Integrating a Supply Chain with Vendor Managed Inventory and Joint Replenishment Policies

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Abstract

We study an integrated supplier chain consisting of a vendor and multiple buyers. We assume that, in order to coordinate the supply chain and achieve high efficiency, the vendor adapts Vendor Managed Inventory (VMI) policy. Demand is modeled as a deterministic linear demand function where buyers’ markets are non-identical. We consider the supply chain problem where the lengths of buyers ordering periods are the decision variables in the optimization model. In our setting the vendor produces multiple items. To facilitate the production planning, the vendor imposes the joint replenishment policy for its buyers. In this paper we present a model of two-echelon single-vendor multiple-buyers supply chain model with multiple items and under joint ordering policy. We analyze the model carried out with a numerical study. To validate the model, we solve some numerical sample problems by GAMS optimization software. Then, we study the sensitive of the model with respect to model’s parameters.

Keywords Supply Chain Management, Vendor Managed Inventory, Joint Replenishment

1. Introduction

The term “supply chain” refers to an integrated view and approach to planning and the control of material and information flows among vendors and buyers. This concept, bridges the inventory management focus in operations management and the analysis of relationships from industrial organization (Lau and Lau, 2003). A vendor usually is the producer of products or items and sells those items to buyers who in turn sells them on their local markets. During the last decade we observed an increasing emphasis on the methods to integrate the decentralized decisions by buyers and vendors. One example of policies designed to integrate the supply chain is Vendor Managed Inventory (VMI) in which the vendor assumes responsibility for maintaining inventory levels and determining order quantities for its buyers, often distributors or retailers (Dong and Xu, 2007). A proper modeling of supply chain can address many issues and inefficiencies in supply chain and can help supply chain managers to enhance their understanding of supply chain. Vendor managed inventory (VMI) is a proven concept for successful collaborative and cooperative agreements in supply chain which efficiently enables a proper information sharing, communication and coordination between the stages of supply chain.

This paper presents an integrated inventory model of VMI system in a two echelon supply chain comprising if multiple buyers who purchase multiple items from a single vendor. The vendor applies joint replenishment policy in which he gets the order for all items at once. Although most of the supply chains in the real-world applications have multiple products, majority of the existing studies only consider a single item. In this paper we focus on concept of VMI and develop a mathematical model for multi-item and multi-buyer supply chain operating under both VMI and joint replenishment policies. The main contribution of this paper is to model the supply chain mentioned under linear demand model and represent the convex mathematical program associated to this supply chain system. We believe this setting is a realistic setup which can capture many of nowadays supply chains.

Our paper relates to three main bodies of literature. First, we contribute to the literature on the coordination methods in supply chain with emphasis on VMI and joint replenishment. Second, our study extensively uses the methods of
deterministic inventory models and more generally inventory theory (including stochastic models). Next, we briefly review the relevant papers in the literature. Third, from solution methodology standpoint, we are also related to those papers which propose algorithms to efficiently solve complicated supply chain models.

Recently, many researchers have paid attention to integrated approaches in supply chain. In the supply chain literature, two different perspectives can be identified: (1) Local optimization and (2) Global or integrated optimization (Aliabadi et al., 2013). Local optimization of each member does not guarantee optimality of the total supply chain. On the other hand, integrated supply chain optimization methods aim to maximize a chain’s revenue or to minimize the cost of a chain considering all members of a supply chain. In this paper we adapt the second approach by introducing the VMI and joint replenishment policies. Darwish and Odah (2010) defined VMI as “an integrated approach for retailer-vendor coordination where the vendor decides on the appropriate inventory levels within bounds that are agreed upon in a contractual agreement between vendor and buyers”.

