Controlling Ground Vehicle Nonlinear Dynamics by the Use of Automobile Traction Models

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Abstract

The objective of this investigation is to assess and comprehend longitudinal ground vehicle traction nonlinear dynamics. Particularly, it explains the operation of vehicles using rubber tires in longitudinal accelerating and braking environments by utilizing two automobile-traction models. Specifically, it considers a two-wheel model and a single-wheel model and assesses their nonlinear properties. In addition, the wheel oscillation rate and forward automobile velocity are considered to be dynamic states. This paper contributes to beneficial formulation in which slip of the wheel, which is a dimensionless value of the variation between the tire circumference velocity and vehicle velocity in relation to center of the wheel substitutes the tire angular speed as dynamic state. This investigation contributes to more additional information about vehicle traction dynamic nature and in every model investigated, the specific properties of the modelling technique enable easy understanding of the dynamic responses of both the two-wheel and single-wheel traction models. Finally, it introduces an ADRC controller to precisely manage the direction and velocity.

Keywords: Direction, speed, ground vehicle, coordinates

1. Introduction

Generally, ground automobiles are classified in two groups namely non-guided and guided. Particularly, non-guided automobiles can maneuver on the ground in any direction while guided ground automobiles like track-levitated trains only maneuver on fixed passages. Specifically, this paper analyses the nonlinear properties in acceleration and longitudinal braking situations of non-guided automobiles using rubber tires. This analysis is highly crucial as ground automobiles dynamics are determined through analyzing ride, handling, and performance parameters. Particularly, automobile performance can be defined as capability of a vehicle to handle forces produced in cornering, braking, and accelerating situations and is dependent on the overall condition of the automobile. Ride parameters constitute of the impacts of vibration and noise caused by factors like engine, tire or road condition and the response of the driver to these conditions (Wong 19). The following section describes the background of the tractive behavior of ground vehicles using rubber tires and how it influences this analysis.

1.1 Background

There is extensive research done on ground vehicle technology due to the huge role ground vehicles play in transportation purposes of both goods and people. In relation to this, advancements in technology have led to manufacture of ground vehicles that are semi-autonomous or autonomous in operation so as to reduce cost and time. Not to mention, military even utilizes unmanned vehicles to undertake casual and risky operations. Safety factor has highly contributed to the rising performance requirements of manned ground vehicles, for example, throttle or brake by-wire technology has enabled safety systems for passengers like stability management and driver aided automated lane keeping to be attained. In addition, performance needs have to be considered in vehicle configurations that utilize control functions like rear steering and differential braking together with by-wire technology. Notably, vehicle stability in sudden movements and preventing rollover has been facilitated by inclusion of differential braking systems. As a result, extensive research has been done on the management of multi-input ground vehicles...
leading to development of techniques utilizing linear quadratic configurations. Consequently, due to the current developments in car technologies various complex accessories like Cameras, Cruise control, and GPS have made car control and modelling processes to advance.

2. Modelling of Systems
Correspondingly, in vehicle traction investigations the wheel and overall vehicle dynamics are assessed using lumped mass models like two-dimensional total four-wheel or two-wheel models and one-wheel model for cornering. In particular, attributes of these systems deal with synergy amidst the road exterior, wheel arrangements, and the vehicle with the force responsible for accelerating or decelerating the vehicle being the longitudinal friction force amidst the tire and road. Notably, this force is represented as a slip situation at the contact therefore composition of the motion equations for vehicles running on tires need to consider the friction force produced at the road or tire interface. Consequently, results of investigations done indicate that friction coefficient acts as the proportionality constant and the longitudinal force of friction is directly proportional to the normal contact force. Not to mention, the friction coefficient can be modified in a specific way that relies on slip, which is a dimensionless property establishing the variation amidst the velocity of vehicle and circumferential velocity of the tire in relation to the center of the wheel. This variation is produced by the wheel brake torque that opposes the vehicle inertia and happens in braking.

Note that the slip affects the vehicle dynamic through force of friction as it relies on the interaction between the wheel and vehicle dynamics. Due to this, it leads to a system with both wheel and vehicle equations which are generated on the basis of the absolute wheel oscillation rate and velocity of the vehicle in relation to the surface. With regards to this, a reference coordinate system is utilized when modeling a moving automobile due to the fact that vehicle inertia moments remaining constant. As a result, the standard coordinate system of the Society of Automotive Engineers (SAE) shown by Figure 1 is utilized.

![Figure 1: Coordinate System fixed on a vehicle](image)

Comparatively, consider a model which sums up the opposing and friction forces to one parameter, specifically of a car driving in a straight direction on a horizontal road. Notably, the engine supplies the driving force which should outweigh the vehicle weight that acts in a downwards direction and is opposed by the inertia, wind resistance, and friction between surface and wheels. This force is represented by the equation below:

\[
F = m \frac{dv}{dt} + b \cdot v
\]

Where V represents the velocity, b the damping force, \( \frac{dv}{dt} \) the acceleration rate, m the vehicle mass, and F the engine force.
3. Problem Statement
Vehicle control is highly crucial in situations like braking and steep landscapes and vehicle dynamics are complex due to varying road and environmental conditions. Comparatively, the car is required to accelerate and attain the required velocity in a 2% accuracy within three seconds.

