Multi-item, Multi-location Transshipment Model for Cross Filling

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Abstract

Distribution centers (DCs) usually receive orders from the customers (mostly retail stores) located in its area; and deliver goods in the following day. In order to secure fulfilling all orders every day, the DCs will have to maintain a high level of inventory, which will deteriorate the profitability. As an alternative to high level of inventory, cross filling is, after closing the daily order receipt, to ship the surplus items to other DCs that lacks the same item during the night to improve the order fulfillment rate. The economic justification of such cross filling will depend on the trade-off between extra cost of transshipment and saving of shortage cost. In this paper, an LP model is proposed, which determines the optimal quantity of items and number of trucks to transship so as to maximize the profit of cross filling for a day. Unlike most existing research on this problem where transshipment cost is proportional to the transshipped quantity, transshipment cost is considered as proportional to the number of trucks used for the transshipment, regardless of the quantity loaded in it, which we believe is more realistic. The proposed LP model also allows the 'simultaneous chain transshipment', which enables distant locations to supply surplus to the location that faces shortage.

Keywords
Transshipment, cross filling, order fulfillment, simultaneous chain transshipment

1. Introduction

1.1 Background

As information technology develops, physical distribution and delivery of goods to the customers is becoming the bottleneck in today's fast business environment. The supply chain of most manufacturers comprises factories, distribution centers (DCs), and retailers. Even on-line commerce employs DCs as an important component of its supply chain for efficient storage and order picking to supply goods. Order fulfillment has been one of the critical performance measures in most DCs. Shortage of a certain item at a DC will very likely lead to the shortage of the item at the customer stores, which will cause potential loss of sales, and jeopardize the satisfactory service for the consumers.

In order to avoid shortage, DCs are pushed to hold high level of inventory, which will incur substantial cost. As an alternative, cross filling has been studied in academia, and practiced in business for decades. Cross filling is, after closing the daily order receipt, to transship the surplus items to other DCs that lacks the same item so as to avoid shortage with relatively lower level of inventory. Such a lateral transshipment is also referred to as lateral resupply, reallocation of stock, and stock transfer (Paterson *et al*., 2011). Shortage can be either backordered or result in the loss of sales.

Such transshipment of surplus items will obviously incur extra cost such as trucking cost and material handling cost. However, depending on the cost parameters, transshipment can be economical, especially where shortage is critical such as special type blood supply in the hospitals. Successful implementation of transshipment can reduce inventory cost, while maintaining higher level of service for the customers.

In this paper, a practical problem is addressed as follows: there are single supplier and multiple non-identical DCs dealing with multiple items. Each DC supplies goods to the customers (retail stores) located in its own geographical region. DCs receive orders from the retail stores during a day, and close the receipt of orders at a certain time in the late afternoon; process the received orders (we assume stationary random daily demand); and try to fulfill the orders with inventory. For shortage items, DCs try to find surplus of the same item from other DCs. After determining how many of which items to transship from which DC to which DC, delivery trucks that DCs operate are used for transshipment between the pairs of DCs during the night. For a pair of DCs to transship items each other, it does not matter for trucks to depart from which DC. Transshipment trucks make round-trip travels, that is, return to the DC the truck departed so that there will be no change in the number of trucks for delivery at the DCs after the transshipment. Transshipment must be finished within a certain amount of time, say three hours, so that transshipped items can be delivered to the retail stores along with the items picked from the DC by the next day morning. Figure 1 illustrates an example of transshipment among nine DCs. Note that some DCs may transship items to multiple locations.

