{t+1}. If size of P_{t+1} is N_{Pop}, the next iteration of the algorithm by generating new offspring starts, otherwise $N_{Pop} - |P_{t+1}|$ members from F_l are chosen based on reference points. To do so, objective values are first normalized as following.

***Normalizing the objectives*:** The objectives are normalized as (8) where $z_j^{min} = \min_{s_i \in S_t} f_j(s_i)$, a_j represents intercept point of objective f_j, f_j^n notes normalized objective f_j and $f_j(.) - z_j^{min}$ is translated objective.

$$f_j^n = \frac{f_j(.) - z_j^{min}}{a_j - z_j^{min}}, j = 1,2,3 \quad (8)$$

In order to find the intercept points, the extreme point z_j^{max} in each translated objective axis needs to be identified based on (9).

$$z_j^{max} = f_j(s^*) - z_j^{min} \quad (9)$$

where s^* is determined as (10) and (11).

$$s^* = \underset{s_i \in S_t}{\operatorname{argmin}} ASF(s_i, \boldsymbol{w}) \quad (10)$$

$$ASF(s_i, \boldsymbol{w}) = \max_{j=1,2,3} (f_j(s_i) - z_j^{min})/w_j \quad (11)$$

Note that $\boldsymbol{w} = (\epsilon, \epsilon, \epsilon)$ where $\epsilon = 10^{-6}$ and $w_j = 1$. After finding z_j^{max}, find a_j, the intersection of the hyper-plane of the objectives and each axis. For more detail see (Deb and Jain, 2014).

***Generating reference points*:** NSGA-III is based on predefined reference points to preserve the diversity of the population. The reference points are either provided by experts or generated by a systematic method such as developed in Das and Dennis (1998). Given the set of reference points $\Re$, each member of S_t is associated to a reference point which has the least Euclidean distance. Hence, the reference points (and their associated members) in F_l that are not well represented in $S_t \backslash F_l$ has higher priority to be in the next generation P_{t+1}.

In this study, the developed method presented in Das and Dennis (1998)0 is utilized to generate reference points $\Re$. In this method reference points are placed on the normalized hyper-plane and the number of reference points *H* depends on the number of objectives (m), in this study $m = 3$, as well as given division $g \in \mathbb{N}^+$ which determines the distance of reference points from each other on the same surface. The total number of reference points is determined as (12).

$$H = \binom{m + g - 1}{m - 1} \quad (12)$$

As suggested in Deb and Jain (2014) two-layered reference points are used in this paper (see Fig.2). Suppose g_1 and g_2 represent the divisions of outer and inner layers, respectively, then the total number of reference points is as (13).

$$H = \binom{m + g_1 - 1}{m - 1} + \binom{m + g_2 - 1}{m - 1} \quad (13)$$

***Niche-Preserving Operation*:** After finding the reference points, the perpendicular distance of each member of S_t from each reference line, the line connecting the origin to the reference point as presented in Fig. 2, needs to be calculated. Then, each member of S_t is associated to the closest reference point.

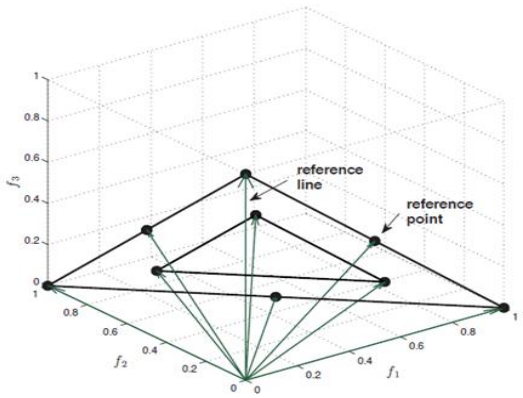

Figure 2: An illustrative example of two-layered reference points 0 et al. 2015)

In the end, some reference points may have one or more associated members and some may not have any associated members. Those members of $S_t \backslash F_l$ are associated to the reference point $j \in \Re$ are counted and presented by niche count ρ_j. Then, in order to have diverse population, the following procedure is used. First, the reference points with the minimum ρ_j is identified, break tie randomly in case there are more than one reference point. If $\rho_j = 0$, it indicates no member of $S_t \backslash F_l$ is associated to the reference point j. In this case, there are two possible cases;

(1) There are members associated with this reference point in F_l, in this case, assign the member with the minimum perpendicular distance to the next generation, P_{t+1}, and set $\rho_j = \rho_j + 1$.

