The Re-formulation For Single Item Capacitated Lot Sizing Problem With Shortage, Inventory And With Strict Carbon Cap

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Abstract

Over the ongoing time we all are facing the impact of 'Climate Change', which is also known as 'Global Warming' or in other words "An unnatural weather change". One of the significant reasons are increase in level of Carbon dioxide in the atmosphere. Now a day, the main concern of the researchers is to discover how to reduce the carbon discharge to decrease the unnatural weather change. In our study, we investigate how firms manage carbon footprint in lot sizing problem under 'Carbon Emission Trading' mechanism. So, we used the concept of Pochet and Wolsey's capacitated lot sizing problem formulation and give a new formulation with carbon footprint constraints. The problems under consideration are single level, the single item production problem with a finite time planning horizon and the demand is also deterministic. We also consider the shortage cost. As per our knowledge in the literature, there is no such work has been done over single item capacitated lot sizing problem with shortage variable and with strict carbon cap constraint. We considered initial inventory and end period shortage is zero ($I_0 = S_T .= 0$) Also, for shortage cost, in earlier formulations are unable to eliminate them; however, our formulation is able to eliminate all shortage (We used the concept from Shrama et al (2017)). This is a solid theoretical contribution as it offers significant computational savings.

Keywords: Lot sizing problem, Carbon footprint, Carbon cap, Carbon emission trading

1. Introduction

As of late, the expression "Carbon footprint" is well-known throughout the world. In the 19th century scientists started observing the change in the climate due to industrial improvements. Researchers observed that the level of greenhouse gases was increasing in the climate. Greenhouse gases are water vapours with CO_2, methane, NO_2 etc. In the literature, there is no such clear definition present for the expression "carbon footprint" but it can be referred as 'A worldwide temperature alteration'. In fact, in the early 90's people had no clue how to quantify or measure the expression "carbon footprint". Generally, the amount of CO_2 emitted into the atmosphere is considered as "carbon footprint". Researchers, general and individuals are worried about this effect. More than 150 countries decided to take steps to reduce the greenhouse gases and they came up with a protocol named the KOYOTO Protocol (United Nation, 1997). In the year 2007, Intergovernmental Panel on Climate Change (IPCC) published the 4th assessment report, which concluded that the greenhouse gas emission creates a negative influence over our environment. European commission (2010) legalized the reduction of carbon emission. Carbon 'cap & trade' and 'carbon tax' are two most popular regulations which were implemented and accepted by different countries. In the annual report of Carbon Disclosure Project (2011) companies like Wal-Mart, Pepsi, Coca-Cola, Volkswagen, Ericsson promised to set up 'carbon target' for themselves. In order to reduce their carbon footprint, companies started using Eco friendly machines, less polluting vehicles and changed their operational decisions. Some companies proposed the idea of green manufacturing. In this paper, we give a concise review of relevant literature starting with different type of lot sizing problems. Considering carbon footprint as a constraint, this study tries to explore its impact over the single stage lot sizing problem.

The paper expresses its line of research in following manner. In section 2, we have given comprehensive literature about lot sizing problem, EOQ models and carbon footprint over supply chain, lot sizing. In section 3, we discuss about our mathematical formulation. In the last section we discuss the possible outcomes and its contribution for the practitioners.