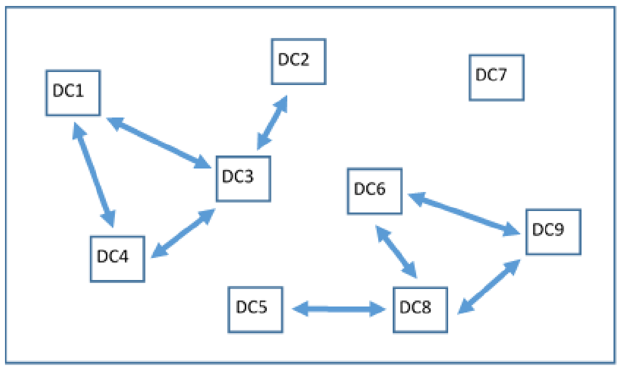

Figure 1. An example of transshipment among nine DCs

Meanwhile, each DC places orders to the supplier for replenishment of items, which arrive in the next day, that is, the replenishment lead time is one day. Each DC may use various replenishment policy such as continuous review, periodic, or order-up-to policy. In this paper, we assume that all DCs use base stock policy, that is, to order up to ***U*** every day since it is known to be optimal for such a case (Herer *et al*., 2006). Transshipment is a viable method to avoid shortage for the time until the items are replenished in the next day from the supplier.

In this paper, we propose a linear model which determines the optimal quantity of items and number of trucks to transship between the pairs of DCs so as to maximize the profit of cross filling for a day. Since the system regenerates every day, optimal decisions for a single day will result in the optimal average performance over infinite horizon (Herer *et al*., 2006).

Three unique contributions of this paper are as follows. Firstly, unlike most existing researches on this problem where transshipment cost is assumed proportional to the quantity transshipped, in this paper we consider transshipment cost is proportional to the number of trucks used for the transshipment, regardless of the quantity loaded in it, which we believe is more realistic, despite it makes the model more complicated. Secondly, the optimal transshipment is determined as a function of the length of time window (T) during which transshipment is carried out. This aspect is practically important, since more distant DCs can be considered for transshipment as T increases by, for example, more efficient order picking in DCs. Thirdly, we consider volume constraint of items to transship, which has not been addressed in the literature. Depending on the size of the items, it is not unusual for the total volume to transship to exceed a volume capacity of a small delivery truck. Furthermore, as a future research, we may exploit possible vehicle routing problem where both pickup and delivery over multiple locations are considered for transshipment, in which case volume constraint will become a critical one. As volume constraint is considered, an index for item is added to the formulation. These three contributions will make our model to be more useful in logistics practice.

1.2 Related Literature

There are numerous research papers reported in the literature regarding the transshipment, which can be classified by many criteria. One classification is that it is either proactive or reactive. Proactive transshipment is to redistribute stocks amongst all stocking points in the same echelon at predetermined point in time before demand is realized such that the stock level is equalized in terms of, for example, the days of supply. For example, van der Heide and Roodbergen (2013) address the problem of transshipping and rebalancing books in the libraries. They present a stochastic dynamic programming formulation and heuristic algorithms. On the other hand, reactive transshipment is to respond to situations where one of the stocking points face a shortage, while another has surplus stock on hand (Paterson *et al*., 2011). Another classification is that it is either complete pooling or partial pooling. Complete pooling is that the transshipping location is willing to share all of its stock (Paterson *et al*., 2011, also named as complete network by Lien *et al*., 2011); whereas partial pooling only agrees to transship while its inventory level is above a fixed threshold (Archibald, 2009). This threshold can be either a certain portion of the current inventory level, or a fixed amount of remaining inventory, or an amount determined as a function of remaining time until the next replenishment.

Papers on the transshipment problem can also be classified as either centralized or decentralized problem. Centralized transshipment problem is to minimize the total cost that incurs over all locations; whereas decentralized one is that each location tries to maximize its own profitability by determining the quantity and price of items to transship.

Transshipment issue has been studied in the area of optimal stocking policy for low usage items. Feng *et al*. (2018) investigates the replenishment and transshipment decisions in a two-retailer inventory system with a single selling season. They investigate emergency lateral transshipment (ELT) and preventive lateral transshipment (PLT); and show the existence of unique Nash equilibrium for the system under each policy.

Order fill rate is defined as the ratio of the number of fulfilled orders to the total number of received orders; whereas item fill rate is defined as the ratio of the total number of items supplied from the inventory to the total number of ordered items. Given a set of received orders and inventory file, Rim and Park (2008) presented a linear programming model to maximize the order fill rate.

Bhatnagar and Lin (2019) study the joint transshipment and production control policies for multi-location production /inventory systems in which items are manufactured and stocked at each location to meet the demand. They formulate the problem as a make-to-stock queue. For the two-location problem, the optimal policy as monotone switching-curve policy is characterized. For the multi-location problem, they develop two heuristic policies.

