

Designing distribution networks of perishable products under stochastic demands and routs

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Abstract

This study investigates an extended Location Routing Inventory problem model of perishable product. The proposed model consists of two phases. In the first phase, the location routing problem is modelled with stochastic customer demands and travel time, and then fixed facility and transportation costs are derived. In the second phase, a queue system is used to model the inventory control problem based on selected locations and routes. Reneging and balking are assumed in the second phase, and holding, shortage, product expiration, customer waiting times, and customer loss costs are calculated. An integral model is proposed based on these two phases. Simulated annealing heuristic is applied in order to solve the model, and one numerical examples are presented.

Keywords

Location-routing-inventory problem, perishable products, queuing system, reneging and balking

1. Introduction

According to Liu and Lin (2005), the Location Routing Inventory Problem (LRIP) involves selecting depots from several potential locations, scheduling vehicle routes in order to meet customer demands, and determining inventory policy. In classical LRIP models, customer demands and travel times of vehicles are deterministic; in reality, though, accurate determination of those two parameter values is difficult and even impossible. A majority of studies on LRIP have modelled the problem with imperishable products, but most real-world products have specific consuming duration after which time the products expire, resulting in financial loss.

Liu and Lee (2003) introduced the classic model of LRIP, and they proposed the location routing problem with stochastic customer demands and consideration of inventory control decisions. Shen and Qi (2007) considered approximate routing costs in the inventory-location model with stochastic customer demands. The objective function of their model was the sum of inventory-location costs and an approximate routing cost that depended on locations of open distribution centers. However, their model optimized only inventory and location decisions and did not determine transportation decisions. Ahmadi Javid and Azad (2010) presented unprecedented integration of LRIP with stochastic customer demand in a single-source supply chain. They integrated location-allocation, routing, and inventory decisions, demonstrating that their model provides a significant advantage over Shen and Qi's (2007) model which only incorporated approximate routing cost.

The aim of this paper is to approximate the LRIP problem as a real case by considering stochastic customer demands and travel time, perishable products, reneging and balking. In this study, the location routing problem is initially modelled with stochastic customer demands and travel time, and then fixed facility and routing costs are derived. Based on selected locations and routes, a queue system is used to model the inventory control problem, and reneging and balking are assumed. Because products in the distribution centers expire at an already-determined rate, the costs

of holding, shortage, product expiration, customer loss, and customer waiting times are calculated. An integral model was proposed based on these two phases.

This paper is organized into descriptive sections. Section 2 presents the mathematical formulation of the Stochastic Location Routing Inventory Problem (SLRIP) with perishable products and renegeing and balking. Detailed implementation of simulated annealing (SA) in order to solve the model is described in Section 3, and Section 4 reports computational results. Section 5 provides conclusions and suggestions for future studies.

2. Mathematical programming formulations

The goals of the model proposed in this work were to choose, locate, and allocate a set of distribution centers, schedule vehicle routes, and determine distribution center capacity and reorder point in order to minimize total cost. For model presentation, the problem was divided into two subproblems: the location-routing problem with uncertain customer demands and travel time and the inventory control problem with perishable products and renegeing and balking.

In general, stochastic programming and queuing systems are the two approaches used for modeling probabilistic problems. In the present study, the probabilistic location-routing subproblem is modelled with chance-constrained programming, and the queuing theory is used to model the inventory control subproblem.

2.1. Probabilistic location-routing subproblem

In this work, after chance-constrained programming is employed to model the probabilistic location-routing subproblem, queuing systems are designed per vehicle, considering a set of customers allocated per vehicle. Related costs are then derived. Charnes and Cooper (1959) introduced chance-constrained programming (CCP), a powerful tool for modelling probabilistic problems in which some constraints are forced to be held with at least a predefined probability.

- **Sets and parameters**

I	Set of customers indexed by i
J	Set of candidate distribution centers sites indexed by j
K	Set of vehicles indexed by k
V	Set of all points: $V = I \cup J$
E	Set of arcs (i,j) connecting every pair of nodes $i, j \in V$
c_{ij}	Cost of traveling associated with arc $(i,j) \in E$
R_j	Capacity of distribution center to be located at candidate site j
o_j	Fixed cost of opening a distribution center at candidate site j
Q	Capacity of vehicles: vehicles are assumed to be homogeneous
d_i	Random variable representing the demand of customer i
λ_i	Demand rate in each retail center i
B_k	Upper limit for service time of vehicle k (This article proposes that all vehicles have upper limit B , which is drivers working time per day [i.e., 8 hours].)
t_k	Random variable representing travel time of vehicle k

