Warranty Analysis of Remanufactured Electrical Products

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Abstract

Warranty issuance is key in ensuring a healthy manufacturer—consumer relationship. Manufacturers hope to minimize warranty costs while consumers believe that good warranty promises better product quality and reliability. This paper presents a framework for optimizing the warranty period from the perspective of a manufacturer to maximize the total expected profits, while ensuring sustained consumer relation. The case study is based on a remanufactured electronic product, and we use real data from a local company with a global supply chain footprint to provide the numerical example.

Keywords

Warranty optimization, remanufactured products, renewal function, Weibull distribution.

1. Introduction

The Remanufacturing Industrial Council defines remanufacturing as a comprehensive and rigorous industrial process by which a previously sold, worn, or non-functional product or component is returned to a "like-new" or "better-than-new" condition and warranted in performance level and quality (R. Steinhilper, 2001) . Different from rebuilding, refurbishing, reconditioning, repaired and other derivatives of product restoration processes, remanufacturing is becoming the standard term for the process of restoring used products to a "like new" condition (R. Steinhilper, 2001). According to Lundand Hauser (Lund & Hauser, 2010), the remanufacturing industry has existed in the United States for at least a century, with strongest application in the automotive industry. During World War II, much of the machinery industry was occupied with the production of much needed weaponry. Therefore, remanufacturing is what sustained the automotive market (Rolf Steinhilper & Brent, 2003). After a century of developments, remanufacturing is currently an important and growing activity in many industrial sectors. Some of the most commonly remanufactured product categories are: aircraft components, automotive parts, electrical and electronic equipment, engines and components, medical equipment, office furniture, printing equipment, restaurant and food-service equipment.

The latest report published by the U.S. International Trade Commission (USITC) (Williamson, Pearson, Aranoff, & Pinkert, 2012), provides an overview of the U.S. and global remanufactured goods industries and markets. According to the report, although the statistical data on global trade in remanufactured goods are largely unavailable, the United States and Europe account for the bulk of remanufacturing and trade in the world. Even though their size of remanufacturing is relative small, other countries like Brazil, India and China are fairly catching on.

In order to comprehend the rapid development of the remanufacturing industry, we begin with discussing the major constituents of the remanufacturing industry. There are three main participants in remanufacturing industry. First are the remanufacturers. They comprise of original equipment manufacturers (OEMs) or independently remanufacturers (IRs). Generally, OEMs produce new products and sells them. After the products worn out, OEM collects the used products and remanufactures them in house. One the other hand, IRs collect used products irrespective of the original OEM, they then remanufacture these used products for resale. In this paper, we consider that the remanufacturing entity is an OEM who sells the remanufactured product with a flat rate warranty agreement. To these manufacturers, this recovery process is profitable from the perspective of the reduced consumption of energy and virgin material.

The second category of key players in the remanufacturing business is the consumer base, which will purchase the remanufactured products. To these consumers, remanufactured products, especially with assured warranties have equal product integrity like new products but at lower prices. Furthermore, with the increasingly environmental awareness, more consumers are embracing remanufacturing as a way to promote global manufacturing sustainability.

The third main participant is the government. The USITC found that: U.S. exports of remanufactured goods total $11.7 billion in 2011, up 50 percent compared with 2009. Canada, the European Union, and Mexico are important markets for U.S. exports of remanufactured goods. During the 2009-2011period, production of remanufactured goods grew by 15 percent to at least $43 billion in U.S., supporting 180,000 full-time jobs (Williamson et al., 2012). This means remanufacturing is not just a concept on environmental protection any more, but a real contributor to the national employment sector and a booster of the country's economy. There are challenges that beleaguer the manufacturing industry, such as uncertainties in the rate and quality of returned products, technological changes and resource allocation such as labor. The challenge that we address in this paper is customer's reduced perception of the quality of remanufactured products. To mitigate this challenge, manufacturers are forced to find ways to assure consumers about their products' quality and reliability with respect to new products. Warranty issuance, one of the ways which manufacturers offer assurance, and has a key role in the trade-off between product quality and consumer impression.