Wei *et al*. (2018) explore the impacts of lateral transshipments on the stability, bullwhip effect, and other performance measurements in the context of a two-tiered supply chain system composed of one supplier and two retailers. They develop a unified discrete-time state space model to address two different scenarios of placing orders. Analytical stability results are derived, through which it is found that inappropriate lateral transshipment policies readily destabilize the supply chain system. Theoretical results are validated through simulation experiments and the influences of system parameters on performance measures are investigated numerically.

Lee (1987) presents a two-echelon repairable item inventory system where emergency lateral transshipment is allowed. He derives an approximation for the expected values of performance measures such as the backorder level and the number of emergency lateral transshipments; and presents a procedure for determining optimal stocking levels. Axsäter (1990) addresses the same two-echelon lateral transshipment problem of repairable items in continuous review inventory systems with one-for-one replenishments and Poisson demand. Van Wijk *et al*. (2019) address an inventory model for repairable parts with two stock points, and characterize the threshold type optimal policy using stochastic dynamic programming. Patriarca *et al*. (2016) define system-approach model for determining the stock levels of repairable items in a complex network, by a genetic algorithm optimization process. Dreyfuss and Giat (2018) address the spares allocation problem in a two-echelon, exchangeable item repair system in which the lower echelon comprises multiple locations and the higher echelon is a single depot. They develop two algorithms; one is formula-based and is suboptimal; and the other combines simulation into the first algorithm and obtains a higher degree of accuracy at the cost of extra computation time.

Archibald *et al.* (1997) address the stock transfer problem between a pair of retailers where stocks at retailers are replenished periodically from an outside source, and customer demand at each retailer follows independent Poisson processes; and characterize an optimal policy. In case of stock-out, a retailer may either place an emergency order to the supplier or call a transfer to the other retailer, which may not respond to the call if its own stock is not enough for the remaining time until the next replenishment (called "partial pooling"). All orders and transfers are assumed to occur instantaneously. The objective is to minimize the expected discounted cost over an infinite horizon.

Transshipments are assumed to occur at any time during a period, whereas, in our paper, transshipment occurs only once in a period (day).

Archibald *et al.* (2009) present an approximate solution method which applies decomposition to reduce a Markov decision process model of a multi-location inventory system into a number of models involving only two locations. Coelho *et al*. (2012) firstly address the multi-period, single item problem of inventory routing and transshipment option simultaneously, namely, inventory routing problem with transshipment (IRPT); and present a mathematical formulation for the problem. A heuristic algorithm is proposed to solve four variants of the IRPT.

Herer *et al*. (2006) address the multi-location transshipment problem, where multiple retailers in the same echelon are replenished from single supplier every period. They prove that order-up-to replenishment policy minimizes the long-run average cost of the retailers. They suggest an infinitesimal perturbation analysis (IPA) procedure, a sample-path optimization technique, in which the gradients of the expected total cost with respect to order-up-to levels are obtained for different demand realizations and for candidate vector **S**. The gradient values are used to update **S**, and the procedure is guaranteed to converge to the optimal order-up-to vector. The operational transshipment problem is then solved for random demand observations to calculate the optimal expected cost.

Smirnov and Gerchak (2014) address the single-period circular unidirectional chaining problem, assuming that a location cannot both receive and ship items. They analyze the optimality of the problem and compare its performance to that of no pooling and 'pooling among all nodes' (they call this 'complete pooling'). Axsäter (2003) address the unidirectional lateral transshipment problem where transshipment is allowed only in one direction. This can also be interpreted as a substitution of higher quality item to lower quality item. Tagaras (1999) and Herer *et al*. (2002) address the problem of group configuration where transshipment is allowed only within groups.

Some papers consider that a limited number of links be established between the pairs of locations since establishing a link between locations requires investments in communication channels, physical distribution systems, and financial and administrative arrangements. Lien *et al*. (2011) address the problem of finding optimal transshipment network having P links; and analytically compare the total cost with unidirectional and bidirectional chain having P links. They demonstrate the efficiency and robustness of chain configuration for more general scenarios.