There are several benefits of VMI highlighted in the literature. The potential improvement of VMI which is enabled by information system and electronic data exchange systems is estimated to be around 42% (Angulo et al., 2004). Besides, some studies show that VMI is an effective way to mitigate the bullwhip effect (Yang and Wee, 2003). Kheljani et al. (2009) considered the coordination problem between one buyer and multiple potential suppliers in the supplier selection process. In the objective function of their model, the total cost of the supply chain is minimized rather than only the buyer’s cost. The total cost of the supply chain includes the buyer’s cost and suppliers’ costs. Using an EOQ model, Pasandideh et al., (2011) developed genetic algorithm to solve integrated multi-product model of a dyadic supply chain. However, they do not consider joint replenishment policy. Darwish et al. (2012) purposed models that include a contract between the vendor and retailer in a two-layer supply chain under VMI initiative. Other examples of papers in this category are Ross et al. (2017), Cai et al. (2017), Sajadifar and Pourghannad (2011) and Bazan et al. (2017).

Our study also relates to the extensive literature on deterministic inventory models. The Economic Order Quantity (EOQ) models have been the central part of supply chain for many years. The EOQ model is the building block of deterministic inventory models. For a complete review of EOQ models, we refer the readers to Choi (2014); For the treatment of EOQ models with general cost functions, Frenk et al. (2014) is a good source. See also Hadley and Whitin (1963) and Zipkin (2000) for some of the fundamental extensions of the EOQ model and its applications to production planning.

There is also an extensive literature on the solution methods for solving the deterministic supply chain models. Yu et al. (2013), and Kuo and Han (2011) considered a hybrid method based on a dynamic programming algorithm and GA. Sadeghi et al. (2014) developed a bi-objective VMI model in a supply chain with one vendor (producer) and several buyers. They showed that their problem is NP-hard and proposed a two multi-objective GAs, namely NSGA-II and NRGA, which were used to find Pareto fronts. Other examples are a GA algorithm application in traditional way (Roy et al., 2008), an innovative 2-level GA (Aliabadi et al., 2013), Neural network (Avşar and Aliabadi, 2015), and implementing agent-based algorithms (Aliabadi et al., 2017). Finally, although we focus on the deterministic models, we acknowledge that there is a body of inventory literature focusing on stochastic inventory models, examples are Axsater (2006), Pourghannad (2013), Bagherpour et al. (2009), Sajadifar and Pourghannad (2010), Shahraki et al. (2015), Voß et al. (2009), Tamjidzad and Mirmohammadi (2017), and Sajadifar and Pourghannad (2012).

This paper presents a model of integrated supply chain under VMI and joint replenishment policies with considering multiple buyers multiple items framework. The rest of paper is organized as follows. In section 2, we present notations and derive the model. In section 3, we illustrate the model by using a numerical example and check sensitivity of the model with respect to fixed ordering cost. Finally, the section 5 concludes the paper and presents the future research directions.

2. Problem Description and Notations
In this section, we present the mathematical formulation of our problem. There is a supply chain with two echelons where the upper echelon consists of a vendor (she) and the lower echelon consists of N buyers (he if referred to a single buyer). We next present the elements of our model. The complete list of notations and parameters can be found in the Appendix A.

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2.1. Demand Model
We assume that the demand in each buyer’s market follows a linear demand function and demands for different items are independent of other items and other markets. This is a common assumption in the literature (see for example Lau and Lau, 2003 and Pourghannad et al., 2015). In particular, if the $j^{th}$ buyer purchases $y_{ij}$ units of the $i^{th}$ item, the demand on the $j^{th}$ buyer market is according to

$$P(y_{ij}) = a_{ij} - b_{ij}y_{ij} \quad \text{for all } i = 1,2,...,M \text{ and } j = 1,2,...,N$$

(1)

Values $a_{ij}$ and $b_{ij}$ are constant and positive coefficients which determine the market’s characteristics. Since the price for each item at each market is always positive, i.e. $P(y_{ij}) > 0$, we should always have

$$y_{ij} \leq \frac{a_{ij}}{b_{ij}} \quad \text{for all } i = 1,2,...,M \text{ and } j = 1,2,...,N$$

(2)