There are two papers in literature, (Pang, Casalin, Papagiannidis, Muyldermans, & Tse, 2015) and (Jimenez-Parra, Rubio, & Vicente-Molina, 2014) that analyze customer behavior when purchasing remanufactured products in the UK and Spain respectively. They use different study methods but have same conclusion that consumers regard both the product's price and the reputation of the remanufacturer as of particular importance in their intention to purchase a remanufactured product. Particularly, a good warranty policy (but not necessarily the length of warranty) has a positive indication on the reputation of a remanufacturer. These findings by (Pang et al., 2015) and (Jimenez-Parra, Rubio, & Vicente-Molina, 2014) address the following research question: What is a good warranty policy for a remanufactured product? It is definitely not "the longer the policy the better" because Pang's study results already show that the warranty length is not a significant factor. From a remanufacturer's perspective an optimal warranty should maximize the remanufacturer's profit while portraying high product quality to the consumers. This paper therefore, seeks to answer this question by determining the most optimal combination of warranty and price for remanufactured products. To make the analysis as realistic as possible, real failure data of remanufactured electronic products obtained from a reputable fortune 500 company based in the U.S. is used as a case numerical example.

Even though there is a huge market opportunity in the remanufacturing industry, we cannot ignore the both current and future challenges, some of which will be discussed the following section.

Lack of regulations and standards
This challenge is government related, especially in emerging economies like India and China. An important barrier in the remanufacturing industry is the lack of regulations, enforceable laws, penalties and incentives to motivate manufacturers. For example, if the government can provide some economic incentives or preferential tax policies, more manufactures are likely to incorporate strategies for responsible manufacturing, such as remanufacturing in their business plans.

Rapid product technology changes
In the U.S., the 1-year rate of change (how long consumers keep a product) for electronic products is almost 9% (Kline, 2014). This percentage is indicative of the overall rate of technology change in the electronic industry. This change means that manufacturers have to invest more money to manufacture and launch new products to the market faster. However, for the remanufacturing industry, the manufacturer has to spend money and time to collect product returns; train operators to enable them work on older versions of the products and create a remanufacturing process that is flexible enough to accommodate a wide range of product returns. The more frequently products are upgraded, the higher cost of remanufacture (Sundin & Dunbäck, 2013).

Consumer concepts

Most literature about remanufacturing focuses on the operational and organization aspects of the process, with less attention being focused on customers' perception of remanufactured products. This is a double-sided challenge not only affecting consumer but also manufacturers (Michaud & Llerena, 2006) . On one hand, consumers lack confidence in the quality of remanufacturing products, and on the other hand the manufacturers incur increasing costs to ensure quality, which in turn increases the price of remanufactured products. This calls for models that are able to strike a cost-quality-warranty tradeoff.

2. Warranty Analysis for Remanufactured Products

Over the past decades, several review papers have been written about product and service warranty policies. A rich compilation of literature on warranty can be found in (Shafiee, Chukova, Saidi-Mehrabad, & Akhavan Niaki, 2011), (Thomas & Rao, 1999), (Murthy & Djamaludin, 2002) and (Karim & Suzuki, 2005) all of which are review articles. An elaborate list of close to 190 warranty articles can be found in (Murthy & Djamaludin, 2002), in which literature on warranty, especially for new products is broadly categorized into the following areas: warranty policies, warranty cost analysis, warranty and engineering, warranty and marketing, warranty and logistics, and warranty and management. In addition, a book by (Wallace, 1993) provides a wide coverage of warranty taxonomy, theories, policies, and models, cost analyses as well as a few case studies. The warranty model that has been used in this study is adopted from Blischke's book (Wallace, 1993).

There is also a substantial amount of literature covering warranty for second hand products. Chattopadhyay and Murthy (Chattopadhyay & Murthy, 2000) are the earliest researchers to publish articles in this area. In their paper "warranty cost analysis for second-hand products", Chattopadhyay and Murthy suggest that dealers of second-hand products should estimate the warranty cost and build this cost into the product price structure. This is because customers of second hand goods are more concerned with protection against product failure. They then provide two approaches to demonstrate how to estimate the expected warranty cost under free-replacement warranty and pro-rata warranty from a system and component level perspectives. With similar goals as Chattopadhyay and Murthy, (Saidi-Mehrabad, Noorossana, & Shafiee, 2010) researched about minimizing the total mean cost of product from a seller's perspective, by choosing the optimal reliability improvement and warranty strategy for second-hand products. They applied two methodologies—the virtual age model and screening test approach to estimate the reliability improvement cost. They then add this improvement cost to the corresponding warranty cost, under a specific warranty policy to optimize the total cost. On one hand, their results show that reliability improvement is usually costly and adds directly to the sale price of the second-hand products. On the other hand, reliability improvement cost reduces the overall warranty cost. So the key is to find the balance between optimal upgrade level of reliability improvement and warranty cost of second-hand products.

