An ILP Model for Healthcare Facility Location Problem with Long Term Demand

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Abstract

Facility location decisions is one of the most crucial commitments that manufacturing and service industries face to impacts their interaction with the end users or customers. This is a long term decision and recourse option is very difficult once the decision has been implemented. By optimally placing the facilities considering the probable future expansion in the market, a business entity can gain the dominance over its competitors. In the face of rapid urbanization and increasing demand, making facility location decision based only on existing demand is not optimal; taking into account possible realization of future demand would result in more robust decisions. The present study focuses on developing a mathematical model for making optimal decisions regarding healthcare facilities taking into account long term demand. It utilizes the concept of grid-based location problems to divide the area of interest into discrete cells. The model provides the optimal locations for facilities to be built at present time and the potential location of facilities in the future. Finally, the model is programmed with a standard modeling language AMPL and solved with the CPLEX solver. Results show that the model is efficient in solving small to moderate sized problems. The developed approach can be used by government or relevant agencies to make optimal location decisions for healthcare or other service facilities.

Keywords
Healthcare facility location, Grid-based location problem, Long term location decision.

1. Introduction

Numerous studies have addressed the facility location problem for both manufacturing and service sectors; many algorithms have been developed for determining the optimal numbers and locations for facilities to be built. The survey done by Brandeau *el al.* (1989) provides an overview of the studies focusing on location decision problems conducted in the earlier part of the twentieth century. According to the survey, one of the most important inputs for the location problem is the demand for the products or services that the facility will provide. But very few of these studies have taken into account the future demand of customers or end users, in addition to current or existing requirements. In one of these studies, done by Brancolini *et al.* (2006), asymptotical location problem was considered; the authors compared long term and short-term strategies and their effects on location decisions. Their study suggested that considering long term demand in making location decisions is beneficial. In a slightly similar manner, Fernández *at el.* (2007) considered a location problem and the price setting in order to maximize profit, in which the authors considered long term competition on prices and showed the effect of it on location decisions. Chou (2009) proposed an integrated short-term and long term MCDM (multiple-criteria decision-making) Model for location problems. It showed the importance of integrating the short and long term evaluation method with examples. Kim and Kim (2010) studied long term healthcare facilities problem, which can balance the numbers of patients assigned to each facility; although cost was not considered in the model, which is a limitation of this study. Marić *et al.* (2015) used Hybrid metaheuristic method for long term health-care facility problem that focused on minimizing the maximum number of patients assigned to established facilities. Carlo *et al.* (2012) discussed and proposed several approaches for the long term location problem with the objective of minimizing the total cost of interacting with a set of existing facilities. Öhman and Lämås (2003) studied the effect of long term location planning on harvest activities. In considering the long term demand, a few studies have been done that strived to make the optimal location decisions in presence of demand uncertainty. Hosseini and MirHassani (2015) proposed a 2-stage stochastic location model for refueling station under uncertainty. But the model is too complicated to be solved by available solvers. Albareda-Sambola *et al.* (2013) presented a compact binary formulation for the deterministic equivalent model of the problem under uncertainty. Temur (2016) presented a multi-attribute decision making approach for location decision under high uncertainty. It showed that location decision is very sensitive to the uncertainty. Bai and Liu (2014) examined the influence of uncertain transportation costs and customers' demands on the location decisions.

Healthcare facility location decision has a significant impact on the effectiveness of a facility to provide reliable and safe services to the patients in the long run. Once such decisions have been made and implemented, it is extremely difficult and costly to take recourse actions. Therefore, in deriving an optimal solution for the facility location problem, considering future demand is required in addition to current demand. To the best of our knowledge, a little effort has been made to incorporate future demand in determining optimal healthcare facility locations. Our objective of this study is to develop a new *integer linear programming* (ILP) model for a long term healthcare facility location problem. This model will consider both, the present and future demand, i.e. total number of patients requires treatment at present and additional number of patients that would require treatment in the future. Therefore, for both the present time and the future time horizon, two types of decision variables will be considered. The first type of decision variables will provide the optimal locations of facilities to be built, while the second type will determine the optimal allocation of patients to the facilities. The objective of the model is to minimize the total cost of building and maintaining all the facilities in the considered time horizon, while meeting all the demands.