Although there are numerous research results reported in the literature, again, our work is unique in three aspects; firstly, we consider transshipment cost proportional to the number of trucks used for the transshipment, instead of the transshipped quantity; secondly, our model considers the length of time window during which transshipment is carried out; and thirdly, we consider volume constraint of items to transship.

2. Modeling

2.1 Notations

We use the following notations to describe the problem

k : item (k=1,2,$\cdots$,K)
i, j : distribution center (i, j =1,2,$\cdots$, W)
d_{ik} : demand of item k that DC i receives during a day
X_{ijk} : quantity of item k to be transshipped from DC i to j (decision variable)
U_{ik} : order-up-to value of item k at DC i
I_{ik}^- : available inventory of item k at DC i before transshipment after assigning on-hand inventory to the daily demand
S_{ik}^- : shortage of item k at DC i before transshipment after assigning on-hand inventory to the daily demand
I_{ik}^+ : inventory of item k at DC i after transshipment
S_{ik}^+ : shortage of item k at DC i after transshipment
m : handling cost per transshipped item
p_k : unit price of item k
h : daily inventory holding cost rate (=0.25/365=0.000684932)
N_{ij} : number of trucks used for transshipment between DC i and j, $1 \leq i \neq j \leq W$ (decision variable)
c_{ij} : trucking cost per round trip between DC i and j

r_k : shortage cost rate of item k
v^o : volume capacity of a truck
v_k : unit volume of item k
t_{ij} : time required for one round trip of a transshipment truck between DC i and j including loading/unloading items (We assume that t_{ij} is independent of the quantity to be transshipped, but only dependent on the distance between the two DCs.)
T: maximum time allowed for round trip transshipments

2.2 Daily Operation Schedule

Under order-up-to policy, the system regenerates every period. Hence minimizing the expected cost for one period is equivalent to minimizing the long-run expected costs (Herer *et al*., 2006). Hence we will focus on the single period problem. Figure 2 illustrates daily operation schedule of DCs, which can be described step-by-step as follows: for all i=1,2,$\cdots$,W and k=1,2,$\cdots$,K,

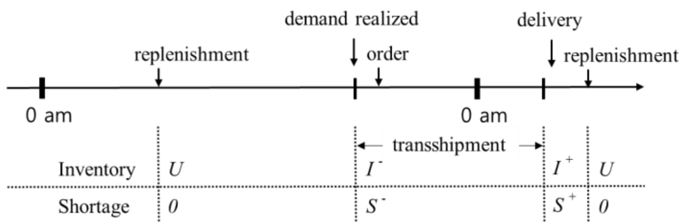

Figure 2. Daily operation schedule and corresponding inventory/shortage notations

Step 1: replenishment ordered in the previous day arrives in each DC from the upstream supplier, then backlogged demand is satisfied, so shortage is zero, and inventory level becomes equal to the order-up-to level U_{ik}.
Step 2: each DC i receives orders from its customers located in its own region during a day; and order receiving is closed at late afternoon and daily demand d_{ik} is realized.
Step 3: each DC i determines the inventory (I^-) and shortage (S^-) quantity of each item before transshipment after assigning its on-hand inventory to the daily demand as follows: if d_{ik} is greater than U_{ik}, then $S_{ik}^- = d_{ik} - U_{ik}$, and $I_{ik}^- = 0$; otherwise $S_{ik}^- = 0$, and $I_{ik}^- = U_{ik} - d_{ik}$.
Step 4: determine the optimal X_{ijk}^*s and N_{ij}^*s using the mathematical model to be presented in the following section; and compute the resulting I_{ik}^+s and S_{ik}^+s as follows:
(1) if $I_{ik}^- > 0$ (that is, $S_{ik}^- = 0$), then $I_{ik}^+ = I_{ik}^- + \sum_{j \neq i} X_{jik} - \sum_{j \neq i} X_{ijk}$, $S_{ik}^+ = 0$; or
(2) if $S_{ik}^- > 0$ (that is, $I_{ik}^- = 0$), then $I_{ik}^+ = 0$, $S_{ik}^+ = S_{ik}^- + \sum_{j \neq i} X_{ijk} - \sum_{j \neq i} X_{jik}$; or
(3) if $I_{ik}^- = S_{ik}^- = 0$, then $I_{ik}^+ = S_{ik}^+ = 0$.
Note that even a DC which face shortage of an item *after* assigning U to the daily demand can still ship out the item as many as U.
Step 5: each DC i places a replenishment order to the supplier as many as $U_{ik} - I_{ik}^+$ if $I_{ik}^+ \geq 0$; or $U_{ik} + S_{ik}^+$ if $S_{ik}^+ \geq 0$, such that inventory level of next day after replenishment will become U_{ik}.
Step 6: transshipments are carried out. Then trucks depart to the customers for delivery.
Step 7: compute the daily total cost of all DCs. Go to step 1 for the next day operation.