As mentioned earlier, this paper is concerned with the analysis of warranty for remanufactured products. There are many studies in the field of remanufacturing as a whole. One of the earliest studies of remanufacturing was by Lund in the early 1970s, and since then, a lot research has addressed remanufacturing, including (Hatcher, Ijomah, & Windmill, 2011)—a review paper within which additional references can be found. However, most researchers are concerned with remanufacturing as a closed-loop supply chain strategy as well as establishing feasible models such as the End-of-life (EOL) models for a variety of products. Quite rarely do studies consider warranty analysis of remanufactured products. In this section, we will discuss the four papers found in literature that directly relate to warranty analysis of remanufactured products.

Impact of quality variations on the warranty cost of remanufactured-products

Yu and Peng (Yu & Peng, 2009) present the impacts of the quality variations on the remanufactured product in the process of remanufacturing and build models for reliability and the warranty cost under the Free Rectification Lifetime Warranty (FRLTW) policy. They list three possible ways that the reliability of remanufactured products can be affected, i.e. in the initial product design stages, presence of non-conforming components and errors during reassembly. They model the failure distribution for each of these causes and integrate them into the warranty cost model. Their goal is to determine how the three process variations affect the warranty cost. They summarize that reassembly errors affect the reliability of the remanufactured product more than non-conforming components.

Warranty policy analysis for end-of-life product in reverse supply chain
One of the earliest research presented in the area of warranty policies in reverse supply chain was by (Glickman & Berger, 1976). They maximize the profit of a product sold under warranty by optimizing the price and warranty length. (Ammar & Gupta, 2015) evaluated the warranty cost for end-of-life products and predict an optimal warranty period for the disassembled components using sensor information embedded in a product. From the sensor information, they track the age and usage of each end-of-life product to meet the material demand while minimizing the warranty cost.

3. Model

Since there is no existing warranty cost model particularly established for remanufactured product, we opt to find an applicable warranty model for new product, and extend it to the remanufacturing field. In our case, we make use of the warranty cost model for repairable items. This is because we can assume that the repair process is able to renew the product—to as good as new condition. The difference therefore, between a repaired and remanufacture is that a repairable item's outcome can range from as "good as new" to "better than old" (minimal repair). Similar to remanufacturing, when the failed product is repaired to as good as new condition, it is assumed that the failure rate of the repaired product is the same as that of a new product.

3.1 Basic Cost Model for Remanufactured Product

According to Blischke and Murthy [14], there is a basic cost model for supplier to replace a single repairable product under Free-Replacement warranty (FRW).

$$C_s(w) = c_s + c_r N_r(w) \qquad (1)$$

Where c_s is the average cost to the seller of providing a new product without warranty and c_r is the expected total cost of supplying a repaired product under warranty. N_r (w) is the expected number of repairs required during the warranty time w. So that $c_r N_r$ (w) is the expected warranty cost and C_s(w) is the estimated total cost for the Original Equipment Manufacturer (OEM) to sell a new product with repairable warranty length w.

As mentioned earlier, the remanufacturing industry normally has two entities carry out the process, i.e. the OEMs who produces both the new product and refurbished used one to new, and the IRs, who only focus on the used product for remanufacture. In reality, whether the process is carried out by an OEM or IR, c_s can be seen as the average cost for initial product without warranty. For OEM, the initial product is the new product, while for an IR, the initial product is a product that has been used once, and brought back for remanufacture. Hence,

$$C_R(w) = c_m + c_r N_r(w) \qquad (2)$$

Where c_m is the average cost to OEM or IR of providing an initial product without warranty and C_R (w) is the total cost for OEM or IR to sell a remanufactured product with warranty W. N_r (w) and c_r retain their notation similar to Equation (1). From the basic cost model of remanufactured product, the initial cost c_m may include manufacturing costs, distribution costs and all other costs associated with providing the item to consumers. Remanufacturing cost c_r includes material costs, labor costs, shipping costs and administrative costs, among other direct and indirect remanufacturing costs. So, $c_r N_r$ (w) is the warranty cost for remanufactured products and c_r can be obtained from company's accounts. However, it is not possible to know the value of N_r (w) deterministically, but it has to be calculated using a stochastic approach. To do this, the failure rate of the product has to first be defined.