- **Decision variables**

$$z_j = \begin{cases} 1 & \text{if we open a distribution center at candidate site } j \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ij} = \begin{cases} 1 & \text{if demands at customer } i \text{ are served by the DC at candidate site } j \\ 0 & \text{otherwise} \end{cases}$$

$$x_{ijk} = \begin{cases} 1 & \text{if vehicle } k \text{ goes directly from node } i \text{ to node } j \\ 0 & \text{otherwise} \end{cases}$$

This section presents the mathematical programming formulation for the probabilistic location-routing subproblem using the CCP approach.

$$\min \sum_{j \in J} O_j z_j + \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} c_{ij} x_{ijk} \quad (1)$$

Subject to:

$$\Pr\left\{\sum_{i \in I} d_i y_{ij} \leq R_j z_j\right\} \geq \alpha \quad \forall j \in J \quad (2)$$

$$\Pr\left\{\sum_{j \in V} \sum_{i \in I} d_i x_{ijk} \leq Q\right\} \geq \alpha \quad \forall k \in K \quad (3)$$

$$\Pr\{t_k \leq B_k\} \geq \alpha \quad \forall k \in K \quad (4)$$

$$\sum_{k \in K} \sum_{j \in V} x_{ijk} = 1 \quad \forall i \in I \quad (5)$$

$$\sum_{i \in V} x_{jik} - \sum_{i \in V} x_{ijk} = 0 \quad \forall j \in V, k \in K \quad (6)$$

$$\sum_{i \in I} \sum_{j \in J} x_{ijk} \leq 1 \quad \forall k \in K \quad (7)$$

$$\sum_{j \in S} \sum_{i \in S} x_{ijk} \leq |S| - 1 \quad \forall S \subseteq I, k \in K \quad (8)$$

$$\sum_{u \in I} x_{juk} + \sum_{u \in V \setminus \{i\}} x_{uik} \leq 1 + y_{ij} \quad \forall i \in I, j \in J, k \in K \quad (9)$$

$$x_{ijk} \in \{0, 1\} \quad \forall i \in I, j \in V, k \in K \quad (10)$$

$$y_{ij} \in \{0, 1\} \quad \forall i \in I \quad (11)$$

$$z_j \in \{0, 1\} \quad \forall j \in V \quad (12)$$

The two terms in the objective function (1) represent the sum of the fixed distribution center location costs and routing costs. Constraints (2) and (3) ensure that the capacity of the vehicles and distribution centers in at least α proportion of the times are not violated. Constraint (4) makes the travel time of each vehicle less than B_k in at least α percent of the time. Constraint (5) ensures that each customer belongs to one and only one route and that each customer has only one predecessor in the route. Constraints (6) and (7) guarantee the continuity of each route and that each route terminates at the depot at which the route begins. Constraint (8) is a subtour elimination constraint. Constraint (9) ensures that a customer is allocated to a depot if a route connects them. Constraints (10), (11), and (12) are integrality constraints.

The traditional method of solving the CCP problem is to convert the chance constraints into their corresponding deterministic equations. Therefore,

$$\min \sum_{j \in J} O_j z_j + \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} c_{ij} x_{ijk} \quad (13)$$

Is subject to:

$$\sum_{i \in I} \lambda_i y_{ij} + \Phi^{-1}(\alpha) \sqrt{\sum_{i \in I} \lambda_i y_{ij}^2} \leq R_j z_j \quad \forall j \in J \quad (14)$$

$$\sum_{j \in V} \sum_{i \in I} \lambda_i x_{ijk} + \Phi^{-1}(\alpha) \sqrt{\sum_{j \in V} \sum_{i \in I} \lambda_i x_{ijk}^2} \leq Q \quad \forall k \in K \quad (15)$$

$$1 - e^{-\frac{x}{\mu_k}} \geq \alpha \quad \forall k \in K \quad (16)$$

(5)-(12)

Since t_k follows a exponential distribution, the left side of Constraint (4) is the cumulative distribution function of the exponential distribution for B_k .

2.2. Inventory control subproblem

The previous section modelled the probabilistic location-routing subproblem with CCP and distribution center locations, allocation of customers and vehicles to distribution centers, and vehicle routes, derived in order to minimize location and routing costs. In this section, the queuing system is used to determine distribution center capacities and reorder points based on minimizing costs associated with holding, shortage and product expiration, customer waiting times, and customer loss.

The model by Teimoury et al. (2011) was used to modeling inventory control with queuing systems. The goals of were to choose, locate, and allocate a set of distribution centers and to determine distribution center capacities such that location, inventory, and customer loss costs were minimized. A queue with customer balking and renegeing was discussed, but no constraint on goods was present. Moreover, vehicle movement was direct contact, meaning that each vehicle per travel serviced only one distribution center. The routing problem was not considered.

