Optimal Policy for a Vendor-buyer Inventory System with Price-dependent Demand, Production Cost Discount and Reliability Consideration: A Geometric Programming Approach

Bibhas C. Giri and Biswarup Samanta
Department of Mathematics
Jadavpur University, Kolkata 700032, India
bcgiri.jumath@gmail.com, biswarupsamanta@gmail.com

Abstract
The article considers a single-vendor single-buyer joint pricing and lot size model in which the vendor’s production process is imperfect. The market demand is a function of the buyer’s selling price. The vendor’s unit production cost is assumed to be a function of lot size of each shipment to the buyer. This paper simultaneously determines shipment size, unit selling price of the buyer, vendor’s set-up cost, reliability of the production process and total number of shipments per lot from the vendor to the buyer. The expected annual total profit function is derived and signomial geometric programming technique is used to find the optimal solutions for different preferences of objective functions. The solution procedure is illustrated by a numerical example.

Keywords
Supply chain, optimal pricing, lot sizing, reliability, geometric programming

1. Introduction
With the growing focus on supply chain management, company managers today increasingly realize that the greater cooperation and better coordination among all the members of a supply chain is more beneficial than those policies obtained separately from each member's perspective. Owing to globalization and competitiveness in the market, all manufacturing sectors want to produce perfect quality items. This is almost impossible due to several factors such as quality of raw materials, efficiency of workers, labor problems, machine breakdowns, etc. Even in stable condition, the manufacturing system may produce defective items together with non-defective ones. These defective items may be rejected, repaired/reworked or vended at lower price. In this paper, we develop a vendor-buyer integrated supply chain model consisting of a vendor and a buyer where the defective items in each lot are rejected at the end of buyer's 100% screening process. The demand rate at the buyer side is dependent on the selling price. We consider a simple practical situation where the delivery quantity to the buyer at each replenishment is identical. Unit production cost is not fixed but discounted with the increase in lot size, which is more realistic in the market. The main objectives of this study are to find the answers of the following questions:
What would be the optimal selling price at the buyer side? What would be the discount price of each defective item from the vendor's side? How much quantity would receive by the buyer from the vendor in each replenishment? How many shipments are made by the vendor to fulfill the buyer's demand?

We organize the paper as follows: Section 2 presents notations and assumptions of the proposed model. Section 3 deals with literature review. Section 4 formulates the mathematical model. Solution methodology for the developed model is discussed in Section 5. Numerical examples are provided in Section 6. The computational results are compared with those of Huang's (2004) model in this section. Section 7 summarizes the paper and suggests future scopes of research.
2. Literature Review

Several researchers have considered pricing strategy in integrated supply chain model where the lot sizing is performed as part of the resulted models (Giri and Roy, 2015; Maiti and Giri, 2017). Joint pricing and lot size model (JPLM) is an approach where product's price and lot size are determined simultaneously. Lee (1993) investigated item's optimum price and optimum lot size by considering price-dependent demand. He applied geometric programming technique to solve the model. Lee and Kim (1993) developed a model for a profit maximizing firm over a planning horizon by considering similar demand function. They simultaneously determined price, marketing expenses, demand or production volume and lot size for a single item with known economies of scale in their model. They also compared the results with some heuristic methods in order to demonstrate the effectiveness of their approach. Sadjadi et al. (2005) considered a linear relationship between demand and production. They formulated a joint production, marketing and inventory model to determine the production lot size, marketing expenditure and product's selling price simultaneously. Sadjadi et al. (2009) addressed a production-marketing problem with unreliable production process consideration.

Over the last few decades, many researchers have proposed production-inventory models with single or multiple items for uniform or variable production rate. In most of these studies, they assumed unit production cost as constant. However, the unit production cost reduces if the number of units produced is large. Abad (1988) considered the production cost per unit as a function of the order quantity. Lee (1994) noticed that increase in the order quantity implies decrease in production cost per unit, and the manufacturer has the advantage to gain more profit. Khouja (1995) developed an economic production lot size model that considers the production rate as a decision variable. Thereafter, Bhandari and Sharma (1999) extended Khouja’s (1995) model by considering a generalized unit production cost function dependent on production rate, cost of raw materials and labor charges. Jung and Klein (2001) analyzed two economic order quantity based inventory models under decreasing cost function via geometric programming technique. Later, Jung and Klein (2005) extended this model into three economic order quantity based inventory models which were also analyzed by geometric programming technique. Jung and Klein (2006) developed three economic order quantity based inventory models by optimizing order quantity and price for each of these models considering production (lot sizing) as well as marketing (pricing) decisions. Esmaeil et al. (2007) developed a JPLM by considering a discount factor. They realized that the demand affects production cost indirectly. Sadjadi et al. (2009) considered lot size dependent unit production cost and developed a production-inventory model under reliability consideration.