3.2 Failure Rate and Expected Numbers of Failure in Warranty

N_r (w) is the number of remanufacturing sessions required for a given product during the warranty period w. In reality, N_r (w) is a random variable that varies from one product to another. We therefore need to find a way to estimate N_r(w). The renewal function of a product offers a credible approach to estimate the expected value of N_r (w) (Wallace, 1993). Since we intend to investigate the warranty cost for remanufactured products, we must be able to estimate the expected number of returned products that are claimed to have failed during the warranty period. Failure rate is a key in estimating the expected number of failures during warranty. Generally, we consider the failure rate as the frequency with which an industrial system or component fails. Let X_1 denote the time of a product's first failure. Let F(x) denote the distribution function for the X_1. Then the probability of product failure in the interval [x, x+t], given that no failure happens before x, is

$$F(t|x) = [F(t+x) - F(x)]/\bar{F}(x) \qquad (3)$$

Failure rate is therefore

$$r(x) = \lim_{t\to 0} \frac{F(t|x)}{t} = \frac{f(x)}{\bar{F}(x)} \qquad (4)$$

For example, if we consider a product's first failure time is exponentially distributed, F(x) = 1- $e^{-\lambda t}$. Then the failure rate for this product becomes

$$r(x) = \lim_{t\to 0} \frac{F(t|x)}{t} = \frac{f(x)}{\bar{F}(x)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda \qquad (5)$$

for $0 \leq x < \infty$, $\lambda > 0$.

After we know the failure rate of a product, we can determine the expected number of failures during a specific time period using the renewal function approach. According to (Wallace, 1993) a counting process {N (t), t ≥ 0} can be used to represent a delayed renewal process if the following conditions hold:

1. N (0) = 0.
2. X_1, the time to first event, is a nonnegative random variable with distribution function F (x).
3. X_j, $j \geq 2$, the time intervals between jth and (j-1)st events, are independent and identically distributed random variables with distribution function G (x), which is different from F (x).
4. N (t) = sup {n: $S_n \leq t$}, where $S_0 = 0$ and, for $n \geq 1$, where S_n is the time instant of the nth renewal (or remanufacture in our application).

$$S_n = \sum_{i=1}^{n} Xi \qquad (6)$$

When G(x) equals F(x), the process becomes an ordinary renewal process.

For a repairable product, it is usually assumed that the repair process is less than perfect and the repaired items are not good as new. An alternative model in which is recognized widely in literature is based on the assumption that the initial product has a lifetime of X_1 with distribution F(.), and the repaired items during warranty are assumed to have lifetimes X_2, X_3…, identically distributed with distribution G(.). Since the definition indicates that the remanufactured products have the same failure function with new one, then the failure rate of remanufactured product will not be changed. We consider this assumption as credible given that the company on which the numerical case example is based upon provides the same warranty for both new and remanufactured products. This means that the initial remanufactured product has a lifetime X_1 with distribution F(.), and the recurrent remanufacture processes of the same item while in warranty, offer X_2, X_3…, lifetimes with the same distribution F(.). Therefore, we conjecture that remanufactured products exhibit an ordinary renewal process.

Blischke and Murthy already proved that the expected number of renewals in [0,t] is

$$\mathrm{M(t)} = \mathrm{F(t)} + \int_0^t \mathrm{M\,(t-x)f(x)dx} \qquad (7)$$

The function in Equation 7 is called the renewal integral equation and M(t) is called the renewal function associated with the distribution function F(t). Considering the remanufacturing industry, M(t) means the expected number of products that need to be remanufactured during warranty time [0,t]. As will be seen in later case study, the renewal integral equation plays an important role in the warranty analysis. The difficulty is figuring out M(t) from Equation 7 in which M(t) appears on both sides of the equation. It is only for a small group of distribution functions F(t) that M(t) can be calculate analytically in closed form. For example, Blischke and Murthy [14] demonstrate that for exponential distribution with parameter λ, the renewal function is M(t) = λt.