The proposed queuing system was designed for each vehicle so that each vehicle could serve all retail centers (customers) allocated in Section 2.1. Figure 1. shows the supply chain with one supplier, two distribution centers, and three vehicles. Distribution Centers 1 and 2 were selected. Vehicles 1 and 3 were allocated to Distribution Center 1, and Vehicle 2 was allocated to Distribution Center 2. Retail Centers 3-5-1, 4-7, and 6-2 were allocated to Vehicles 1, 3, and 2, respectively. As mentioned in Section 2, the demand rate in each retail center was λ_i ; therefore, the demand rate in each vehicle was equal to the sum of the demand rate of the correspondingly allocated retail centers.

One assumption was used to model the inventory subproblem with the queuing system: after the distribution centers were constructed, its warehouse was partitioned for nourishing each vehicle.

Customers entered each retail center following a Poisson distribution with the determined rate, but in the case of the long queue, they did not enter the system. As a result of customers waiting in the queue for a long time, they renegeed. The waiting time before the customer became impatient was a random variable that followed an exponential distribution with β parameter. Vehicles served customers in bulk service, and they waited until the number of customers in the queue became more than q ; then vehicles began to move. When accounting for the capacity constraint in Section 2.1, vehicles served all customers in one travel when the number of customers in the queue became more than q .

The duration time before expiring was a random variable that followed an exponential distribution with θ parameter. Inventory control policy in the specified warehouse of vehicles was continuous-review (R, r). Therefore, if the number of products was less than the reorder point, products were ordered until the warehouse was filled.

• Parameters

- N_i Customer population of retail center i
- r_k Reorder point of specified warehouse of vehicle k
- R_k Capacity of specified warehouse of vehicle k
- ν_j Products' arrival rate in distribution center j
- h_j Holding cost per unit product per unit time in distribution center j
- c_{w_i} Customer waiting time cost per unit time in retail center i
- c_b Shortage cost per unit product per unit time

- c_p Expiration cost per unit product per unit time
- c_l Loss cost of one customer (renege or balk) per unit time
- N_k Customer population allocated to vehicle k ($N_k = \sum_{i \in I} N_i f_{ik}$)
- λ_k Demand rate in vehicle k ($\lambda_k = \sum_{i \in I} \lambda_i f_{ik}$)

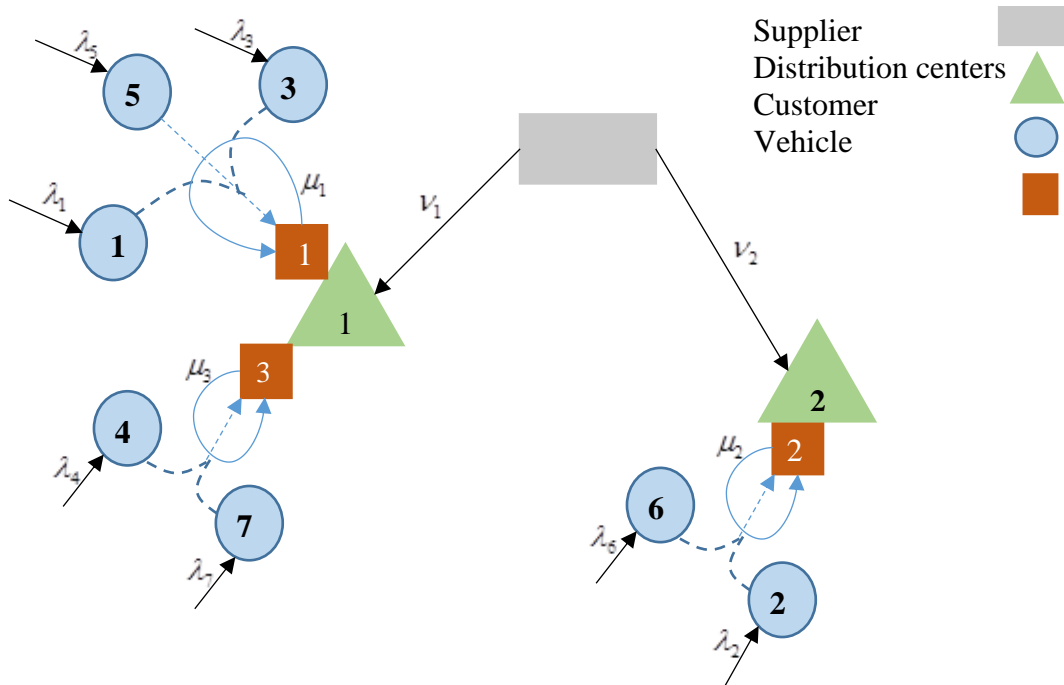


Figure 1. Supply chain with one supplier, two distribution centers, and three vehicles

• **Decision variable**

$$a_{jk} = \begin{cases} 1 & \text{if vehicle } k \text{ allocate to distribution center } j \\ 0 & \text{otherwise} \end{cases}$$