2. Problem description

In order to formulate a mathematical model for the long term healthcare location problem, a region is considered, where the patients are known to be present. The total number of patients can be treated as the demand and their locations as the demand location for services. The region of interest can be divided into two-dimensional grids to consider the problem as *grid-based location problems* (Noor-E-Alam et al., 2012), where each cell with the same dimension; patients located at a particular grid can be represented by the coordinates of a cell. Cell coordinates (i, j) serve as the demand locations, and number of patients present at each cell determines the demand. For each cell (i, j), demand is a_{ij} during the time interval beginning from time point a to time point b, and during the time interval T' (beginning from time point b to the end of planning time horizon), demand is b_{ij}. These demands are assumed to be determined and will be considered as parameters in our model. To serve the patients, healthcare facilities need to be built, for which fixed costs of building facilities (c) and maintenance cost per unit time (c') will be incurred. Each facility is capacitated (maximum capacity is L), and patients from each cell can only go to the facility that is nearest to them. For the facility locations, the same region is considered, divided into two-dimensional grids, where cells are indicated by a different set of coordinates (x, y). If a facility is to be built at (x, y) at time point a, the binary variable associated with this decision ρ_{xy} takes a value of 1, and if a patient located at (i, j) goes to facility located at (x, y) during the time interval beginning from time point a to time point b, then the binary decision variable γ_{ijxy} takes a value of 1. The corresponding binary decision variables for time point b and time interval T', i.e. ρ'_{xy} and γ'_{ijxy} follow the same logic. While making these decisions, capacity restrictions of facilities and conditions for allocation of patients to the nearest facilities must be met. The problem is to decide optimal numbers and locations for facilities to be built now and in the future, and allocate patients to their nearest facilities. The objective is to keep the total cost of building and maintaining the facilities at the minimum level.

3. Location model

In the following subsections, the assumptions, parameters and variables included in the model are described. Figure 1 shows the time horizon considered in developing the model. Following that, objective function and the constraints of the developed model are presented and explained in detail.

3.1 Assumptions

We have made the following assumptions to simplify our decision problem:

- All the current patients at each cell go to one facility in the entire planning horizon.
- All the future patients estimated at each cell at time point b will also go to a single facility during the time interval beginning from time point b to the end of planning time horizon.
- All the patients will go to the nearest facility for the entire planning time horizon.
- All the facilities have the same capacity.
- Cost of maintenance for a facility will remain same in the planning time horizon.
- Costs of building a facility at time point a is the same as that at time point b.

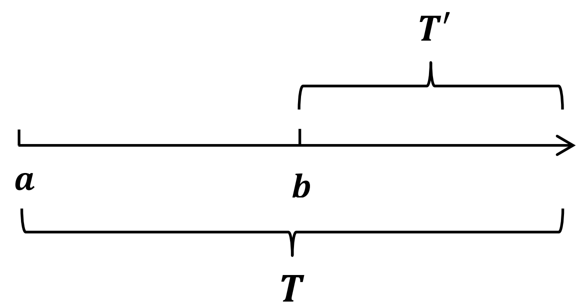

Figure 1: Planning horizon to develop optimization model.

3.2 Parameters

T	total time interval under consideration (from time point a to the end of planning time horizon)
T'	time interval from time point b to the end of planning time horizon
a_{ij}	current demand of cell (i, j) (demand during time interval beginning from time point a to time point b)
b_{ij}	future demand of cell (i, j) during time interval T', beginning from time point b to the end of planning time horizon
c	fixed cost of building a facility
c'	cost of maintain one facility per unit time
d_{ijxy}	distance between points (i, j) and (x, y)
L	maximum capacity of one facility
M	large cost parameter

3.3 Variables

γ_{ijxy}	if patients from (i, j) go to facility (x, y) during time interval beginning from time point a to time point b, then $\gamma_{ijxy} = 1$, else $\gamma_{ijxy} = 0$
ρ_{xy}	if a facility is built at (x, y) at time point a, then $\rho_{xy} = 1$, else $\rho_{xy} = 0$
γ'_{ijxy}	if patients from (i, j) go to facility (x, y) during time interval T',then $\gamma'_{ijxy} = 1$, else $\gamma'_{ijxy} = 0$
ρ'_{xy}	if a facility is built at (x, y) at time point b, then $\rho'_{xy} = 1$, else $\rho'_{xy} = 0$