Since all DCs employ order-up-to policy, the total replenishment order is equal to the total demand realized. As the replenishment order cost is assumed to be equal for all DCs, it does not affect the transshipment decisions and can be omitted from our model.

2.3 Cost Factors

We will first define the cost factors as follows.
(1) trucking cost
The trucks for transshipment may depart from any DC, but has to return to its origin, so that the same number of trucks be available for regional delivery in the following day morning. During the round trip between a pair of DCs, the transshipment truck(s) ship the items in both directions. The trucking cost between a pair of DCs will differ based on

the distance between the two DCs, so the total trucking cost is defined as in eq.(1).

$$C_t = \sum\sum c_{ij}N_{ij} \quad \text{for } 1\leq i<j\leq W \tag{1}$$

(2) handling cost
The extra material handling for transshipment includes picking, loading, and unloading. We assume that the handling cost for unit item to transship is equal for all items in all DCs, so the total extra handling cost is defined as in eq.(2).

$$C_h = m\sum\sum\sum X_{ijk} \quad \text{for } 1\leq i\neq j\leq W,\ k=1,2,\cdots,K \tag{2}$$

(3) shortage cost
Some DCs may still face shortage of some items even after receiving the transshipped items. We assume it incurs a shortage cost of certain portion of the item price, so the shortage cost is defined as in eq.(3).

$$C_s = \sum\sum p_k r_k S_{ik}^{+} \quad \text{for all } i=1,2,\cdots,W;\ k=1,2,\cdots,K \tag{3}$$

(4) inventory cost
Unlike most of other studies such as Herer et al. (2006) in which transshipment is performed immediately, our daily operation schedule shown in Figure 2 indicates that the inventory level remains as U from the time the replenishment arrives to the time delivery trucks depart. Since the length of time between the departure of delivery trucks and arrival of the next replenishment is negligible, we regard the inventory level as U, instead of I+ as presented in most previous studies. So the approximate daily total inventory cost is defined as in eq.(4)

$$C_i = \sum\sum hp_k U_{ik} \text{ for all } i=1,2,\cdots,W;\ k=1,2,\cdots,K \tag{4}$$

2.4 Mathematical Formulation

Using the above notations and cost factors, we formulate the optimal transshipment problem for cross filling among multiple DCs of a day as follows.

$$\text{minimize } Z = \sum\sum c_{ij}N_{ij} + m\sum\sum\sum X_{ijk} + \sum\sum p_k r_k S_{ik}^{+} + \sum\sum hp_k U_{ik} \tag{5}$$

subject to

$$\sum_{j\neq i} X_{ijk} \leq U_{ik} \quad \text{for all } 1\leq i\leq W;\ k=1,2,\cdots,K \tag{6}$$

$$S_{ik}^{+} \geq d_{ik} - U_{ik} + \sum_{j\neq i} X_{ijk} - \sum_{j\neq i} X_{jik} \quad \text{for all } 1\leq i\leq W;\ k=1,2,\cdots,K \tag{7}$$

$$\sum_k v_k X_{ijk} \leq v^o N_{ij} \quad \text{for all } 1\leq i\neq j\leq W \tag{8}$$

$$N_{ij} = N_{ji} \quad \text{for all } 1\leq i<j\leq W \tag{9}$$

$$(t_{ij}-T)N_{ij} \leq 0 \quad \text{for all } 1\leq i<j\leq W \tag{10}$$

$$\text{where } X_{ijk}, N_{ij}, S_{ik}^{+} \text{ are nonnegative integer variables for all } i=1,2,\cdots,W;\ k=1,2,\cdots,K. \tag{11}$$