• **The Markov Chain**

The vehicle queue system can be described by a birth-and-death Markov process with states (n, p) , where n is the number of customers in the system, including one being served, and p is the number of products available in the specified warehouse of vehicles. Five state transitions in the system are presented below.

$$(n, p) \xrightarrow{b_n \lambda_k} (n + 1, p) \text{ for } n = 0, 1, \dots, N_k - 1, p = 0, 1, \dots, R_k.$$

This state transition occurs with customer arrival at system. Let b_n denote the probability that a customer enters the queue when n customers are in the system. b_n is defined as follows (Bhat,2008):

$$b_n = \begin{cases} 1 & n = 0, \\ e^{-n/\mu} & 1 \leq n \leq N - 1, \\ 0 & n \geq N \end{cases}$$

Therefore, the customer arrival rate in each retail center is $b_n \lambda_i$, and the customer arrival rate in each vehicle's specified warehouse is equal to the sum of customer arrival rates in each retail center, $b_n \lambda_k$.

$$(n, p) \xrightarrow{n \beta} (n - 1, p) \text{ for } n = 1, \dots, N_k, p = 0, 1, \dots, R_k.$$

If more than one customer is present in the system, each customer with probability β will renege. Therefore, with respect to the presence of customers in the system, the above mentioned state transition occurs with $n\beta$ probability.

$$(n, p) \xrightarrow{p\theta} (n, p - 1) \text{ for } n = 0, 1, \dots, N_k, p = 1, 2, \dots, R_k.$$

Each product in the warehouse expires with θ probability. Therefore, if p products are in the warehouse, the probability of reducing one product is $p\theta$.

$$(n, p) \xrightarrow{v_j} (n, R_k) \text{ for } n = 0, 1, \dots, N_k, p = 0, 1, \dots, r_k.$$

This state transition occurs with replenishment warehouse.

$$(n, p) \xrightarrow{\mu_k} (n - p, 0) \text{ for } n = q, q + 1, \dots, N_k, p = 1, \dots, R_k, n \geq p$$

$$\text{And } (n, p) \xrightarrow{\mu_k} (0, p - n) \text{ for } n = q, q + 1, \dots, N_k, p = 1, \dots, R_k, n < p.$$

The above state transitions occur with shipment products to customers.

In order to derive system performance measures, steady-state probabilities must be calculated. $\pi_{(n,p)}$ denotes the steady-state probability associated with the condition in which there are n customers and p products in the specified warehouse of vehicles. Referring to the state transition diagram, the following balance equations are provided:

$$(b_n \lambda_k + v_j) \pi_{(n,p)} = \beta \pi_{(n+1,p)} + (p + 1) \theta \pi_{(n,p+1)} + \mu_k \left[\sum_{i=q}^{R_k} \pi_{(i,i)} \right] \quad n = 0, p = 0 \quad (17)$$

$$(b_n \lambda_k + v_j + p\theta) \pi_{(n,p)} = \beta \pi_{(n+1,p)} + (p + 1) \theta \pi_{(n,p+1)} + \mu_k \left[\sum_{i=q}^{R_k-p} \pi_{(i,i+p)} \right] \quad n = 0, 1 \leq p \leq r_k \quad (18)$$

$$(b_n \lambda_k + p\theta) \pi_{(n,p)} = \beta \pi_{(n+1,p)} + (p + 1) \theta \pi_{(n,p+1)} + \mu_k \left[\sum_{i=q}^{R_k-p} \pi_{(i,i+p)} \right] \quad n = 0, r_k < p < R_k \quad (19)$$

$$(b_n \lambda_k + p\theta) \pi_{(n,p)} = \beta \pi_{(n+1,p)} + v_j \left[\sum_{i=0}^{r_k} \pi_{(n,i)} \right] \quad n = 0, p = R_k \quad (20)$$

$$(b_n \lambda_k + v_j + n\beta) \pi_{(n,p)} = (n + 1) \beta \pi_{(n+1,p)} + \theta \pi_{(n,p+1)} + b_{(n-1)} \lambda_k \pi_{(n-1,p)} + \mu_k \left[\sum_{i=q-n}^{N_k-n} \pi_{(i+n,i)} \right] \quad (21)$$

$$1 \leq n < q, p = 0$$

$$(b_n \lambda_k + v_j + n\beta) \pi_{(n,p)} = (n + 1) \beta \pi_{(n+1,p)} + \theta \pi_{(n,p+1)} + b_{(n-1)} \lambda_k \pi_{(n-1,p)} + \mu_k \left[\sum_{i=1}^{N_k-n} \pi_{(i+n,i)} \right] \quad (22)$$

$$q \leq n < N_k, p = 0$$

$$(v_j + n\beta) \pi_{(n,p)} = \theta \pi_{(n,p+1)} + b_{(n-1)} \lambda_k \pi_{(n-1,p)} \quad n = N_k, p = 0 \quad (23)$$