(2) There is no member associated to this reference point in F_l, in this case, do nothing and go to another reference point with the minimum ρ_j.

If $\rho_j \geq 1$, the reference point *j* has more than one associated member, and in this case add an associated member from F_l, if exists, with the minimum perpendicular distance to the next generation, P_{t+1}, and set $\rho_j = \rho_j + 1$. The above procedure is repeated until population size reaches N_{Pop}.

2.2. Data Envelopment Analysis (DEA)

Data Envelopment Analysis (DEA), introduced in Charnes et al. (1978), is a powerful non-parametric approach to evaluate the relative efficiency of a group of decision making units (DMUs) with multiple inputs (i.e., cost type criteria) and outputs (i.e., benefit type criteria). Thus, it is of interest to calculate the relative efficiency of each DMU as the ratio of a weighted sum of outputs to a weighted sum of inputs. Assuming a set of *n* DMUs, each with m inputs and s outputs, the efficiency $(\bar{e}_j)$ for DMU_j is equal to $\sum_r u_r y_{rj} / \sum_i v_i x_{ij}$ when the weights u_r and v_i associated with output r and input i, respectively, are known. Based on this definition of efficiency, Charnes et al. (1978) proposed the following fractional programming problem to measure the technical efficiency of DMU_0:

$$\begin{aligned} & e_0 = \max \textstyle\sum_r u_r y_{r0} / \sum_i v_i x_{i0} \\ & \text{s.t.} \quad \textstyle\sum_r u_r y_{rj} - \sum_i v_i x_{ij} \leq 0, all\ j \\ & \qquad u_r, v_i \geq \varepsilon\ , \forall\ r, i. \end{aligned} \tag{14}$$

where ε is a non-Archimedean element smaller than any positive real number. Since this model is involving the ratio of outputs to inputs, it is referred to as the input-oriented model. The output oriented model is the inverted form of this ratio with minimization objective.

By making some changes in variables, the previous fractional programming problem can be converted to linear programming (LP) model, known as the primal problem. The duality of that is also a linear programming problem known as the multiplier or dual problem which provides detailed information for relative efficiency measure:

$$\begin{aligned} & min\ \theta_0 - \varepsilon \left(\textstyle\sum_r S_r^+ + \sum_i S_i^-\right) \\ & \text{s.t.} \quad \textstyle\sum_j \lambda_j x_{ij} + S_i^- = \theta_0 x_{i0}\ , i = 1, \dots, m \\ & \qquad \textstyle\sum_j \lambda_j y_{rj} + S_r^+ = y_{r0}\ , r = 1, \dots, s \\ & \qquad \lambda_j, S_i^-, S_r^+ \geq 0\ , \forall i, j, r \\ & \qquad \theta_0\ unconstrained \end{aligned} \tag{15}$$

where S_i^- and S_r^+ are slack variables. A comprehensive description of the DEA method is provided in Tavana et al., 2016, and Li et al. 2016.