Process reliability is an important factor in imperfect production environment and it has significant impact on costs and profits. There are only a few works where the reliability of the production process has been considered. Cheng (1989) considered process reliability in a single-period inventory system which was formulated as an unconstrained Geometric Programming problem, which was later used by Leung (2007) and Maiti and Maiti (2005) for their respective models. Cheng’s (1989) model was extended by Bag et al. (2009) to consider product demand as a fuzzy random variable.

Among nonlinear optimization methods, Geometric Programming (GP) is an efficient and effective method when decision variables interact in a non-linear manner. Islam (2008) determined optimal solution for a multi-objective marketing planning inventory model by using GP technique. Jung and Klein (2001) used GP method to find the optimal order quantity in a profit maximization model. Similarly, Abad (1988), Lee (1993), Lee et al. (1996), Kim and Lee (1998) and Sadjadi et al. (2009) applied the primal and dual of GP to determine the optimal price and lot size. This article incorporates the view of the integrated vendor-buyer approach into the supply chain model with reliability consideration, price dependent demand and lot size dependent unit production cost herein. The article extends the model of Huang (2004). The demand function, unit production cost function, interest and depreciation cost function are considered same as adopted in Sadjadi et al. (2009). This model also considers that the delivery quantity to the buyer at each shipment is identical as in Huang (2004).

3. Assumption and Notations

The notations used throughout the paper are as follows.
Table 1. Notations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$</td>
<td>size of each shipment from the vendor to the buyer (decision variable)</td>
</tr>
<tr>
<td>$p$</td>
<td>selling price per item at the buyer (decision variable)</td>
</tr>
<tr>
<td>$S_v$</td>
<td>set up cost per setup for the vendor (decision variable)</td>
</tr>
<tr>
<td>$n$</td>
<td>total number of shipments per lot from the vendor to the buyer (decision variable)</td>
</tr>
<tr>
<td>$Q_p$</td>
<td>lot size</td>
</tr>
<tr>
<td>$D$</td>
<td>demand rate</td>
</tr>
<tr>
<td>$k$</td>
<td>scaling constant for demand</td>
</tr>
<tr>
<td>$P$</td>
<td>production rate ($P &gt; D$)</td>
</tr>
<tr>
<td>$S_b$</td>
<td>ordering cost per order for the buyer</td>
</tr>
<tr>
<td>$h_b$</td>
<td>unit stock holding cost per item per year for the buyer</td>
</tr>
<tr>
<td>$h_v$</td>
<td>unit stock holding cost per item per year for the vendor</td>
</tr>
<tr>
<td>$T$</td>
<td>time interval between successive deliveries</td>
</tr>
<tr>
<td>$T_c$</td>
<td>cycle time</td>
</tr>
<tr>
<td>$F$</td>
<td>transportation cost per shipment</td>
</tr>
<tr>
<td>$C$</td>
<td>unit production cost</td>
</tr>
<tr>
<td>$r$</td>
<td>scaling constant for unit production cost</td>
</tr>
<tr>
<td>$\beta$</td>
<td>lot size elasticity to unit production cost</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>price elasticity to demand</td>
</tr>
<tr>
<td>$R$</td>
<td>reliability of the production process</td>
</tr>
<tr>
<td>$V$</td>
<td>maximum reliability of the production process</td>
</tr>
<tr>
<td>$T(R, S_v)$</td>
<td>total cost of interest and depreciation per production cycle</td>
</tr>
<tr>
<td>$l$</td>
<td>scaling cost for interest and depreciation cost</td>
</tr>
<tr>
<td>$x$</td>
<td>reliability elasticity to interest and depreciation cost</td>
</tr>
<tr>
<td>$y$</td>
<td>set up cost (per set up) elasticity to $T(R, S_v)$</td>
</tr>
<tr>
<td>$z$</td>
<td>screening rate</td>
</tr>
<tr>
<td>$d$</td>
<td>unit screening cost</td>
</tr>
<tr>
<td>$W$</td>
<td>wholesale price per item for the vendor</td>
</tr>
<tr>
<td>$w$</td>
<td>discount price per defective item at the vendor</td>
</tr>
</tbody>
</table>

The following assumptions are made to develop the proposed integrated vendor-buyer inventory model.