2. Literature Review

Wilson (1934) proposed a statistical approach to find ordering points, thereby promoting the Economic Order Quantity (EOQ) equation in practice. This strategy accepts a steady or constant demand calculating a single ordering point or the total cost of the problem, when the demand rate shifts in various periods the outcomes from the EOQ recipe can be tricky. Wagner and Whitin (1958) proposed a different approach which can be performed on variable demand, but the algorithm was restricted to be used over large finite planning horizon also a dynamic programming-based algorithm requires an enormous measure of CPU processing time. If firms or plants directly supply the commodities to the markets, then it is referred as Simple Plant Location Problem (SPLP). Comuejols et al. (1983) proved that term SPLP is NP-Hard (A problem is NP-hard if a calculation for fathoming it tends to be converted into one for solving any nondeterministic polynomial time problem.) and the scientist used heuristic methodology for finding the best arrangements. Sharma and Murlidhar (2009) proposed a new formulation of SPLP, and also stated "SPLP is to choose a sufficient number of plants with unlimited capacities so that given the number of markets can be serviced while minimizing the sum of the fixed costs of plant location and transportation cost of shipping goods from plant to markets". In capacitated plant location problem (CPLP) the capacity of the plants is finite. For literature on reformulation of Capacitated Lot Sizing Problem (CLSP) Pochet and Wolsey (1991) and Miller et al. (2000) can be referred. In a paper written by Wolsey (2002), nothing was mentioned about stronger formulation using backlog. Verma and Sharma (2009) developed a relaxed formulation for the capacitated lot sizing problem in which backorder and setup cost was considered. Verma and Sharma (2010) extended their previous work and proposed Lagrangian Relaxation based algorithm to solve the backlogging problem for lot sizing case. Verma and Sharma (2015) improved their work by applying Lagrangian Relaxation over multi period and multi item capacitated lot sizing problem which convert it into a single constraint knapsack problem from an NP hard problem.

Wiedmann and Minx (2008) proposed a systematic definition of carbon footprint, "carbon footprint is a measure of the total amount of CO_2 emissions that directly and indirectly caused by an activity or is accumulated over the life stages of a product". The researchers also mentioned that all the direct and indirect or internal and external carbon emissions are required to be considered. Sundarakani et al. (2010) proposed a mathematical formulation to check carbon footprints over the supply chain and the same concept is also applied for green supply chain management. The work shows the usage of Eulerian and Lagrangian transport models to analyze the results. The method used was proposed by Lee et al. (2000) for measuring different types pollution, water quality etc. It was noticed that for the first time Sundarkani et al. used this concept to measure carbon footprint over the supply chain. In a paper, Hua and Cheng (2011) examined the effect of carbon cap and trade over the inventory management decision. The paper described how an optimal ordering quantity is affected because of carbon emission trading. Benjaafar et al. (2013) described how CO_2 emanation concerns could be coordinated into operational decision for production, procurement and inventory management. Absi et al. (2013) introduced different types of carbon emission constraints for single item incapacitated lot sizing problem. The constraints focused on the limitation of carbon emission for each unit product shipment and mode of shipments. This paper mentions four new emission constraints (periodic, cumulative, global and rolling carbon emission) but models only with periodic carbon emission constraint can be solved in polynomial time and for that too author had to develop a dynamic program. Chen et al. (2013) proposed if firms modified the ordering quantity under given conditions, then the carbon emission reduction is possible without significant increase in the total cost. Hovelaque and Bironneau (2015) advanced the work by proposing carbon constrained EOQ model with demand variable which is dependent on environment. That EOQ model takes environment policies, total carbon emissions, price and environmental dependent demands with it. The work discusses different conditions where companies can increase their profit and reduce the carbon emissions. In the same year Joana et al. (2015) proposed a separate mathematical model. The model works with uncertain demand with different types of carbon caps and tax over the supply chain and market considering different carbon policies. That model analyzes the effect of different environmental policies over the supply chain and helped them to take some decision on product types and innovation over its. That model was verified by some numerical examples describing how various carbon taxes, carbon cap, environment policies affect the supply chain and product innovation. Zakeri et al. (2015) focused his work on how carbon pricing and the carbon emission trading effect supply chain performance at the operational planning level using the empirical data. The data were collected from an Australian company where carbon regulatory policy schemes were maintained. Later in the year 2016 Absi et al. extended the study to real time situation where the concept of fixed carbon emission with single item green lot sizing problem is used for which dynamic programing algorithm was developed. Chai et al. (2018) analyzed the uses of carbon cap and trade mechanism for remanufacturing in both ordinary market and green market. In recent years, Liao and Deng (2018) considered uncertain demand for remanufactured product and proposed a mathematical model for carbon constrained EOQ model. It is acknowledged all over the world that carbon emission trading is one of the compelling methods to reduce the carbon emanation in various firms. As per the existing literature, there is no such work available where different fixed carbon emission and carbon cap is considered as a constraint for single stage lot sizing problem with shortage. In our study, we investigate how companies should manage carbon footprints in lot sizing problem under the 'Carbon Emission Trading' mechanism.