2.3. VIKOR

VIKOR is based on the compromise programming of MCDM. The name VIKOR is from Serbian: VIseKriterijumska Optimizacija I Kompromisno Resenje, that means: Multi-criteria Optimization and Compromise Solution (Mobin et al., 2014). The basic mechanism to this approach is to calculate the "distance" from each alternative to a "Positive Ideal Solution" (PIS) and a "Negative Ideal Solution" (NIS) that are defined in "n-dimensional" space, where n represent the number of criterion in the decision problem. The chosen alternative should have the smallest vector distance from the PIS and the greatest from the NIS. In VIKOR, the PIS indicate the alternative with the highest value while the NIS indicates the alternative with the lowest value. The steps of VIKOR process presented by Mobin et al., 2014 are summarized as follows:

Step 1: Calculate the normalized values (f_{ij}). In VIKOR, the normalized values are calculated by using (16):

$$f_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, i = 1,2, \dots, m; \;\; j = 1,2, \dots, n. \qquad (16)$$

Step 2: Determine the best and worst values (f_i^* and f_j^-). The best and the worst values are calculated using (17) and (18), where $f_j^*$is the positive ideal solution for the j^{th}criteria and $f_j^-$is the negative ideal solution for thej^{th} criteria:

$$f_j^* = Max_i f_{ij}, i = 1,2, \dots, m \qquad (17)$$

$$f_j^- = Min_i f_{ij}, i = 1,2, \dots, m \qquad (18)$$

Step 3: Distance and calculation of final value.

Step 3-1: Compute the values $s_i$and R_i for $i = 1, \dots, I$. In this step, first the distance from each alternative to the PIS is calculated and then summed them up in order to obtain the final values as (19) and (20):

$$S_i = \sum_j^n \frac{w_i(f_j^* - f_{ij})}{(f_j^* - f_j^-)} \qquad (19)$$

$$R_i = Max_j \left[\frac{w_i(f_j^* - f_{ij})}{(f_j^* - f_j^-)}\right] \qquad (20)$$

where S_i is the distance rate of the i^{th}alternative to the PIS (best combination) and R_i is the distance rate of the i^{th}alternative to the NIS (worst combination). The excellence ranking will be based on S_i values and the worst rankings will be based on R_i values. Then, final value will be calculated by using (21).

***Step 3-2*:** Compute the values I_i for $i = 1, \dots, I$ as follow:

$$\mathrm{I_i} = \mathrm{v}\left[\frac{\mathrm{S_i} - \mathrm{S^*}}{\mathrm{S^-} - \mathrm{S^*}}\right] + (1 + \mathrm{v})\left[\frac{\mathrm{R_i} - \mathrm{R^*}}{\mathrm{R^-} - \mathrm{R^*}}\right] \qquad (21)$$

where $\mathrm{S^*} = \mathrm{Min}\ \mathrm{s_i}$, $\mathrm{S\_} = \mathrm{Max}\ \mathrm{s_i}$, $\mathrm{R^*} = \mathrm{Min}\ \mathrm{R_i}$, $\mathrm{R^-} = \mathrm{Max}\ \mathrm{R_i}$, and v: a weighting reference (usually consider as 0.5).

3. Case study

In this section, the proposed integrated optimization model is applied to the industrial case, which is borrowed from Mobin et al. (2015). The proposed integrated NSGA-III/DEA/VIKOR optimization model is applied in the following case study to show the efficiency of this approach in terms of quality and quality of final optimal solutions.

The case study is about the process of producing electrolytic capacitors. The parameters of this case study are summarized as follows: λ=0.25, a_1=1, a_2=0.1, g=0.01, δ=1, D=2, a_3= a_4 =50, a_5=200, $\alpha_U = 0.005, p_L = 0.95$.

Complex MODM methods involving software could be used to decide the best compromise solution for the foregoing mathematical model. The NSGA-III optimization algorithms, integrated with the DEA and VIKOR model, are applied in this paper to first generate optimal frontier, then obtain the best DMUs in terms of relative efficiency, and finally ranked efficient solutions for better decision making.

In this step, NSGA-III algorithms are used to solve the foregoing mathematical model. There are 3 decision variables in this multi-objective X-bar control chart design which are presented as vector $s = (n, h, k)$. NSGA-III algorithm is modified to deal with both discrete (n) and continues (h *and* k) decision variables. The proper scope for each design parameter is considered in potential solution initialization step of algorithms. For comparison propose, the scope of sample size (n) is confined as from 20 to 30, the same interval as considered in Mobin et al. (2015). Similarly, the scope of sampling time interval (h) is set as from 0.4 to 0.5 hours, while the scope for control limit width is from 2.9 to 3.8 in terms of standard deviation σ.