2.2. Overview of Supply Chain’s Costs
Before proceeding with the detailed derivation of supply chain costs, we present an overview of costs that we consider in this paper. Under VMI policy, the vendor takes the leadership and manages the replenishment systems for buyers. The costs that we consider in this paper are similar to those considered in some similar studies including Pourghannad et al. (2015). The following summarizes the breakdown of costs and revenues in the supply chain we study

A. Buyers’
   a. Purchasing Costs
   b. Revenue from sales

B. Vendor
   a. Production costs
   b. Ordering costs incurred by the buyers
   c. Holding costs at its own warehouse as well as at the buyers’ facilities
   d. Distribution/transportation costs

2.3. Buyer’s Costs and Revenue
When the vendor implements VMI policy, she takes the responsibility for all inventory costs and a buyer’s profit is simply the difference between the purchasing costs and his revenue from selling items on his market. When the contract price between the vendor and the $j^{th}$ buyer for the $i^{th}$ item is $W_{ij}$, the profit of $j^{th}$ buyer from selling the $i^{th}$ item is as follows

$$P_{bij} = P(y_{ij})y_{ij} - W_{ij}y_{ij} \quad \text{for all } i = 1,2,...,M \text{ and } j = 1,2,...,N$$

(3)

$$P_{bij} = (a_{ij} - b_{ij}y_{ij})y_{ij} - W_{ij}y_{ij} \quad \text{for all } i = 1,2,...,M \text{ and } j = 1,2,...,N$$

(4)

$$y_{ijmin} \leq y_{ij} \leq y_{ijmax} \quad \text{for all } i = 1,2,...,M \text{ and } j = 1,2,...,N$$

(5)

Note that the equations (2) and (5) should be satisfied simultaneously, hence we will have

$$y_{ijmin} \leq y_{ij} \leq \min\left\{y_{ijmax}, \frac{a_{ij}}{b_{ij}}\right\} \quad \text{for all } i = 1,2,...,M \text{ and } j = 1,2,...,N$$

(6)

To have an efficient formulation we introduce an intermediate variable $x_{ij}$ as

$$x_{ij} = \min\left\{y_{ijmax}, \frac{a_{ij}}{b_{ij}}\right\} \quad \text{for all } i = 1,2,...,M \text{ and } j = 1,2,...,N$$

(7)

This helps us to write Equation (6) as Equations (8) to (10) as follows.
2.4. The Vendor’s Costs and Revenue

Producing each unit of the $i^{th}$ item costs $\delta_i$ for the vendor. We assume there is no economy of scale. Therefore, the total production cost of the $i^{th}$ item for the vendor is $\sum_{j=1}^{N} y_{ij}$. This is a common way of modeling production costs in similar inventory-production problems (see for example Choi, 2014 and Pourghannad et al., 2015).

As we stated earlier, the buyers are not responsible for ordering in a VMI system, the vendor takes the responsibility for ordering the items. The ordering costs associated with each channel for the vendor is denoted by $A_{j}^{VMI}$ and is calculated according to Equation (11) which is the sum of the vendor and the $j^{th}$ buyer ordering costs. Note that since the order for all items are placed at once the costs does not depend on items and only capture the index for the buyer.

$$A_{j}^{VMI} = A_{V} + A_{j}^{B}$$

In a similar way, the holding costs associated to the $i^{th}$ item in the $j^{th}$ channel, denoted by $H_{ij}^{VMI}$, is the sum of the holding costs at the vendor and at the buyer (see Equation (12)).

$$H_{ij}^{VMI} = H_{ij}^{V} + H_{ij}^{B}$$

Given the Equations (1) to (12), now we can express the costs associated with each channel and calculate the optimal costs for each channel. Note that the whole supply chain is optimal if and only if each individual channel acts optimally. Next Theorem formalizes this idea.