The objective function (5) is the sum of extra transportation and handling cost plus inventory and resulting shortage cost. Eq.(6) represents that a DC can ship out each item at most as much as U, the order-up-to value. This opens the room for simultaneous ship-out and ship-in at a series of DCs, that is, in multiple adjacent locations so as to reduce shortage at the 'terminal' DC. We name this as the "simultaneous chain transshipment" since the same items located in a series of locations move to the adjacent locations simultaneously, just like all links in a chain move to the same direction simultaneously. To the best of our knowledge, the only paper that has addressed this 'simultaneous chain transshipment' is Lien *et al*. (2011), in which no further exploitation on this issue is made. So, we will characterize the simultaneous chain transshipment further in section 2.5.

Eq.(7) comes from the relation $S_{ik}^{+} = \max\{-(U_{ik} - d_{ik} - \sum_{j\neq i}X_{ijk} + \sum_{j\neq i}X_{jik}), 0\}$, which can be rewritten as eq.(7) and $S_{ik}^{+}\geq 0$, where the latter is given in eq.(11). Eq.(8) and (9) force that the total volume of items to transship between a pair of DCs in either direction must not exceed the total volume capacity of the trucks that move between the pair of DCs. Note that for most consumer products, weight constraint is not applied. Eq.(10) forces that transshipment is not allowed between a pair of DCs if they are prohibitively distant. As T increases as the result of more efficient works in the DCs such as order picking, more pairs of DCs located far away from each other will be allowed to tranship items, which may further reduce the total cost.

Since the above mathematical formulation is a linear one, it can be readily solved by various optimizers. The number of decision variables N_{ij} is $W(W\text{-}1)$; and the number of X_{ijk} is $W(W\text{-}1)K$. The number of constraints is WK from eq.(6) and (7), respectively; and $W(W\text{-}1)$ from eq.(8); $W(W\text{-}1)/2$ from eq.(9) and eq.(10), respectively.

2.5 Simultaneous Chain Transshipment

Note that it can be worthwhile to ship-out and ship-in the same item simultaneously at a DC. This is because transshipment is allowed only within a predetermined time window T, as given in eq.(10), due to the daily operation schedule of DCs. But, even for a pair of DCs i and j whose t_{ij} exceeds T, an effective transshipment from i to j is possible if DC i ships item k to DC u and DC u ships item k to DC j at the same time, instead of 'relaying' the items, as long as both DC i and DC u have enough available inventory of item k to ship out simultaneously.

Note that such a simultaneous chain transshipment can be economical for more than one intermediary DCs if the saving of shortage cost at the terminal DC is still greater than possible increase of transshipment cost and extra handling cost at more intermediary DCs. Recall that since we assume all transshipment trucks have to depart almost at the same time, instead of 'relaying', the initial and all intermediary DCs must have the available inventory at least as much as to transship to the next DC at the time trucks depart. For example, in Figure 3, without simultaneous chain transshipment, DC_j could have received at most one unit from the preceding DC_q, but using the simultaneous chain transshipment from DC_i via DC_p and DC_q, DC_j could receive three units and completely avoid the shortage. Note that DC_i can ship seven units to satisfy the shortage at DC_j and DC_m simultaneously, but the above optimization model will determine the optimal transshipment quantities as shown in Figure 3 such that all shortages are satisfied while minimizing the extra cost. Since order-up-to replenishment is implemented every day, it is better to make S_{ik}^+ to be zero at more DCs.

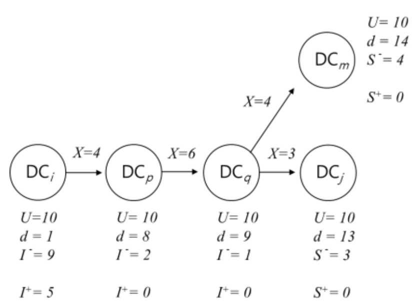

Figure 3. An example of 'simultaneous chain transshipment'