4. History
In past times, there were numerous controllers utilized to control ground vehicles. Some of these controllers were the Model Predictive Controller, the Quadratic Programming Based Control Allocation, and the PID Controller. The PID controller can be tuned easily and has fair tracking capability however, it relies on the dynamics of the system and has low noise exclusion capability. Additionally, the classic CA measures rely on the least squares concept which utilizes a reference model pseudo-inverse. Despite this technique having a high efficiency and being easy to implement, it does not recognize effector command weaknesses. Due to these reasons, Model predictive control (MPC) has highly been adopted in vehicle control developments due its ability to decrease the time required in computing. In addition, MPC has been widely advanced in the chemical industry as dynamics of plants create adequate time for computing and current MPC developments enable acquirement of swift online quick fixing by moving some of the computational tasks off-line. Even though MPC is highly effective, it is also complex in nature and requires additional accessories to store the results of off-line computations. In summary, these classical controllers have low tracking capabilities and noise cancellation abilities.

5. ADRC Controller Design
ADRC is a new technology that possess a swift controller with minimal plant information requirements. It utilizes ESO principles and its management algorithm swiftly approximates and substitutes instantly the impacts of unidentified interruptions and dynamics compelling unidentified plant to act like a normal one. This controller can be utilized to solve various problems such as nonlinear plants, multi-input multi-output (MIMO), and single-input single-output (SISO). Below is the description of an ADRC for the second order dynamic system structure:

\[ y(t) = f(t, y, y, w) + bu \]  
(3)

In this equation, \( b \) represents large frequency addition, \( u \) the input, \( f \) is the mixed impact of both the internal and external dynamics and \( Y \) is the determined output to be managed. Practically, there is no mathematical determination of \( f \) therefore ADRC offers a solution to this issue under the concept that if \( f \) is approximated in a particular duration then a control signal can be utilized to remove it decreasing equation (3) to a double integral plant. Hence it produces a linear cascade integral plant which is can be managed using a PD controller. This concept alters this conflict to an approximation giving the following solution;

I. Extended State Observer
Notably, the reliability of ADRC relies on the accuracy of determining \( f \) through an Extended State Observer (ESO) at a particular time. This can be obtained by enhancing the state variables to add \( f \) and utilising a drive axis model represented in a linear state space. Assuming \( f \) is unidentified, \( X = [x_1, x_2, x_3]^T \), and \( x_1=x, x_2=\dot{x}, x_3=f; \)

\[
\begin{align*}
\dot{X} &= AX + Bu + Eh \\
z &= CX
\end{align*}
\]
(4)

Note that

\[
A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad h = f
\]

Since \( h \) is unidentified, the state observation becomes;

\[
\begin{align*}
\dot{X} &= AX + Bu + L(z - \hat{z}) \\
\dot{z} &= CX
\end{align*}
\]  
(5)

In addition, the ESO observer gain is \( L = [l_1, l_2, l_3]^T \) and because it is adequately designed and resolved, observer state (5) will trail that of plant (1). To simplify it, take \( \hat{z}(z) = [1 - (A - LC)]^{-1} (z + \omega_o)^T \). Note that the ESO consists of a single adjusting property \( \omega_o \) and is an observer bandwidth. By modifying \( \omega_o \) a settlement is easily
attained amidst noise-sensitivity and performance, with a higher observer bandwidth giving a more precise approximation. Nonetheless, a higher observer bandwidth heightens sensitivity of noise thus there is need for an adequate observer bandwidth to be chosen.

II. Design of Controller

Below is a control law that can be established using the ESO;

\[ u = (-\dot{x}_s + u_b) / b \] ........................ (6)

This law can be managed through a PD controller of the following form;

\[ u = k_p (r - \dot{x}_s) + k_d (r - \dot{x}_s) \] ........................ (7)

Notably, \( r \) represents the required trajectory and a feed forward system is incorporated in (7) in a bid to decrease the error in tracking. Additionally, for Equation (7) \( k_p \) and \( k_d \) are the chosen controller gain properties producing \((s + \omega_0)^{-1}\) Hurwitz. Alternatively, \( k_p = \omega_i^2 \), \( k_d = 2\omega_i \) and the main concept in ADRC is to approximate \( f \) in a particular duration and erase it from the law of the controller. Comparatively, a high controller bandwidth heightens the velocity of response but also can cause the system to become unstable hence modification of the controller bandwidth ought to be done carefully.

6. Results of the Simulations

a) Open loop response

The figure below represents an open loop response of the system showing instability of the system

![Open loop response](image)

Figure 2: Open loop response

b) ADRC controller closed loop response

The ADRC controller is adjusted to give required system performance through trial and error technique using the following values

\[ \omega_0 = 26 \]

\[ \omega_e = 7 \]

Figure 3 represents system response when an ADRC controller is utilized. Note that the system attains steady state accuracy of 2% in less than 2 seconds.
c) **Interruption Rejection capability**
Correspondingly, the system input is subjected to a disturbance after 4 seconds in a bid to show the interruption rejection capability of the system. Figure 4 represents a disturbance value of 1000 units and indicates a response that is even.

In a similar fashion, the figure below illustrates the reaction of the closed loop configuration to disturbance at a value of $T_d = 1e5$;
d) Rejection Capability to External Disturbance

In a similar manner, the closed loop configuration feedback is subjected to an external disturbance at a sampling duration of 0.001 seconds. As demonstrated by the figure below, the output possesses no noise despite the system being subjected to noise power of $10^{-5}$.

![Figure 6: Closed Loop System Reaction to Noise Power 1e^{-5}](image)

In contrast to the response above, the output demonstrated a noisy response when the system was subjected to a disturbance with Noise power value of $10^{-4}$. It is thus evident that the ADRC is only capable of cancelling external disturbances of maximum Noise power of $10^{-5}$. It is demonstrated by the figure below:

![Figure 7: Closed Loop System Reaction to Noise Power 1e^{-4}](image)

7. Conclusion

It is evident that there is instability in the system when based on an open loop. However, the closed loop system using an ADRC controller is more reliable as it possesses large external noise cancellation capability and disturbance cancellation capability. Therefore, the ADRC controller is most suitable in system management.

References

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