(i) The supply chain consists of a single-vendor and a single-buyer to deal with a single product.
(ii) The demand is affected by the selling price. We assume that the demand rate $D = kp^\alpha$; $\alpha > 1$. (Sadjadi et al., 2009). Here $\alpha > 1$ confirms that the demand increases when the selling price decreases.
(iii) The production rate $P$ is uniform and finite such that $P > D$.
(iv) Successive deliveries are scheduled so that the next one arrives at the buyer when his/her stock from previous shipment has just been finished.
(v) The number of perfect units is at least equal to the demand during the screening time.
(vi) Shortages are not allowed.
(vii) The time horizon is infinite.
(viii) Unit production cost can be discounted with $\beta$ according to $C = rQ^\beta$; $0 < \beta < 1$.
(ix) Total cost of interest and depreciation cost per production cycle is inversely related to set-up cost and directly related to production process reliability according to the following relation:

$$T(R, S_v) = lR^x S_v^{-y}; l, x, y \geq 0.$$  

(x) Product quality may be imperfect. In other-words, only $R\%$ of all produced items meet the demand while $(1 - R)\%$ of items are defective. We assume that $R \leq v$ due to production constraint. It is apparent that the maximum reliability of the production process cannot exceed 1 ($v \leq 1$).

Equation (1) implies that an increase in the reliability of the production process leads to growth in total interest and depreciation cost. This relationship can be realized easily considering the fact that high reliability can only be achieved with additional cost in practice i.e. significant investment is usually required to improve the reliability of the production. As a result, interest and depreciation cost of the high reliability production is much higher than low...
level one. On the other hand, when the set-up cost is reduced, the total cost of interest and depreciation is raised. This relationship is obvious and is based on the fact that the vendor has to invest more in order that set-up cost per set up is reduced. For instance, it may cost much more to decrease the unit set-up cost, since we need to acquire expensive facilities.

4. Mathematical Model

Since only $R\%$ of total products is acceptable, therefore, in a production cycle, $D/R$ products must be produced to meet the whole demand. Since the vendor produced $nQ$ number of products with production rate $P$, the total time of production is $nQ/P$. In a production cycle, the total time $= nT$, the total demand $= nDT$ and the total acceptable products $= nQR$. Since the total acceptable products fulfills the buyer’s total demand, so $nDT = nQR$, which implies that $T = QR/D$. For the vendor, sales revenue per year $= WD$, production cost per year $= nQCD/(nT)$, set-up cost per year $= S_v/(nT) = S_vD/(nQR)$, holding cost per year $= (QR/2 + QD(1-R)/(zR))h_b$, interest and depreciation cost per year $= nQRSD/(nT)$, discount cost per defective item per year $= wD/(nT)$, therefore, the vendor’s annual total profit is

$$TP_v(n, Q, p, S_v, R) = \text{sales revenue} - \text{production cost} - \text{set-up cost} - \text{holding cost} - \text{discount cost}$$

For the buyer, sales revenue per year $= nDTp/(nT) = pD$, purchase cost per year $= nDTW/(nT) = WD$, holding cost per year $= (QR/2 + QD(1-R)/(zR))h_b$, transportation cost per year $= nF/(nT) = FD/(QR)$, screening cost per year $= dD/(nT) = dD/(nQR)$, and ordering cost per year $= S_b/(nT) = S_bD/(nQR)$. Therefore, the buyer’s annual total cost is

$$TP_b(n, Q, p, R) = \text{sales revenue} - \text{purchase cost} - \text{holding cost} - \text{transportation cost} - \text{screening cost} - \text{ordering cost}$$

Hence, the annual total profit of the supply chain is given by

$$\Pi(n, Q, p, S_v, R) = TP_v(n, Q, p, S_v, R) + TP_b(n, Q, p, R)$$

subject to, $R/v \leq 1$ where, $Q, p, S_v, R, w > 0$ and $n \in N$. The constraint $R/v \leq 1$ i.e., $R \leq v$ indicates that the reliability of system is limited and cannot exceed $v$.