Before running the proposed NSGA-III algorithm, its parameters need to be tuned to control its convergence and running time. Based on the initial experiments, the parameters of the algorithms are set as following: the probability of mutation $p_m = 1/55$, the probability of crossover $p_c = 1$, the number of population of the algorithm $N_{Pop} = 55$, and maximum number of iteration $It_{max} = 60$, and $g_1 = g_2 = 6$. Since combination of g_1 and g_2 using (13) generates 56 reference points, one point more than population size, at each iteration one of the extreme reference points is randomly chosen and ignored in associating the population to the reference points. The computation time of NSGA-III algorithm for 55 population is obtained as 40.4596 seconds.

We defined 55 as the number of population. Multiple runs of the NSGA-III algorithm to the MODM X-bar control chart design formulation generate a very stable Pareto frontier as shown in Fig. 3.

In the next step, data envelopment analysis (DEA) is performed to compare the relative efficiency of the 55 Pareto optimal solutions obtained from NSGA-III algorithm such that a few workable Pareto optimal solutions can be presented for decision making for implementing control charts. When applying the DEA method, all the Pareto optimal solutions are considered as DMUs. The cost objective is considered as input variable and the average run length and the detection power objectives are considered as output variable. To obtain the efficiencies of the 55 DMUs for each algorithm, a linear program needs to be solved for each DMU. Obviously, as the objective function changes from each linear program to another one, the weights for each DMU may be different. Furthermore, in DEA method, there may be more than one efficient DMU with relative efficiency equal to one, as each individual DMU is trying to select a preferable weight set when evaluating the efficiency of this DMU. The higher relative efficiency value represent that a higher output value can be obtained under a relatively lower amount of weighted inputs.

In this study, CCR input oriented (CCR-io) DEA model is applied to evaluate the relative efficiency of the 55 solutions obtained from NSGA-III. The input of this model is considered as cost objective, while the outputs are average run length and detection power objectives utilizing the CCR-io DEA model, the relative efficiency of Pareto results are obtained. The results revealed that out of 55 Pareto solutions obtained from NSGA-III algorithm, 21 solutions are identified as fully efficient ($\theta_0^* = 1 \; and \; S_i^{-*} = S_r^{+*} = 0$) in CCR-io DEA model. Fig. 3 shows the solutions that are identified as efficient solutions from CCR- io DEA model.

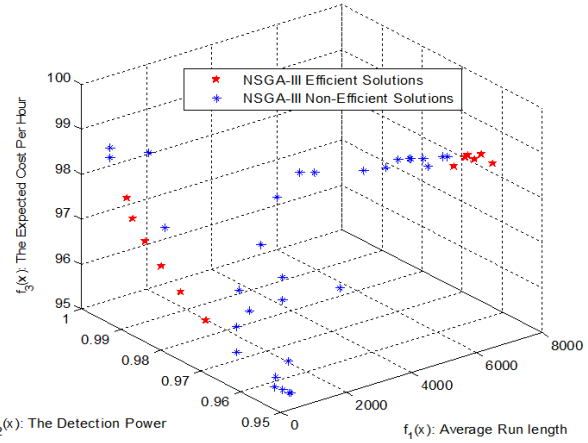

Figure 3: Efficient versus non-efficient solutions (NSGA-III)

In summary, the DEA model can significantly reduce the large Pareto optimal solutions to a few implementable efficient solutions from an economic perspective of considering all three objectives. The information from both the original Pareto frontier and the efficient solutions from DEA can be used to select final solutions for control chart implementation.