$$(b_n \lambda_k + v_j + n\beta + p\theta) \pi_{(n,p)} = (n + 1) \beta \pi_{(n+1,p)} + (p + 1) \theta \pi_{(n,p+1)} + b_{(n-1)} \lambda_k \pi_{(n-1,p)} \quad (24)$$

$$1 \leq n < q, 1 \leq p \leq r_k$$

$$(b_n \lambda_k + v_j + n\beta + p\theta + \mu_k) \pi_{(n,p)} = (n + 1) \beta \pi_{(n+1,p)} + (p + 1) \theta \pi_{(n,p+1)} + b_{(n-1)} \lambda_k \pi_{(n-1,p)} \quad (25)$$

$$q \leq n < N_k, 1 \leq p \leq r_k$$

$$(b_n \lambda_k + n\beta + p\theta)\pi_{(n,p)} = (n+1)\beta\pi_{(n+1,p)} + (p+1)\theta\pi_{(n,p+1)} + b_{(n-1)}\lambda_k\pi_{(n-1,p)} \quad (26)$$

$$1 \leq n < q, r_k < p < R_k$$

$$(b_n \lambda_k + n\beta + p\theta + \mu_k)\pi_{(n,p)} = (n+1)\beta\pi_{(n+1,p)} + (p+1)\theta\pi_{(n,p+1)} + b_{(n-1)}\lambda_k\pi_{(n-1,p)} \quad (27)$$

$$q \leq n < N_k, r_k < p < R_k$$

$$(b_n \lambda_k + n\beta + p\theta)\pi_{(n,p)} = (n+1)\beta\pi_{(n+1,p)} + b_{(n-1)}\lambda_k\pi_{(n-1,p)} + v_j \left(\sum_{i=0}^{r_k} \pi_{(n,i)} \right) \quad (28)$$

$$1 \leq n < q, p = R_k$$

$$(b_n \lambda_k + n\beta + p\theta + \mu_k)\pi_{(n,p)} = (n+1)\beta\pi_{(n+1,p)} + b_{(n-1)}\lambda_k\pi_{(n-1,p)} + v_j \left(\sum_{i=0}^{r_k} \pi_{(n,i)} \right) \quad (29)$$

$$q \leq n < N_k, p = R_k$$

$$(n\beta + p\theta + \mu_k + v_j)\pi_{(n,p)} = (p+1)\theta\pi_{(n,p+1)} + b_{(n-1)}\lambda_k\pi_{(n-1,p)} \quad n = N_k, 1 \leq p \leq r_k \quad (30)$$

$$(n\beta + p\theta + \mu_k)\pi_{(n,p)} = (p+1)\theta\pi_{(n,p+1)} + b_{(n-1)}\lambda_k\pi_{(n-1,p)} \quad n = N_k, r_k < p < R_k \quad (31)$$

$$(n\beta + p\theta + \mu_k)\pi_{(n,p)} = b_{(n-1)}\lambda_k\pi_{(n-1,p)} + v_j \sum_{i=0}^{r_k} \pi_{(n,i)} \quad n = N_k, p = R_k \quad (32)$$

In order to calculate steady-state probabilities, transition matrix Q was defined. Q is the $(N_k+1)*(R_k+1)$ matrix. The balance equations were then solved by MATLAB 7.1. In order to calculate the costs of holding, shortage, product expiration, customer waiting times, and customer loss, system performance measures are determined in the following sections.

- **System performance measures**

This section presents five performance measures of the system under steady-state consideration.

- **Mean inventory level in distribution centers**

If $E(I_j)$ represents the average inventory of products in distribution center j in the steady-state, then the following formula can be presented:

$$E(I_j) = \sum_{k \in K} a_{jk} \left(\sum_{n=0}^{N_k} \sum_{p=0}^{R_k} p \pi_{(n,p)} \right) \quad (33)$$

- **Mean expired products in distribution centers**

If $E(P_j)$ denotes the mean number of expired products in distribution center j in the steady-state, then the following formula can be provided:

$$E(P_j) = \sum_{k \in K} a_{jk} \left(\sum_{p=0}^{R_k} \theta p \pi_{(n,p)} \right) \quad (34)$$