3. Problem Formulation

Sharma and Ali (2017) proposed a mathematical formulation (reformulation) for lot sizing problem, where they reduced the number of variables. In our paper, we used the same problem notations.

3.1. Mathematical model

Indices Used:

t: Set of Time period from 1….T

Constant:

f_t : fixed cost in time period 't';

p_t : per unit variable (production) cost in time period 't';

c_t : production capacity in time period 't';

D_t : demand in time period 't';

h_t : per unit inventory carrying cost in time period 't';

sh_t : per unit shortage cost in time period 't';

f_{ct} : amount of fixed carbon emission (like transportation emission etc.)

p_{ct} : variable amount of carbon emission (like emission due to production handling etc.)

h_{ct} : amount of carbon emissions per unit of inventory held per period

C : fixed carbon cap over the entire planning horizon (set up by carbon regulatory board)

Variables:

x_t : amount produced in time period 't';

y_t : 1, if machine setup to produce in time period 't', 0, otherwise;

s_t : shortage in time period 't';

I_t : Inventory in time period 't';

Mathematical model of SLCLSP (Single Level Capacitated Lot Sizing Problem) with strict carbon cap constraint

$$Min\ z = \sum_{t=1}^{T} f_t * y_t + \sum_{t=1}^{T} p_t * x_t + \sum_{t=1}^{T} h_t * I_t + \sum_{t=1}^{T} sh_t * s_t \ldots\ldots\ldots\ldots 1$$

subject to,

$$I_t - s_t = I_{t-1} + s_{t-1} + x_t - D_t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

$$\sum_{t=1}^{T} (f_{ct} yt + p_{ct} x_t + h_{ct} I_t) \leq C \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3$$

$$x_t \leq c_t y_t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4$$

$$I_t, s_t, x_t \geq 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 5$$

$$y_t \in \{0,1\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 6$$

The objective function in (1) minimizes the total cost over the entire planning horizon. Constraint (2) is an inventory balance constraint. Constraint (3) ensures the cap on carbon emission over the planning horizon is not exceeded. (4) th constraint is the capacity constraint. (5) and (6) are non-negativity constraints.

There are four cost elements present in the above formulation. The summation of those is the total cost of production or the objective function of the formulation.

- **Setup cost:** The setup cost is characterized as the cost inured at the season of beginnings of creation or generation changeover between items. The setup cost is the expense related to setting up a bit of generation hardware. This would join the expense of booking, record keeping, the setup repairman, moving the material, and testing the initial couple of units of yield to be sure the hardware is set up appropriately. We utilize a double factor in the demonstrating to speak to the need of a setup. On the off chance that the esteem if the parallel variable is one, the setup cost is caused and it speaks to that the office is fully operational for the time span. In the event that it is zero, the office isn't being utilized for that time span.

- **Production cost**: Production cost is the cost incurred to produce one unit of the product. It includes costs such as raw material cost, cost of labor etc.

*Production cost = cost of production per unit * no of units produced*

- **Inventory carrying cost**: Inventory holding cost represents the cost of storing and safekeeping the excess amounts that has been produced during a time period. It is the balance quantity of products after meeting the demand at the end of a time period.

*Inventory carrying cost = no of units in the inventory * unit holding cost*

- **Shortage cost**: When customer demand for a particular period is not met, it leads to shortage loss. There is a cost associated with it due to customer loss.

*Shortage cost = no of shortages * cost of the unit shortage item*

Sharma, R.R.K. et al (2017) proposed the new formulation for the Single Level Capacitated Lot Sizing Problem and they reduced the number of variables in the new formulation. This prompt calculation points of interest, and the contention is upheld by empirical investigation. The reformulation is better than the old one in terms of different details like the execution time, number of nodes assessed in the search tree and number of iterations. The definition of constants and variables are the same as above. In the reformulation the capacity constraint is modified to $\mathbf{x_t = c_t * y_t}$. It forces the formulation to produce the maximum capacity if the setup variable is one. Using the above expression, the production variables and shortage variables are replaced in the reformulation.