**Theorem 1.** For each channel $j$, the optimal channel cost can be calculated as

$$TRC_{j}^{*} = A_{j}^{VMI} \left( \frac{2A_{j}^{VMI}}{\sum_{i=1}^{M} (y_{ij}^{VMI}y_{ij})} \right)^{-1} + \frac{1}{2} \sum_{i=1}^{M} \left( H_{ij}^{VMI}y_{ij} \right) \left( \frac{2A_{j}^{VMI}}{\sum_{i=1}^{M} (y_{ij}^{VMI}y_{ij})} \right)$$

**Proof.** See the Appendix B.

Now we can calculate the optimal profit associated with each channel when the vendor makes optimal decisions to achieve the lowest costs in each channel.

**Theorem 2.** For each channel $j$, the channel profit can be expressed as

$$P_{cj} = \sum_{i=1}^{M} \left( a_{ij}y_{ij} - b_{ij}y_{ij}^{2} - TRC_{j}^{*} - \delta_i y_{ij} \right)$$

**Proof.** See the Appendix B.

2.5. Final Mathematical Program

Now that we have calculated all the costs and profit components, we can present the final mathematical program for Vendor Managed Inventory system with joint ordering of multiple items and multiple buyers. The objective function is to maximize the channel profit which is the sum of profits generated by each channel (a vendor-buyer pair). As in Equation (8), we have minimum and maximum supply of the $i^{th}$ item in the $j^{th}$ market. Also, to linearize the model we
used an intermediate variable $x_{ij}$ which should satisfy the last three constraints in the mathematical program below in order to match the nonlinear term we had in the Equation (6). Finally, the quantities and prices of $i^{th}$ item in the $j^{th}$ market should be non-negative which is implied by the last two constraints in the formulation.

$$\begin{align*}
\text{Max } Z &= \sum_{j=1}^{N} P_{cj} \\
&= \sum_{j=1}^{N} \sum_{i=1}^{M} \left( a_{ij} y_{ij} - b_{ij} y_{ij}^2 - TRC_j^* - \delta_i y_{ij} \right) \\
y_{ij\min} \leq y_{ij} \leq x_{ij} &\quad \text{for all } i, j = 1, 2, ..., M, N \\
x_{ij} \leq y_{ij\max} &\quad \text{for all } i, j = 1, 2, ..., M, N \\
x_{ij} \leq \frac{a_{ij}}{b_{ij}} &\quad \text{for all } i, j = 1, 2, ..., M, N \\
y_{ij} \geq 0 &\quad \text{for all } i, j = 1, 2, ..., M, N 
\end{align*}$$

3. Numerical Illustration

In this section, we present a numerical example to illustrate the mathematical model we derived in sections 2 and 3. We consider a setting in which the vendor sells 4 products (or items) to 4 buyers (or markets). Table 1 shows all the parameters associated to markets/buyers. In Table 2, we resented the parameters associated to the vendor for each item. Note that since the vendor works under joint replenishment policy, there is only one fixed ordering cost for all products. Finally, the optimal solutions obtained from solving the problem by GAMS optimization software is given in the Table 3.

<table>
<thead>
<tr>
<th>Table 1. The sample problem’s data for buyers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$a_{ij}$</td>
</tr>
<tr>
<td>$b_{ij}$</td>
</tr>
<tr>
<td>$y_{ij\min}$</td>
</tr>
<tr>
<td>$y_{ij\max}$</td>
</tr>
<tr>
<td>$H_{ij}$</td>
</tr>
<tr>
<td>$A_{ij}^d$</td>
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<tr>
<td></td>
</tr>
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<td></td>
</tr>
<tr>
<td>$a_{ij}$</td>
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<tr>
<td>$b_{ij}$</td>
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<tr>
<td>$y_{ij\min}$</td>
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<tr>
<td>$y_{ij\max}$</td>
</tr>
<tr>
<td>$H_{ij}$</td>
</tr>
<tr>
<td>$A_{ij}^d$</td>
</tr>
</tbody>
</table>

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Next, we study how the optimal supply chain profit changes with respect to the parameter $A_{jVMI}$ (the fixed ordering cost, under VMI policy, for the $j$th buyer). As before, Tables 1 and 2 show the main parameter settings we consider. The sensitivity analysis is based on the 3 levels of parameters (as in Table 4). The middle range is the main data, lower level data is obtained by reducing the value of parameter by 20%, and higher level is obtained by increasing the level of data by 20%. The last raw in the Table 4, we show how the optimal supply chain profit changes as we change the values of $A_{jVMI}$.