- **Mean backorder level in specified warehouse of vehicle**

If $E(B_k)$ represents the average number of backorders in the specified warehouse of vehicle k in the steady-state, then

$$E(B_k) = \sum_{n=1}^{N_k} n \pi_{(n,0)} \quad (35)$$

- **Mean customer waiting time in the specified warehouse of vehicles**

Let $E(W_k)$ denote the mean customer waiting time in the specified warehouse of vehicle k in the steady-state. In order to calculate $E(W_k)$, the mean queue length is obtained as follows:

$$E(L_k) = \sum_{p=0}^{R_k} \sum_{n=1}^{N_k} n \pi_{(n,p)} \quad (36)$$

Utilization of Little's law allows

$$E(W_k) = E(L_k) / \lambda'_k = \frac{1}{\lambda'_k} E(L_k) = \frac{1}{\lambda'_k} \sum_{p=0}^{R_k} \sum_{n=1}^{N_k} n \pi_{(n,p)} \quad (37)$$

In queue systems that demonstrate balking with an increased number of customers in the system, the probability of entering new customers to the system decreases and the customer arrival rate decreases. Therefore,

$$\lambda'_k = \lambda_k \left(1 - \sum_{i=0}^{R_k} \pi_{(N_k,i)} \right) \quad (38)$$

▪ **Mean rate of customer loss in specified warehouse of vehicle**

Let $E(BA_k)$, $E(RE_k)$, and $E(LO_k)$ denote the average balking rate, average renegeing rate, and average rate of customer loss, respectively. These average rates are obtained as follows:

$$E(BA_k) = \sum_{p=0}^{R_k} \sum_{n=1}^{N_k} (1 - b_n) \lambda_k \pi_{(n,p)} \quad (39)$$

$$E(RE_k) = \sum_{p=0}^{R_k} \sum_{n=1}^{N_k} n \beta \pi_{(n,p)} \quad (40)$$

$$E(LO_k) = E(BA_k) + E(RE_k) \quad (41)$$

▪ **Inventory costs**

The expected total inventory cost of the steady-state is

$$TCI(R_k, r_k) = \sum_{j \in J} h_j E(I_j) + \sum_{k \in K} c_b E(B_k) + \sum_{j \in J} c_p E(P_j) + \sum_{k \in K} c_{w_i} E(W_k) + \sum_{k \in K} c_l E(LO_k) \quad (42)$$

$$TCI(R_k, r_k) = \sum_{j \in J} h_j \sum_{k \in K} a_{jk} \left[\sum_{n=0}^{N_k} \sum_{p=0}^{R_k} p \pi_{(n,p)} \right] + \sum_{k \in K} c_b \left[\sum_{n=1}^{N_k} n \pi_{(n,0)} \right] + \sum_{j \in J} c_p \sum_{k \in K} a_{jk} \left[\sum_{p=0}^{R_k} \theta p \pi_{(n,p)} \right] + \sum_{k \in K} c_{w_i} \left[\frac{1}{\lambda'_k} \sum_{p=0}^{R_k} \sum_{n=1}^{N_k} n \pi_{(n,p)} \right] + \sum_{k \in K} c_l \left[\sum_{p=0}^{R_k} \sum_{n=1}^{N_k} (1 - b_n) \lambda_k \pi_{(n,p)} + \sum_{p=0}^{R_k} \sum_{n=1}^{N_k} n \beta \pi_{(n,p)} \right] \quad (43)$$

Taking into account Sections 2.1 and 2.2, the mathematical model for the probabilistic LRIP with perishable products and renegeing and balking can be proposed. Applying this model, distribution center locations, allocation of customers and vehicles to distribution centers, and vehicle routes were derived. In order to determine distribution center capacities and specified vehicle warehouse reorder points, a direct search was used. Therefore, the model ran for various values of r_k and R_j , and the best solution was chosen.

3. Solution approach

Because location-routing problems have proven to be NP-hard (Cornuéjolset al. (1977), Karp (1972), Lenstra and Rinnooy (1981)), the presented problem can be classified as an NP-hard problem. Therefore, this section develops a metaheuristic method based on SA.

3.1. Solution representation

A solution representation in LRIP must determine established depots, assigned customers to depots and vehicles, and vehicle schedules. Therefore, solution representation by Zarandi et al. (2011) was used. A solution is represented by a string of numbers. Considering n customers, m vehicles, and d candidate locations for distribution centers, solution representation is comprised of $n+2m$ elements that are incorporated into three parts. Initial n elements show the sequence of customers to be served by vehicles. The second part (m elements) determines customer indices to be served by a vehicle, and the third part (m elements) identifies vehicles starting from each established distribution center.

3.2. Generating initial solution

Initialization of a solution significantly contributes to the acquisition of appropriate solutions in metaheuristic algorithms. Sajjadi and Cheraghi (2011) presented an algorithm to generate the initial solution. The algorithm begins by randomly selecting a candidate customer. If the demand of the candidate customer is less than or equal to the capacity of the selected vehicle, the customer is assigned to the selected vehicle. The closest customer to the selected customer is then chosen, and if the candidate has a demand less than or equal to the remaining vehicle capacity, it is assigned to the same route. Otherwise, the next closest customer is selected as the new candidate. The process is repeated until the entire capacity of the vehicle is satisfied. The method iterates until all customers are assigned to vehicles.