This means that if a kind of remanufactured product's lifetime follows an exponential distribution with parameter λ. Then,

$$\mathrm{M(t)} = 1 - e^{-\lambda t} + \int_0^t \lambda(\mathrm{t-x})\,\lambda e^{-\lambda x}\mathrm{dx} = \lambda t \qquad (8)$$

3.3 Optimal Warranty Period Model

Once we know the lifetime distribution, failure rate and expected number of returns during warranty length of a product, we can start to build mathematical model to seek the optimal warranty period for remanufactured products. The warranty length is a critical factor. A longer warranty not only means greater protection to consumers, it also indicates a better quality. That means warranty length will affect marketing demand, hence it is also a factor in determining the profitability of a product.

Generally the demand for remanufactured product depends on the marketing strategies. We use the general demand function Q (C_p, w) proposed by (Glickman & Berger, 1976) as shown in Equation (9)

$$\mathrm{Q}\,(C_p, \mathrm{w}) = k_1 {C_p}^{-a}(\mathrm{w} + k_2)^b \qquad (9)$$

Where w is the length of the warranty period, C_p is the sale price of product, k_1> 0 is a an arbitrary constant known as an amplitude factor, k_2≥0 is a constant of time displacement which allows for non-zero demand with no warranty offered, a>1 is the parameter of the price elasticity, and 0<b<1 is the parameter of warranty length elasticity. In this demand function, we see that the product demand decreases exponentially with respect to price and increases exponentially with warranty period.

As mentioned in Section 3.1, C_R(w) is the total cost for OEM or IR to sell a remanufactured product with warranty w, so, E[C_R(w)] denote the expected cost per unit sale.

$$E[C_R(w)] = c_m + c_r E[N_r(w)] \qquad (10)$$

Then the expected profit per unit sale $\pi(C_p, w)$ is given by

$$\pi(C_p, w) = C_p - E[C_R(w)] \qquad (11)$$

Multiplied by the demand function (9), we can get the total expected profit

$$\Pi\,(C_p, w) = Q(C_p, w)\pi(C_p, w) \qquad (12)$$

Using Equations (10) and (11) in (12), we have the profit calculated as

$$\Pi\,(C_p, w) = k_1 {C_p}^{-a}(\mathrm{w} + k_2)^b\{C_p - c_m - c_r E[N_r(w)]\} \qquad (13)$$

What we need is to maximize the profit, $\Pi\,(C_p, w)$ and subsequently obtain the optimal value of w* and C_p*, which denote warranty length and price per item respectively.

For example, assume the failure distribution of a product is exponential distribution with parameter λ. Then according to Equation (8), the expected value of $N_r(w)$ is given by $E[N_r(w)]$ = M(w)=λw. The total expected profit is given by

$$\Pi\,(C_p, w) = k_1 {C_p}^{-a}(\mathrm{w} + k_2)^b\{C_p - c_m + c_r \lambda \mathrm{w}\} \qquad (14)$$

4. Case Study

We use raw data received from the partner company which cannot be displayed in this paper due to proprietary agreements. In total, there were 5192 product-return records from April 2012 to April 2015 in the database. The data includes a family of an electronic product, which is identifiable using their catalogue identities (CatID). Each individual product has an exclusive serial number, which is used to track the number of times a product has been remanufactured. In addition, the serial numbers are used to track and estimate the time intervals between failures (returns) for each product. Once the product fails, the customer claims a failure under warranty and receives a new product or places a remanufacturing order (RO). In the database, RO creation date is the time when a return order is created. Since RO is the closest the actual failure time, we used RO date to represent the failure time.

After pre-processing the raw data, 1168 products were found to have been returned more than once for remanufacture. These 1168 data points were most useful in this study, from which we estimated the time between failures. Among the 1168 returned products, 28 were returned 4 times, 156 were returned 3 times and the rest (984 products) were returned twice during April 2012 to April 2015. Next step was to calculate interval days between each return for every product. There were a total of 613 interval data. These were used as the actual time between failure data. The Weibull distribution was the most suitable function for the time between failure data, with the highest P-value of 0.4851 at a 95% confidence limit. The second most fitting distribution was the Gamma distribution with a P-value of 0.2568. Hence we resolved use the Weibull distribution with the suggested shape parameter $\beta = 1.1794$ and scale parameter of λ of 269.93, thus $1/\lambda \approx 0.0037$. Figure 2 is the frequency curve of the actual data.