Reformulation of SLCLSP (Single Level Capacitated Lot Sizing Problem) with strict carbon cap constraint

$$Min\ z = \sum_{t=1}^{T} f_t * y_t + \sum_{t=1}^{T} p_t * c_t y_t + \sum_{t=1}^{T} h_t * I_t + \sum_{t=1}^{T} sh_t * (\sum_{t=1}^{T} D_t + I_{t^1} - I_0 \sum_{t=1}^{T} c_t y_t) \text{............7}$$

subject to,

$$I_0 + \sum_{t=1}^{T} c_t y_t + s_{t^t} = \sum_{t=1}^{T} D_t + I_{t^1} \text{..........................8}$$

$$x_t = c_t y_t \text{..9}$$

$$\sum_{t=1}^{T} f_{ct} yt + \sum_{t=1}^{T} p_{ct} c_t y_t + \sum_{t=1}^{T} h_{ct} I_t) \le C \text{.........................10}$$

$$I_0 = s_T = 0 \text{..11}$$

$$I_t \ge 0 \text{...12}$$

$$y_t \in \{0,1\} \text{....................................13}$$

4. Discussion and Conclusion

In the literature, Lagrangian Relaxation has been used to solve simple plant location problem with carbon footprints constraint. We extend the work by introducing shortage variable. The target of the traditional lot sizing problem where carbon cap constraint was not considered, was to minimize the total cost (variable and fixed cost) of the supply chain. We consider the carbon cap constraint to check how significantly carbon emission reduction is possible without significantly increasing the total cost. In the presence of carbon emission considerations, the firm must account for the emissions associated with various decisions regarding ordering, production, and inventory holding. Also, by using variable reduction technique, we have reduced the number of variables. In addition, the reformulation has a smaller number of factors than traditional formulation since production and shortage variables are replaced by setup and inventory variables. In this way, our reformulation and computational time can be better than the traditional model, also some improvisation can be expected in terms of number of iterations and the number of node calculation. Considering all the factors, a huge diminishing in computational effort can be expected.

It has been noticed that all constraints are linear in nature. We are undertaking an empirical study that plans to determine the efficiency of both formulations (Traditional and the new one).

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RRK Sharma has had 30 years of career to date. Started as a graduate engineer trainee with TELCO (PUNE) (now TATA MOTORS INDIA) during 1980-82, and later went on to do Ph.D. in management at I.I.M., Ahmedabad, INDIA. After Ph. D. in management, he worked with TVS Suzuki (for 9 months) as executive assistant to GM (marketing). Now he has 26 years of teaching and research experience at the department of Industrial and Management Engineering, I.I.T., Kanpur, INDIA. He has taught over 22 different courses in management at IIT Kanpur, INDIA (to B. Tech., M. Tech. and M.B.A. students) and is well versed with all the facets of management and has unique ability to integrate different areas of the subject. To date he has 811 (published/under review) publications (227 Full Length Papers and 569 Extended Abstracts Outlining Theoretical Framework) in international/national journals and eight research monographs). He has developed over 8 software products. Till date he has guided 60 M TECH and 18 Ph D theses at IIT Kanpur. He has been Sanjay Mittal Chair Professor at IIT KANPUR (15.09.2015 to 14.09.2018). He has won several international awards: American Medal of Honor and the 2006 and the 2008 Man of the Year award given by the American Biographical Institute, Inc., 5126 Bur Oak Circle, PO Box 31226, North Carolina, 27622, USA. Web: www.abiworldwide.com. He was elected FELLOW of AIMS International in 2012. He is the recipient of the Dr Manubhai M Shah Memorial Award 2013 (instituted by the Indian Commerce Association) & it carried a cash prize of Rs. 1 Lakh, trophy and a certificate of excellence. In 2015: "Membership Award" for Prof. RRK Sharma; given by IABE USA (International Academy of Business and Economics). Prof. RRK Sharma, recognized as "Distinguished Scientist", by Venus International Foundation Research Awards 2015 (VIFRA 2015) at CENTRE FOR ADVANCED RESEARCH AND DESIGN; Chennai: Dec 19; 2015. In 2016: Prof. RRK Sharma, "Distinguished Educator Award", selected by IEOM (Industrial Engineering and Operations Management) Society, USA; given Kuala Lumpur on MAR 09, 2016.