5. Solution Methodology

Since $n \in N$, we consider three cases: (I) $n = 1$ (II) $n = 2$ and (III) $n \geq 3$.

5.1 Case I: $n = 1$

In this case, the model (4) is transformed into
we consider the coefficient matrix (non-singular) of six dependent variables. The problem (5) can be rewritten as

$$\text{Max } \Pi_1(Q, p, S, R, w) = k p^{\alpha-1} + \frac{k h_b}{z} Q p^{-\alpha} - \left(\frac{h_v}{2 p} + \frac{h_b}{z}\right) Q p^{-\alpha} R^{-1} - k Q^{-1} p^{-\alpha} R^{-1} - k (S_b + F) Q^{-1} p^{-\alpha} R^{-1} - b Q^{-1} p^{-\alpha} S^{-1} R^{-1} - \frac{h_b}{2} QR + k w p^{-\alpha} - \frac{kw p^{-\alpha}}{R}$$

subject to, $R \vee \leq 1$ where, $Q, p, S, R, w > 0$.

The problem (5) can be rewritten as

$$\text{Max } \Pi_1(Q, p, S, R, w) = -\text{Max } \Pi_1(Q, p, S, R, w)$$

$$= -k p^{\alpha-1} - \frac{k h_b}{z} Q p^{-\alpha} + \left(\frac{h_v}{2 p} + \frac{h_b}{z}\right) Q p^{-\alpha} R^{-1} + k Q^{-1} p^{-\alpha} R^{-1} + p h p^{-\alpha} R^{-1}$$

subject to, $R \vee \leq 1$, where, $Q, p, S, R, w > 0$.

The dual of (6) is given by

$$\text{Max } F(W) = \left(\frac{k}{w_5}\right)^w_1 \left(\frac{k h_b}{2w_2}\right)^w_2 \left(\frac{k(h_b/(2P) + h_b/z)}{w_3}\right)^w_3 \left(\frac{kr}{w_4}\right)^w_4 \left(\frac{dk}{w_5}\right)^w_5 \left(\frac{k(S_b + F)}{w_7}\right)^w_6 \left(\frac{kr}{w_8}\right)^w_8 \left(\frac{h_b}{2w_{10}}\right)^w_9 \left(\frac{k}{w_{10}}\right)^w_{10}$$

subject to the normality and orthogonality conditions

$$-w_1 - w_2 + w_3 + w_4 + w_5 + w_6 + w_7 + w_8 + w_9 - w_{10} + w_{11} = 1$$

$$-w_2 - w_3 - \beta w_4 - w_5 - w_6 - w_7 - w_8 + w_9 = 0$$

$$(\alpha - 1) w_1 + \alpha w_2 - \alpha w_3 - \alpha w_4 - \alpha w_5 - \alpha w_6 - \alpha w_7 - \alpha w_8 + \alpha w_{10} - \alpha w_{11} = 0$$

$$w_6 - w_8 = 0$$

$$-w_3 - w_4 - w_5 - w_6 - w_7 + (x - 1) w_8 + w_9 - w_{11} + w_{12} = 0$$

$$-w_{10} + w_{11} = 0$$

The non-negativity conditions are

$$-w_1 - w_2 + w_3 + w_4 + w_5 + w_6 + w_7 + w_8 + w_9 - w_{10} + w_{11} \geq 0$$

$$w_j \geq 0, \forall j = 1, 2, 3, \ldots, 12$$

Solving the above five equations (8) in terms of seven independent variables (DD=7) we get the optimal values of the primal variables. The system of dual equations in (8) are six linear equations in 13 variables which can be written as $A^T W = B$, where

$$A^T = \begin{bmatrix} -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ \alpha - 1 & \alpha - \alpha & -\alpha & -\alpha & -\alpha & -\alpha & 0 & \alpha & -\alpha & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -y & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & -1 & -1 & x - 1 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 \end{bmatrix}$$

$$W = [w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8, w_9, w_{10}, w_{11}, w_{12}]^T$$