Then, the efficient solutions which are obtained by CCR-io DEA models are prioritized using VIKOR technique. For this purpose, objective functions are considered as criteria. The first and second criteria are considered as positive criteria, since the first and second objectives are maximization. The third function is cost which is considered as a negative criterion. For the sake of complexity, the weights of criteria are considered as equal, although different values can be considered as weights of criteria based on the firm's priority. After applying VIKOR technique, the rank of each DMU is obtained. Top 10 optimal efficient solutions obtained by VIKOR are identified as (Rank 1 to 10): DMU35, DMU05, DMU30, DMU42, DMU04, DMU29, DMU36, DMU49, DMU12 and DMU14. Fig. 4, presents the top 10 ranked efficient optimal solutions.

In addition to the VIKOR method, there are other MCDM approaches that can be used to rank the optimal solutions obtained by evolutionary algorithm. For example, TOPSIS method, presented in Salmon et al. (2015) is used by Tavana et al. (2016) to rank the Pareto optimal solutions. Other MCDM approaches that can be used to rank the alternatives with defined criteria, i.e, the optimal solutions with different objective function values, can be listed as: Analytical Hierarchy Process presented in Malek et al. (2016), COPRAS method described by Mobin et al. (2015), and the DEMATEL method described in Vafadarnikjoo et al. (2015). In addition, other evolutionary algorithm such as imperialist competitive algorithm (Borghei et al., 2015), the ant colony algorithm (Vafadarnikjoo et al., 2015), and the general variable neighborhood search algorithm (Komaki et al., 2015) can be considered to find the optimal solutions. Furthermore, different clustering techniques such as k-means (Rastegari et al. 2016) can be used to group

the optimal solutions with similar properties to have more efficient decision making process.

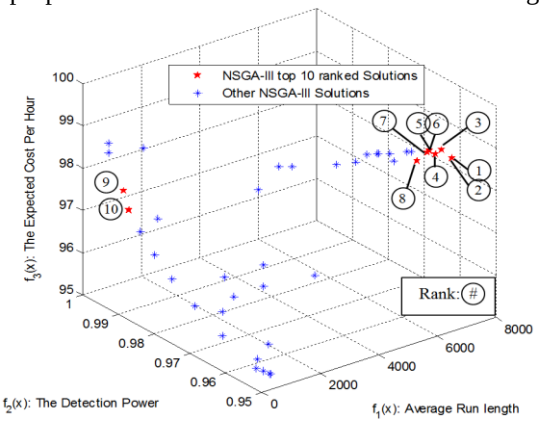

Figure 4: Top 10 ranked efficient solutions obtained by NSGA-III / DEA CR-io / VIKOR methods

4. Conclusion

A new multi-stage multi-objective optimization approach in designing an X-bar control chart is proposed in this paper. The proposed integrated optimization approach starts with NSGA-III algorithm to find the Pareto frontiers. NSGA-III performance in Pareto frontiers has been proven in recent studies. The main difference of the NSGA-III and its earlier version, NSGA-II, is in the selection mechanism where the former one is based on reference points to maintain diversity of the population, and the original NSGA-II is based on crowding distance. Reference points are sophisticated and innovative method to have diverse and well-distributed solution. At second stage, the DEA optimization tool is utilized to evaluate the efficiency of Pareto optimal solutions. Applying DEA tools provide a set of efficient optimal solutions. Although DEA provides a workable size of design of X-bar control chart, but it's still difficult for decision makers to find the best design. Therefore, one MCDM method called VIKOR is used to rank the final set of efficient optimal solutions. The advantages of this proposed three-stage approach is to have better exploration in the feasible solution area using NSGA-III, obtaining all efficient optimal Pareto frontier solutions using DEA approach, as well as ranking the efficient optimal solution to have more productive decision making. A numerical example of multi-objective design of an X-bar control chart is solved using the proposed optimization approach. The ranked efficient optimal solution provide the QC manager a variety of appropriate design of X-bar control chart in terms of optimal allocation of the sample size (n), control limits (k) in terms of a known process standard deviation (σ) and the sampling frequency in terms of the interval (h) of two successive samples. For future research, the sensitivity analysis of the parameters of the multi-objective design of an X-bar control chart can be conducted to investigate the effect of model parameters on the solutions.

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