3.4 ILP model

With the above assumptions, we have proposed the following ILP model to solve our long term healthcare facility location problems:

$$Minimize \sum_{x} \sum_{y} \rho_{xy} (c + c'T) + \sum_{x} \sum_{y} \rho'_{xy} (c + c'T') \qquad (1)$$

$$\gamma_{ijxy} \leq \rho_{xy} \quad (2)$$

$$d_{ijmn}\gamma_{ijmn} \leq d_{ijxy} + M(1 - \rho_{xy}) \quad (3)$$

$$\sum_{i}\sum_{j} a_{ij}\gamma_{ijxy} \leq L \quad (4)$$

$$\sum_{x}\sum_{y} \gamma_{ijxy} = 1 \quad (5)$$

$$\rho_{xy} + \rho'_{xy} \leq 1 \quad (6)$$

$$\gamma'_{ijxy} \leq \rho_{xy} + \rho'_{xy} \quad (7)$$

$$d_{ijmn}\gamma'_{ijmn} \leq d_{ijxy} + M(1 - \rho_{xy} - \rho'_{xy}) \quad (8)$$

$$\sum_{i}\sum_{j} b_{ij}\gamma'_{ijxy} \leq L \quad (9)$$

$$\sum_{x}\sum_{y} \gamma'_{ijxy} = 1 \quad (10)$$

The objective function (1) is a cost minimization function that aims to minimize the total cost of building and maintaining the facilities. It is assumed that the building cost c is the same at time points a and time b. The term $c'T$ indicates the maintenance cost of one facilities built at time point a for time length T, whereas the term $c'T'$ indicates the maintenance cost of one facilities setting at time point b for time length T'. Constraint (2) describe the relation between the variables γ_{ijxy} and ρ_{xy}, that is a patient from (i, j) can go to (x, y) during time interval beginning from time point a to time point b for service only if a facility is located at (x, y). Constraint (7) specify the same relation during time point b to the end of planning horizon between the variables γ'_{ijxy} and ρ'_{xy}. Constraint (3) restricts the patients to go only to the nearest facility built at time point a. As Figure 2 indicates if there is a facility built at (x, y) at time point a, then for any other point (m, n), γ_{ijmn}=1 if and only if distance $d_{ijmn} \leq d_{ijxy}$. Constraint (8) represents the same relation for facilities and patients for time point b. Constraint (4) restricts the total number of patients that can go to a facility to the maximum facility capacity for the current time period and constraint (9) does the same for future time period. Constraints (5) and (10) restrict the number of facilities that patients at each grid cell can visit to one. Constraint (6) specify that at any location there can be at most one facility, i.e. if a facility is built at (x, y) at time point a, then no facility can be built at the same location at time point b.

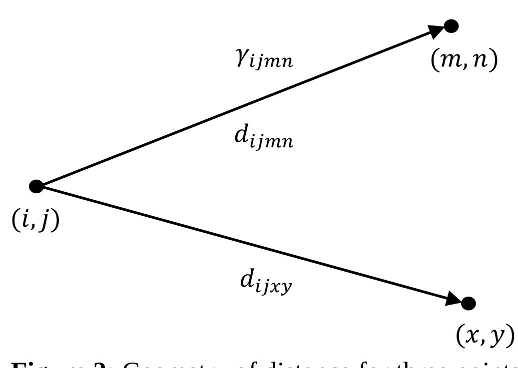

Figure 2: Geometry of distance for three points.