We can write six equations in six unknowns in terms of 12-6 = 6 independent variables. Without loss of generality, we consider $w_1, w_2, w_5, w_6, w_7, w_{10}$ as dependent variables and remaining variables $w_3, w_4, w_8, w_9, w_{11}, w_{12}$ as independent variables. We consider $W_1 = [w_1, w_2, w_5, w_6, w_7, w_{10}]^T$ and $W_2 = [w_3, w_4, w_8, w_9, w_{11}, w_{12}]^T$ and $A_1$ = Coefficient matrix (non-singular) of six dependent variables.

$$A_1 = \begin{bmatrix} -1 & -1 & 1 & 1 & 1 & -1 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ \alpha - 1 & \alpha & -\alpha & -\alpha & -\alpha & -\alpha \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$A_2$ = Coefficient matrix (non-singular) of six independent variables.
Then the relation $A^TW = B$ is transformed into $A_1W_1 + A_2W_2 = B$ i.e., $W_1 = A_1^{-1}B - A_1^{-1}A_2W_2$. Considering suitable values of $w_3, w_4, w_5, w_9, w_{11}, w_{12}$ satisfying normality, orthogonality and non-negativity conditions as stated in (8) we can find the optimal values of $w_1, w_2, w_5, w_6, w_7, w_{10}$. Let these optimal values be $w_1^*, w_2^*, w_5^*, w_6^*, w_7^*, w_{10}^*$. Then the relation (9) is transformed into

$$k_{p'}R^{-\alpha} = \frac{dp_{p'}R^{-\alpha}R^{-1}}{w_2} = \frac{kQ^{-\alpha}p^{-\alpha}S^{-1}}{w_5} = \frac{k(S_6 + F)p^{-\alpha}R^{-1}}{w_7}$$

subject to, $W_{1}/\gamma \leq 1$, $w_{1}, w_{2}, w_{5}, w_{6}, W_{10}$.

Thus, the optimal solutions of the problem are summarized as follows:

$$Q^* = \frac{k(S_6 + F)w_6^*}{dkw_5}, \quad p^* = \frac{h_w^*w_7^*}{tw_6}, \quad R^* = \frac{dw_2^*}{h_w^*w_6}, \quad w^* = \frac{h_w^*w_6w_{10}Q^*}{zw_2^*w_5}$$

### 5.2 Case II: $n = 2$

In this case, the model (4) is transformed into

$$\begin{align*}
\text{Max} \quad & \Pi_2(Q, p, S, R, w) = k_{p'}^{-\alpha} + \frac{k_{b}Q^\alpha}{z} + \frac{h_w}{z}Q^{\alpha}R^{-1} + h_{Q}/(2 - krQ^\beta R^{-1} - dp_{p'}^\alpha R^{-1} - (k/2)Q^{-\alpha}p^{-\alpha}R^{-1} - (k/2)Q^{-\alpha}S^{-1}R^{-1} - h_{Q}/2 + QR - kwp^\alpha - kwp^\alpha - kwp^\alpha) \quad (9)
\end{align*}$$

subject to, $R/\gamma \leq 1$ where, $Q, p, S, R, w > 0$.

The above problem can be rewritten as

$$\begin{align*}
\text{Max} \quad & \Pi_2(Q, p, S, R, w) = -\text{Max} \quad & \Pi_2(Q, p, S, R, w)
\end{align*}$$

$$+ (k/2)Q^{-\alpha}p^{-\alpha}S^{-1}R^{-1} + k(S_6 + F)Q^{-\alpha}p^{-\alpha}R^{-1} + (k/2)Q^{-\alpha}S^{-1}R^{-1} + h_{Q}/2 + QR - kwp^\alpha + kwp^\alpha) \quad (10)
$$

subject to, $R/\gamma \leq 1$ where, $Q, p, S, R, w > 0$.