Routes are assigned to depots that are physically closest to them. Therefore, using Equation (44), an (x, y) coordinate is assigned to each route. The equation identifies the weighted average coordinate of a route. Using this coordinate value, average distances from a given route to all depots are calculated, and then the shortest path is chosen.

$$X_k = \frac{\sum_{i \in I} f_{ik} \lambda_i x_i}{\sum_{i \in I} f_{ik} \lambda_i}, \quad Y_k = \frac{\sum_{i \in I} f_{ik} \lambda_i y_i}{\sum_{i \in I} f_{ik} \lambda_i} \quad (44)$$

Where X_v and Y_v indicate the route coordinate v ; (x_i, y_i) shows the coordinate of customer i , and N_v is the number of customers assigned to route v . Since the capacity of depots is limited, a penalty cost is charged to the network if the total demand assigned to a depot exceeds depot capacity.

3.3. Neighborhood structure

A Neighborhood Search Structure (NSS) is a mechanism to obtain new solutions by slightly changing the current solution. The NSS mechanism of Zarandi et. al. (2011) was used in the present study. Four NSS types were applied: 2-opt, Shuffle (used for the first part), Reorder (used for the second part), and Mutate (used for the third part). Overall, r values of randomly chosen elements are replaced in an r -opt move. The same rule is followed by a 2-opt move. Two random indices are chosen in a shuffle move, and the values between these moves are randomly shuffled in order to achieve a new solution. The second part also changes when a solution is reordered, and the group of customers served by each vehicle is changed via this move. In order to change vehicle allocation to depots, a mutation is employed. The value of an element is changed to mutate; therefore, a new allocation plan of vehicles to depots is obtained. Reorder and mutate are employed every time, and one of 2-opt and shuffle moves is applied according to a Monte-Carlo approach.

4. Numerical example

Because this model is a novel, unprecedented version of the family of LRIPs, no test problem instance is available in the literature in order to fit its specifications. Therefore, new test instances were generated. Numerical experiment were performed on a personal computer with CPU 2.66 GHz and memory of 4 GB with a Microsoft operating system. The proposed algorithm was coded in MATLAB R2011a in order to solve the problem.

In the numerical example, five potential locations for the construction of distribution centers and 16 retail centers were specified. Information regarding location coordinates of potential sites for construction of the centers, arrival

rate of customers to the retail centers, potential population of customers in the retail centers, and other required information is provided in Tables 1 and 2.

Table 1. Customer data in the large numerical example

Customer number	Coordinate x	Coordinate y	Arrival rate of customers	Potential population of customers	Customer waiting time cost per unit time
1	20	35	17	5	18
2	8	31	18	7	13
3	29	43	13	5	19
4	18	39	19	2	16
5	19	47	12	6	11
6	31	24	18	7	13
7	38	50	13	2	19
8	33	21	13	9	22
9	2	27	17	8	10
10	1	12	20	5	14
11	26	20	16	7	20
12	20	33	18	8	15
13	15	46	15	6	23
14	20	26	11	3	23
15	17	19	18	2	13
16	15	12	16	6	15

Table 2. Data of potential sites for construction in the large numerical example

Number	Coordinate x	Coordinate y	Construction cost	Product arrival rate to the center	Holding cost per unit product per unit time
1	6	7	10841	8	19
2	19	44	11961	7	26
3	37	23	6091	8	22
4	35	6	7570	7	30
5	5	8	7497	8	35

The capacity of each vehicle was considered to be 70 units and the transportation cost of each unit product was 10. The index q , which represents the minimum number of people in the queue in order to begin servicing vehicles, was considered equal to 5.

The proposed algorithm in Section 3.2 was employed in order to produce the initial result. In the obtained result, Distribution Centers 2 and 3 were selected for construction, covering Customers 10-9-8-7, 3-5-13-2 and 14-12-1-4 with three vehicles covered by Distribution Center 2 and Customers 15-16-11-6 with one vehicle covered by Distribution Center 3.

As mentioned in Section 2, one expected outcome of the proposed problem was the determination of warehouse capacity of distribution centers and the reorder point of products for warehouses designated for vehicles. The direct search method was utilized in order to achieve this outcome. After several implementations of the algorithm for various amounts of capacity and the reorder point, the variation range of these two decision variables was determined. The variation range of distribution center warehouse capacity was estimated to 100-150, and the variation range of distribution center reorder points was estimated to be 0-10. In order to determine the optimal amount of these two variables, six levels of variation for warehouse capacity and 11 levels of variation for reorder point were considered. A 5-unit interval was determined for vehicle warehouse capacity and a 1-unit interval was

determined for the reorder point. The algorithm was implemented for various values of reorder points and each level of warehouse capacity. The related cost graph is presented in Figure 2.