### Table 2: The sample problem’s data for the vendor

<table>
<thead>
<tr>
<th>items</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{ij}^V$</td>
<td>9</td>
<td>10</td>
<td>9</td>
<td>12</td>
</tr>
<tr>
<td>$\delta_i$</td>
<td>7</td>
<td>8</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>$A^V$</td>
<td>70</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 3: The optimal quantities and the optimal supply chain profit

<table>
<thead>
<tr>
<th>items</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buyer 1</td>
<td>2000</td>
<td>866</td>
<td>500</td>
<td>2089</td>
</tr>
<tr>
<td>Buyer 2</td>
<td>735</td>
<td>1000</td>
<td>3137</td>
<td>1925</td>
</tr>
<tr>
<td>Buyer 3</td>
<td>639</td>
<td>1007</td>
<td>500</td>
<td>847</td>
</tr>
<tr>
<td>Buyer 4</td>
<td>1666</td>
<td>800</td>
<td>800</td>
<td>1588</td>
</tr>
<tr>
<td>SC profit</td>
<td></td>
<td></td>
<td></td>
<td>39500</td>
</tr>
</tbody>
</table>

### Table 4. Sensitivity analysis parameters and the optimal profit

<table>
<thead>
<tr>
<th>$A_{jVMI}$</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buyer 1</td>
<td>76</td>
<td>95</td>
<td>114</td>
</tr>
<tr>
<td>Buyer 2</td>
<td>64</td>
<td>80</td>
<td>96</td>
</tr>
<tr>
<td>Buyer 3</td>
<td>74</td>
<td>93</td>
<td>112</td>
</tr>
<tr>
<td>Buyer 4</td>
<td>84</td>
<td>105</td>
<td>126</td>
</tr>
<tr>
<td>Supply chain optimal profit</td>
<td>62216</td>
<td>39500</td>
<td>17473</td>
</tr>
</tbody>
</table>

### 4. Conclusion and Future Research Directions

In this paper, we considered an integrated supply chain which operates under Vendor Managed Inventory system with joint replenishment policy. The model is two-echelon supply chain consisting of a vendor and N buyers. The vendor produces M products and the demand at each market for these products is according to a linear demand model. To the best of our knowledge, this paper is the first attempt in the literature to model an integrated inventory model under VMI policy with considering joint replenishment constraint. The model proposed in this paper is a good approximation for many supply chains such as agricultural, dairy, and auto manufacturing which are dealing with multiple products and multiple buyers/markets. We illustrated the model by a numerical example and presented a sensitivity analysis with respect to the buyers’ fixed ordering cost.

As one part of future research, one can focus on developing algorithms to solve the problem we study in this paper for large scale supply chains with thousands of buyers and hundreds of items. Examples of such algorithms in supply chain settings are to use GA in traditional way like GA developed by Roy et al. (2008), an innovative way similar to...
Aliabadi et al., 2013, Neural Network algorithm (Avsar and Aliabadi, 2015), or agent-based algorithms (Aliabadi et al., 2017). The model presented here, when the customer’s valuation for products is different, can be integrated with a revenue management method (see Pourghannad 2013). Another extension which is worth to mention is considering the vendor’s budget limitations and its effects on the supply chain.