The above problem is a signomial Geometric Programming problem with DD=7. The dual of (10) is given by

$$\begin{align*}
\text{Max} \quad & F(\delta) = \frac{k}{\delta_4} - \frac{h_{b}Q^\alpha}{z}\delta_5 - \frac{h_{w}}{z}\delta_6 - \frac{h_{Q}}{2}\delta_7
\end{align*}$$

subject to the normality and orthogonality conditions

$$-\delta_1 - \delta_2 + \delta_3 - \delta_4 + \delta_5 + \delta_6 + \delta_7 + \delta_8 + \delta_9 + \delta_{10} - \delta_{11} + \delta_{12} = 1
$$

$$-\delta_2 + \delta_3 - \delta_4 - \beta \delta_5 + \delta_7 - \delta_8 - \delta_9 + \delta_{10} = 0
$$

$$-\delta_3 - \delta_4 - \delta_5 + (x - 1)\delta_9 + \delta_{10} - \delta_{11} + \delta_{12} = 0
$$

$$\delta_7 - \gamma \delta_9 = 0
$$

$$\delta_9 - \delta_6 - \delta_7 - \delta_8 + (x - 1)\delta_9 + \delta_{10} - \delta_{11} + \delta_{12} = 0
$$

$$\delta_{11} + \delta_{12} = 0
$$

The non-negativity conditions are

$$\delta_{j} \geq 0, \quad \forall j = 1,2,3,13
$$

We consider dependent variables $\delta_2, \delta_3, \delta_4, \delta_7, \delta_8, \delta_{11}$ in terms of seven independent variables $\delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6, \delta_7$ satisfying normality, orthogonality and non-negativity conditions stated in (12). Let $\delta_2, \delta_3, \delta_4, \delta_7, \delta_8, \delta_{11}$ be respective optimal values of dependent variables for suitably chosen $\delta_1,\delta_2,\delta_3,\delta_4,\delta_5,\delta_6,\delta_{10},\delta_{12},\delta_{13}$. One of the optimal solutions of the problem is found as
for (13) Programming problem with degree of difficulties 13-5-1=7. Its dual is given by subject to the normality and orthogonality conditions one of the optimal solutions is found as

In this case, without loss of generality, we consider six dependent variables subject to,

where the primal problem is

subject to, \( R/v \leq 1 \) where, \( Q, p, S, R, w > 0 \). In this case, without loss of generality, we consider six dependent variables \( \gamma_2, \gamma_3, \gamma_4, \gamma_7, \gamma_8, \gamma_{11} \) in terms of seven independent variables \( \gamma_1, \gamma_5, \gamma_6, \gamma_9, \gamma_{10}, \gamma_{12}, \gamma_{13} \) satisfying normality, orthogonality and non-negativity conditions as stated in (13). Let \( \gamma_2^*, \gamma_3^*, \gamma_4^*, \gamma_7^*, \gamma_8^*, \gamma_{11}^* \) be the respective optimal values of dependent variables for suitable choice of \( \gamma_1, \gamma_5, \gamma_6, \gamma_9, \gamma_{10}, \gamma_{12}, \gamma_{13} \).

Then from the relations

one of the optimal solutions is found as

\[ Q^* = \left( \frac{z(S_h/2 + F)\delta_4}{h_0/\delta_2} \right)^{1/\alpha}, \quad p = \left( \frac{2kh_0^{1/\alpha}b}{z} \right), \quad S_v^* = (S_h/n + F)^{1/\alpha}, \quad R^* = k(S_h/n + F)^{1/\alpha}, \quad w = \frac{h_0^{1/\alpha}Q^*}{z} \]
In the subcase(ii): \( \frac{(n-2)h_v}{2p} < \frac{h_v}{z} \) for \( n = n^* \geq 3 \), proceeding similarly as before, we can find the optimal values of \( Q, p, S_v, R, w \) as follows:

\[
Q^* = \frac{S_v^*\mu_3^*}{n\mu_7^*(-n^*-2)h_v/(2P)+h_v/z}, \quad p^* = \left( \frac{2kh_v\mu_4^*}{(n^*-1)z\mu_1^*\mu_2^*} \right)^{1/\alpha}, \quad S_v^* = \frac{(S_v+n^*P)\mu_2^*}{\mu_8}, \quad R^* = \frac{z\mu_2^*(-n^*-2)h_v/(2P)+h_v/z}{\mu_1^*\mu_3^*}, \quad w^* = \frac{h_v\mu_1^*Q^*}{z\mu_2^*}
\]