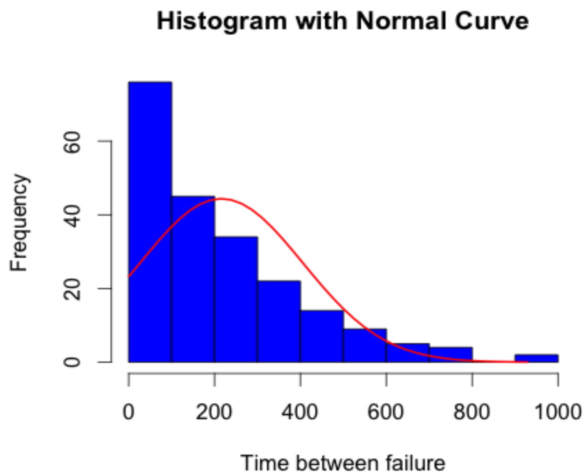

Figure 1: Product Returns Frequency Graph of Time Between Failures (in days)

Figure 2 shows the probability distribution function (pdf) of the failure data.

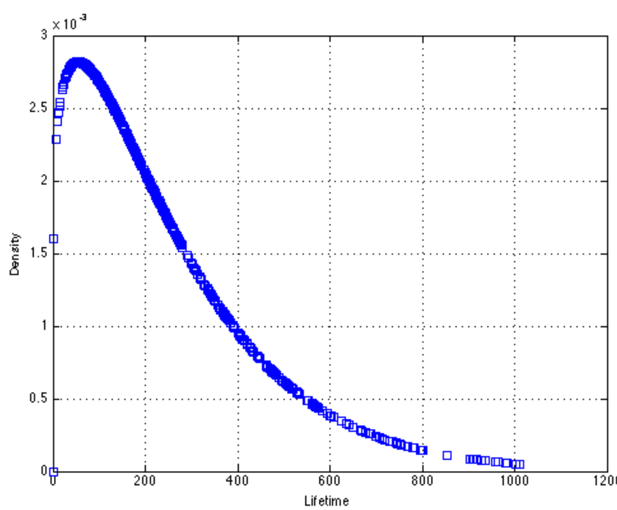

Figure 2: Plot of Probability Density Function of Product's Time Between Failures (in days)

Once we know the probability distribution function, the next step is to calculate the expected number of returns during warranty by using the renewal function M(.). However, as mentioned in Section 3.1, only a small group of distribution's renewal function can be solved analytically in closed form. Unfortunately, the Weibull distribution is not among them. Several researchers have proposed various methods to solve the renewal function of the Weibull distribution. We chose to consider the approximation method by (Jiang, 2010) because his approximation methodology is not only appropriate for our assumptions, but also sufficiently accurate and relatively computationally simple to execute. In Jiang's method, the renewal function of Weibull distribution can be approximated by

$$M(t) = F^p(t)H^q(t) \qquad (15)$$

where F(t) is the cumulative Weibull distribution given by

$$F(t) = 1 - \exp[-\left(\frac{t}{\lambda}\right)^{\beta}] \qquad (16)$$

H(t) is the cumulative hazard function which is given by

$$\mathrm{H(t)} = \left(\frac{t}{\lambda}\right)^{\beta} \qquad (17)$$

Here p>0 and q > 0 are parameters to be determined from the Weibull F(t) as follows:

$$p(\beta) = 1 - \exp[-\left(\frac{\beta-1}{a}\right)^{b}] \qquad (18)$$

Where a and b are hyper-parameters of the prior distribution of β, the shape parameter of the Weibull distribution. In our case study the values are solved to be a = 1.0571, b = 1.0518. Then q = 1- p(β). Applying Jiang's approximation methodology in our case we get,

$$M(t) = \{ 1 - \exp\left[-\left(\frac{w}{\lambda}\right)^{\beta}\right]\}^{p} * [\left(\frac{w}{\lambda}\right)^{\beta}]^{q} \qquad (19)$$

Where w is the warranty length variable.

We then substitute the values of λ = 269.9341, β =1.1794, p = 0.1434, q = 0.8566 into Equation (19), to obtained the numerical approximation of the expected number of returns as a function of the warranty period.

We built the optimization model in Matlab that maximizes the total profit. For company in this case study the average initial sale cost c_m=c_r=$737.17. The expected number of returns $E\ [N_r(w)]$=M(t). Where t is considered as the warranty period w. The total profit to be maximized is therefore expressed as

$$\Pi\left(C_p, w\right) = k_1 C_p{}^{-a}(\mathrm{w} + k_2)^b\{C_p - c_r - c_r \mathrm{M(w)}\} \quad (20)$$

The values of $C_p *\ and\ w *$ that maximize the total expected profit $\Pi\left(C_p, w\right)$ are obtained by calculating their partial derivatives and equating them to zero.