Appendix A: Summary of Notations
Below is the list of notations we used in this paper.

\[ a_{ij} \] : The maximum demand possible for the \( i \)th product at the \( j \)th market
\[ b_{ij} \] : The slope of demand function of the \( i \)th product at the \( j \)th market
\[ y_{ij} \] : The order quantity (or supply) of the \( i \)th product at the \( j \)th market
\[ P(y_{ij}) \] : The price of the \( i \)th product at the \( j \)th market
\[ y_{ijmin} \] : The minimum sale quantity (or supply) of the \( i \)th product at the \( j \)th market
\[ y_{ijmax} \] : The maximum sale quantity (or supply) of the \( i \)th product at the \( j \)th market
\[ W_{ij} \] : The contract price between for the \( i \)th product with the \( j \)th buyer
\[ \delta_{ij} \] : The \( i \)th product’s production cost
\[ A^v \] : The fixed ordering cost of the vendor which is indipende of items and includes all items ordering (which are going to be ordered together)
\[ A^b_j \] : The fixed ordering cost of the \( j \)th buyer
\[ A^VMI_j \] : The fixed ordering cost for the \( j \)th channel when the supply chain operates under VMI
\[ H^v_{ij} \] : Unit holding cost of \( i \)th product with the \( j \)th buyer
\[ H^VMI_{ij} \] : Unit holding cost of \( i \)th product with the \( j \)th buyer under VMI policy
\[ T^* \] : The optimal replenishment period for the \( j \)th buyer
\[ TRC_j \] : The total holding and ordering cost associated to the \( j \)th channel
\[ P_{cij} \] : The supply chain profit associated to the \( j \)th channel
\[ P_{b_{ij}} \] : the \( j \)th buyer’s profit from selling the \( i \)th product
\[ P_{s_{ij}} \] : The vendor’s profit from his interaction by the \( j \)th buyer
\[ M \] : The number of products or items
\[ N \] : The number of buyers or markets

Appendix B: Omitted Proofs
Proof of Theorem 1. Consider a channel between the vendor and the \( j \)th buyer. Since the supply chain operates under joint replenishment, the inventory holding and ordering costs associated by \( j \)th channel is as follows.

\[ TRC_j(T_j) = A^VMI_j T_j + \frac{T_j}{2} \sum_{i=1}^{M} (H_{ij}^{VMI} y_{ij}) \]

The function \( TRC_j(T_j) \) is convex with respect to \( T_j \) and hence there is unique minimizer of this function which could be obtained as

\[ \frac{\partial TRC_j}{\partial T_j} = 0 \rightarrow T_j^* = \frac{2A^VMI_j}{\sum_{i=1}^{M} (H_{ij}^{VMI} y_{ij})} \]

Now if we plug back the value of \( T_j^* \) obtained above into \( TRC_j(T_j) \) we get

\[ TRC_j^* = A^VMI_j \left( \frac{2A^VMI_j}{\sum_{i=1}^{M} (H_{ij}^{VMI} y_{ij})} \right)^{-1} + \frac{1}{2} \sum_{i=1}^{M} (H_{ij}^{VMI} y_{ij}) \left( \frac{2A^VMI_j}{\sum_{i=1}^{M} (H_{ij}^{VMI} y_{ij})} \right) \]

which completes the proof.
Proof of Theorem 2. Given the derivations in the section 3, the profit that supply chain earns from the $j^{th}$ channel is

$$P_{s_j} = \sum_{i=1}^{M} (P(y_{ij}) y_{ij} - TRC^*_j - \delta_i y_{ij})$$

Now plug back the value of $P(y_{ij})$ from Equation (1) and simplify the equations which give us the Equation (14) in the Theorem 2.

References


**Biography**

Milad Khajehnejad is currently a fulltime senior credit risk modeler in Realtyshares, a leader in real estate crowdfunding. Mr. Khajehnezad holds a Master of Science degree in Industrial Engineering from University of Wisconsin Milwaukee. He is a credit and portfolio risk manager with about 4 years of experience in working with large banks and peer-to-peer lending platforms. He has also accomplished various operations research and data analytics projects in supply chain networks design (location-inventory models) and high-volume production lines simulation. He has served as a reviewer for Transportation Research Part E: Logistics and Transportation Review journal.