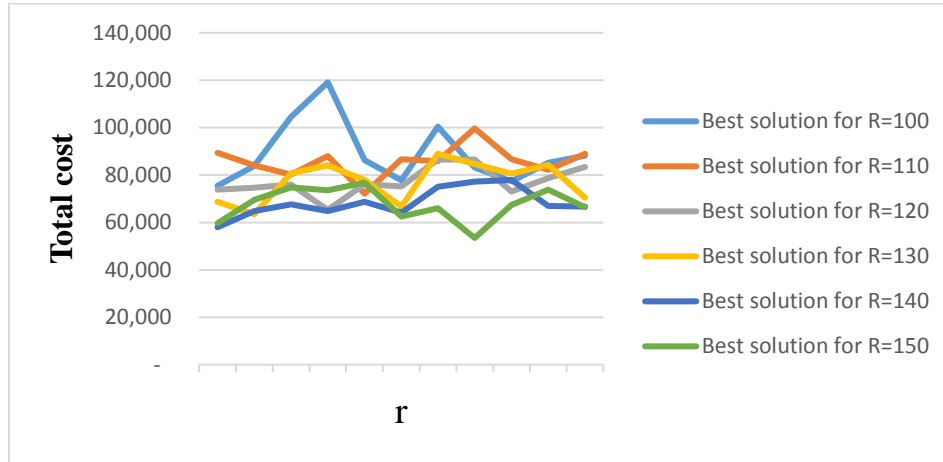


Figure 2. Cost graph of large numerical example for values of reorder point and warehouse capacity

According to Figure 2, the lowest cost of the large numerical example occurred for reorder point 7 and warehouse capacity of 150. The optimum result of this combination is construction of Distribution Centers 2, 3, and 5, assignment of Customers 1-12-14 and 2-13-5-7 with two vehicles to Distribution Center 2, assignment of Customers 4-15 with one vehicle to Distribution Center 3, and assignment of Customers 3-6-11-16 and 8-9-10 with two vehicles to Distribution Center 5.

5. Conclusion

The LRIP is a recent crucial problem for analysts of logistic systems with regard to integration of decisions. In order to realistically resolve the problem, a number of hypotheses in problem modelling, including probability of customer demands, travel time of vehicles, perishability of products, and customer refusal right, were discussed in this paper. The research problem was subcategorized into the location-routing problem with regard to uncertainty of customer demand and travel time and the inventory control problem with regard to perishability of products and customer refusal right. The mathematical model of location-routing was developed using the CCP approach, and the inventory control subproblem was modelled using queue systems. Their related costs were measured. Finally, the SA algorithm was provided for problem solving.

The main distinctive achievements of this study are as follows:

- (1) The current study conducted unprecedented evaluation of the probability of vehicle travel time in location-routing problem modelling.
- (2) Due to probability of customer demands, inventory control was modelled using queue systems. In temporal-themed literature, scholars have considered demand probability and employed the stochastic planning method for modelling. However, this study utilized queue systems for inventory problem modelling due to probable planning limitations and complexity of its calculations.
- (3) In a diversion from real-world experience, customer refusal rights have been ignored in all studies of inventory location-routing problems. Consequently, we are diverted from the real world. Therefore, for the first time, this right was considered for retail center customers in this study.
- (4) Only two previous studies investigated perishability of products in their models, but they failed to achieve efficiency because they ignored other assumptions such as probable travel time and customer refusal right. The LRIP model of perishable products with multiple constraints such as uncertain demands and travel time, renegeing and balking is presented in this article for the first time.

Suggestions for further study

Additional studies could refer to the following guidelines:

- (1) This study investigated inventory control only in distribution centers and evaluated reorder points only in established distribution centers. This research could be developed by designing queue systems for retail centers and setting reorder points according to least inventory control costs. Queue systems could be designed by determining the state of the system as i , j , and k , which are the number of goods and products in retail depots, the number of present customers in the system, and available goods in distribution depots, respectively.
- (2) One basic hypothesis in this study was imperishability of products along the routes. However, food may perish along routes during real distribution. Therefore, focused attention on the perishability of goods would increase practical efficiencies despite high complexities.
- (3) Inventory control in distribution centers pursues a continuous-review policy (R, r) so that suppliers can always fulfill orders of goods and products in any quantity. Indeed, suppliers' considerations are neglected in the case of product supply. In reality, continuous-review is carried out for distribution centers based on fixed order, and suppliers are ready to produce goods according to distributors' orders. Therefore, the order-fixed continuous-review policy increases practical efficiencies.
- (4) In the current study, a customer demanded one product unit per one visit, but real-world ordering does not contain a clear number of ordered goods. Therefore, further inquiries could assess random numbers of goods and products ordered.

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