3. Illustrative example

In this section, a numerical example is given to illustrate the mathematical model we presented in the previous section. A small case is considered with three DCs (W=3) and two items (K=2). Table 1 shows the initial inventory and daily demand at each DC for each item. Other parameter values are used as follows: p_1=200, p_2=150, r_1=0.2, r_2=0.3, v^o=6, v_1=0.4, v_2=0.6, m=1, t_{12}=5, t_{13}=7, t_{23}=4, T=6 (that is, direct transshipment between DC 1 and 3 is not allowed due to time limit), c_{12}=100, c_{13}=130, and c_{23}=70. Note that the model has six N_{ij}s, twelve X_{ijk}s, and 24 constraints.

item (k)	DC (i)	i=1	i=2	i=3
1	order-up-to level (U_{i1})	20	40	70
	daily demand (d_{i1})	40	80	50
2	order-up-to level (U_{i2})	40	90	50
	daily demand (d_{i2})	30	50	70

Table 1. Beginning inventory and daily demand of each item in each DC.

Using Excel to solve the proposed linear programming problem, we obtain X_{232}=20, X_{321}=20; and $N_{23} = N_{32}$ =2, that is, to transship 20 units of item 2 from DC2 to DC3; and 20 units of item 1 from DC3 to DC2 using two trucks. The minimized total cost of 1,816.04 is obtained, whereas the total cost without any transshipment is 3,300 so the total cost is reduced by 45 percent using the optimal transshipment method that we proposed. The computation time of Excel to solve this example is less than 2 seconds.

4. Conclusion

In this paper, we address the transshipment problem for a common configuration comprising single supplier and multiple non-identical DCs, each of which supply goods to the customers (retail stores) located in its geographical region; and each DC orders to the supplier every day using order-up-to policy. A linear programming model is proposed which determines the optimal quantity of items to transship, number of trucks to transship between pairs of DCs so as to maximize the profit of cross filling for a day.

Three features that make this paper a unique one are as follows. Firstly, unlike most existing researches on this problem where transshipment cost is assumed proportional to the quantity transshipped, in this paper transshipment cost is considered proportional to the number of trucks used for the transshipment, regardless of the quantity loaded in it, which we believe is more realistic, despite it makes the model more complicated. Secondly, unlike some studies where possible links are limited to a certain number, in this paper, possible links between DCs are limited based on the distance (or, correspondingly, travel time) between them. That is, the optimal transshipment is constrained by the length of time window (T) during which transshipment is carried out. This aspect is practically important, since more distant DCs can be considered for transshipment as T increases by, for example, more efficient order picking in DCs. Thirdly, volume constraint of items to transship is considered, which has not been addressed in the literature. Depending on the size of the items, it is not unusual for the total volume to transship to exceed a volume capacity of a small delivery truck. Furthermore, authors are exploiting the vehicle routing problem where both pickup and delivery over multiple locations is considered for transshipment, in which case volume constraint becomes a critical one.

Another fruit from the model is that ‘simultaneous chain transshipment’ is formalized, which has not received appropriate attention in the literature. This simultaneous chain transshipment of items can be of practical tool for the cases where surplus is not available in its vicinity, but surplus from far distant site can reach the needing site by simultaneously moving the same quantity in a supply chain.

One may criticize the proposed model in that transshipping multiple truckloads between DCs every day is unrealistic in the real world logistics practice. However, under order-up-to replenishment policy and highly variable demand, transshipping a relatively large amount can reduce more shortage with substantially lower inventory level. Also, in the near future some new transportation tools such as drone may become a common carrier, in which case multiple carriers to transship among multiple DCs or locations can be a practical tool for cross filling.

For further research, a solution procedure needs to be developed to find the optimal order-up-to value for our transshipment model. One may extend the problem to the case when transshipment cost is negligible, for example, where there already exist trucks visiting routinely a sequence of locations. One possible research avenue is to implement replenishment policy other than order-up-to policy, such as reorder point policy or periodic order policy, which we believe is more practical since most upstream replenishment from factory to DCs are unlikely to occur every day. For these cases, one may develop a simulation model in which the proposed optimal daily transshipment repeats over a long time so as to evaluate the average performance of the proposed model. Using the simulation model, the value range of the parameters that makes this transshipment economical can be identified; and the impact of the replenishment policy on the average performance of the transshipment can be better understood. Also the simultaneous chain transshipment can be further exploited to identify its characteristics.

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