Let us assume that the sale price C_p range includes the actual remanufacturing cost of $737.17 (i.e. zero profit) to an arbitrary upper limit of $30000. Let the warranty period also vary from 180 days (half year) to 1080 days (3 years)—this range subsumes the actual warranty periods offered by the company. Let k_1= 80000, k_2= 2, a ε {1, 1.5, 2} and b ε { 0.8, 0.85, 0.90}, then the maximum value of total profits are listed in the sensitive analysis Table 1. From Table 1 we can see that when the warranty period is w=360 days, a=1.5 or 2, the maximum total profit is the highest $7,015,200 and the optimal C_p is $29,000. Table 1 also indicates that if the sale price ranges from $737.17 to $30000, then 1-year warranty is the optimal choice for the remanufactured products of the company.

Table 1: Sensitivity to Elasticity's Parameters

		w=180 days		w=360 days		w=540 days	
		Total Profit	Cp	Total Profit	Cp	Total Profit	Cp
a=1	b=0.80	$160,200	$29,000	$3,892,000	$29,000	$3,255,100	$29,000
	b=0.85	$207,900	$29,000	$5,225,200	$29,000	$4,459,300	$29,000
	b=0.90	$269,600	$29,000	**$7,015,200**	$29,000	$6,109,000	$29,000
a=1.5	b=0.80	$269,600	$29,000	**$7,015,200**	$29,000	$6,109,000	$29,000
	b=0.85	$269,600	$29,000	**$7,015,200**	$29,000	$6,109,000	$29,000
	b=0.90	$269,600	$29,000	**$7,015,200**	$29,000	$6,109,000	$29,000
a=2	b=0.80	$269,600	$29,000	**$7,015,200**	$29,000	$6,109,000	$29,000
	b=0.85	$269,600	$29,000	**$7,015,200**	$29,000	$6,109,000	$29,000
	b=0.90	$269,600	$29,000	**$7,015,200**	$29,000	$6,109,000	$29,000
		w=720 days		w=900 days		w=1080 days	
		Total Profit	Cp	Total Profit	Cp	Total Profit	Cp
a=1	b=0.80	$1,443,300	$29,000	-$142,180.00	$29,000	-$5,268,900	$29,000
	b=0.85	$2,005,800	$29,000	-$142,180.00	$29,000	-$5,268,900	$29,000
	b=0.90	$2,787,500	$29,000	-$142,180.00	$29,000	-$5,268,900	$29,000
a=1.5	b=0.80	$2,787,500	$29,000	-$26,000.00	$29,000	-$96,400	$29,000
	b=0.85	$2,787,500	$29,000	-$26,000.00	$29,000	-$96,400	$29,000
	b=0.90	$2,787,500	$29,000	-$26,000.00	$29,000	-$96,400	$29,000
a=2	b=0.80	$2,787,500	$29,000	-$500.00	$29,000	-$1,800	$29,000
	b=0.85	$2,787,500	$29,000	-$500.00	$29,000	-$1,800	$29,000
	b=0.90	$2,787,500	$29,000	-$500.00	$29,000	-$1,800	$29,000

Table 1 indicates that the objective function is optimized by the highest possible sale price. Though intuitive, further studies will address the role that C_p has is the model to ascertain if the behavior of C_p in Table 1 is indeed expected.

5. Conclusion

In this paper, we present a new exploratory study that attempts to find optimal warranty period for remanufactured products under free-replacement policy. We develop a stochastic process model, based on the renewal function from manufacturers' point of view, to optimize the warranty period and price of remanufactured products. The objective function is the manufacturer' total expected profit. For calculation purposes, the total cost function is assumed to only depend on the demand function, sale price and warranty period. In the quest to determine the optimal solution of case study, we apply a simple and accurate approach to approximate the renewal function for the Weibull distribution. There are also some uncertain factors in our model, such as amplitude parameter, price-demand elasticity and warranty-demand elasticity factors all of which are attributes of the demand function, hence they change with the real market conditions. The next steps of this study, will address the sensitivity analysis of the model given variations in the demand parameters. As well as variations in product failure, i.e. exhibiting decreasing failure rates.

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