4. Computational results

The model is solved on a computer with Intel(R) Core(TM) i5 CPU running at 3.19 GHz with 4 GB memory and a 64-bit operating system. For implementation, AMPL programming language is used, and CPLEX 12.6.3 solver is utilized to solve. Three different instances are solved using the model, e.g. 5 × 5 Grid (Shown in Fig. 3), 5 × 8 Grid

(Shown in Fig. 4) and 10 × 10 (Shown in Fig. 5) Grid. In these instances, other parameter are assumed as follows: $c = c' = 10$, $T = 20$, $T' = 10$, $L = 10$ and $d_{ijxy} = \sqrt{(i-x)^2 + (j-y)^2}$. For a_{ij}, values were randomly generated ranging from 0 to 5 and for b_{ij}, random values range from 0 to 8. In Figures 3-5, the numbers in each cell before the sign '/' specify the demand at time a, and the number after the sign '/' indicate the demand at time b. The grey grid cells represent facilities are built at those locations at time a, while the black grid cells represent building of facilities at time b. The white gird cells indicate that no facility is required at those locations.

4/6	0/6	1/7	1/4	3/4
5/6	1/0	5/6	2/3	0/4
1/3	3/2	5/3	5/6	4/5
5/5	5/0	2/8	4/6	5/6
3/1	5/1	4/0	5/1	3/6

Figure 3: Demand distribution and result for 5×5 grid.

4/1	1/4	3/1	4/3	2/7	1/3	4/6	4/2
5/6	5/1	0/8	0/7	2/5	3/0	1/3	1/3
1/2	5/2	4/0	1/1	4/3	3/7	3/2	4/7
5/4	2/7	5/6	0/2	4/4	1/8	3/3	1/0
3/1	4/1	3/7	0/1	1/3	1/4	4/1	5/0

Figure 4: Demand distribution and result for 5×8 grid.

4/1	1/4	3/1	4/3	2/7	1/3	4/6	4/2	2/4	0/7
5/6	5/1	0/8	0/7	2/5	3/0	1/3	1/3	4/2	0/6
1/2	5/2	4/0	1/1	4/3	3/7	3/2	4/7	3/6	3/4
5/4	2/7	5/6	0/2	4/4	1/8	3/3	1/0	3/2	4/3
3/1	4/1	3/7	0/1	1/3	1/4	4/1	5/0	5/5	5/4
0/5	1/7	4/7	4/1	2/1	2/4	5/1	2/1	1/1	1/2
1/2	2/4	4/1	3/7	2/2	5/3	3/8	1/5	4/3	3/4
3/5	5/8	2/3	2/5	3/1	2/7	1/8	1/6	4/5	2/4
5/6	5/1	1/2	0/4	4/1	1/3	1/5	2/5	3/6	0/7
5/6	5/4	1/6	0/1	4/2	1/1	1/0	2/4	3/1	2/6

Figure 5: Demand distribution and result for 10×10 grid.

Table 1: Runtime statistics.

Grid size	Number of facilities at time a	Number of facilities at time b	CPU time (seconds)	Simplex iterations	Branch and bound nodes	MIP Gap
5×5	9	4	5.834	38360	1388	0.001
5×8	11	5	281.005	865350	6091	0.001
10×10	27	16	>36338.577	21005880	27202	0.05

Table 1 summarizes the runtimes and other measures that were obtained through solving for these three instances using the developed model. From our preliminary experiment we see that our proposed model is capable to identify the optimum healthcare facility locations and assigned facilities for the patients located in each grid for a long-term planning horizon within a reasonable runtime at least for 5x5 and 5X8 grids. However, when we try to solve larger grids, it takes very long time to reach the optimality (after running a 10x10 grid for more than 10 hours, we saw that the mipgap was 5%). Therefore, we future plan is to reformulate the problem such that it will be scalable and capable to solve large-scale instances.

5. Conclusion

In this paper, a grid-based integer linear programming model is developed for identifying optimal location of healthcare facilities to be built now and in the future. It takes into consideration the long term, i.e. future demand realizations in addition to the existing demand. Although this study focuses on the healthcare facilities, this model can also be used for a variety of service sectors, for example parks and recreational centers, as well as in placing retail stores. The model is scalable up to a degree, but for solving the large instances it takes longer time than we expect. To rectify this limitation, future extensions would consider relaxation of some constraints and addition of some logical constraints. Furthermore, another extension of the model will consider multiple periods and demands as functions of time periods, which will make the model more robust.

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Biography

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