6. Numerical Example

In this section, we illustrate the developed models through a numerical example. We assume the parameter-values as \( \alpha = 2.5, \beta = 0.02, \ r = 5, \ k = 10^6, \ l = 80000, \ x = 1.2, \ y = 4.8, \ P = 160000, \ S_b = 30, \ h_v = 2, \ h_b = 5, \ F = 25, \ z = 175200, \ d = 0.5, \ W = 6, \ v = 0.99 \) in appropriate units. For this chosen data, we see that integrated profit is maximum for \( n^* = 7, Q^* = 155.36 \) units, \( p^* = 8.69 \), \( w^* = 3.09 \) (each defective item is sold at \( (6-3.09) = 2.91 \)), \( S_v = 9.1794 \), and \( R^* = 0.99 \). We observe that annual demand of the buyer is 4492.37 units. So, the vendor has to produce 4537.75 units in a year. Here, unit production cost per item reduces to 4.52 and hence the total production cost per year is 20510.85. In a year, the vendor earns 26954.22 from selling products whereas his/her total expenses is 21607.41 and hence the vendor’s total annual profit reduces to 5346.81. On the other hand, the buyer earns 39037.80 from selling products whereas his/her total expense is 30463.19 and hence the buyer’s total annual profit reduces to 8574.61. As a result, annual profit of the supply chain becomes 13921.42.

6.1 The case of \( \beta = 0 \) (unit production cost does not depend on lot size)

Here we assume that unit production cost is fixed and it is 5. We observe that annual demand of the buyer decreases in this case and it is 3451.85 units. So the vendor has to produce 3486.72 units in a year and hence total production cost per year is 1743.58. Since demand is less, the buyer wants to receive less items from the vendor in each replenishment. The vendor’s set-up cost remains unchanged but the defective items are sold with more discount. The buyer compensates this loss by selling the good items with higher price. In this case, sales revenues of the vendor and buyer are 20711.10 (23% decrease) and 33329.68 (14.62% decrease), respectively, and the profits of the vendor, the buyer and the whole supply chain are 2306.91 (56.85% decrease), 9634.52 (12.36% increase) and 11941.44 (14.22% increase), respectively. It is clear that the model with lot size dependent unit production cost is more profitable than fixed unit production cost.

6.2 The case of \( R = 1 \) (all items produced by the vendor are good)

In real market, increase in reliability of a product increases the demand of the product. We observe that annual demand of the buyer is 4719.55 units, which is larger than that of our proposed model. Here the vendor produces no defective item and hence, all the products meet to customer demand. So the vendor need not produce extra units. Therefore, the vendor has to produce 4719.55 units in a year. Here unit production cost reduces to 4.43 and hence the total production cost per year becomes 20905.88. Since customers’ demand is high, so the buyer wants to receive more items in each replenishment from the vendor. The vendor or the buyer has to pay no extra cost to the customer as there is no defective items. As the vendor has to produce more good items, he/she has to set-up super-structural machine and use new/ultramodern technique for which vendor’s set-up cost increases. Here the sales revenues of the vendor and the buyer are 28317.33(5% increase) and 40210.60 (3% increase), respectively, and the profits of the vendor, the buyer and the integrated supply chain are 7185.86 (34.4% increase), 7858.41(8.35% increase) and 15044.27 (8.07% increase), respectively. It is clear from the above results that if a product is highly reliable then profit of the supply chain will be more.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Our proposed model</th>
<th>Different cases</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \beta = 0 )</td>
<td>( R = 1 )</td>
</tr>
<tr>
<td>( n )</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>( p )</td>
<td>8.6898</td>
<td>9.6556(11.11%)</td>
</tr>
<tr>
<td>( Q )</td>
<td>155.3596</td>
<td>99.3046(-36.08%)</td>
</tr>
<tr>
<td>( S_v )</td>
<td>9.1794</td>
<td>9.1790(0.0%)</td>
</tr>
<tr>
<td>( w )</td>
<td>3.0908</td>
<td>4.2484(37.45%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8.5155(-2.01%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>426.7090(174.66%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>19.2026(+109.19%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0000(-100.00%)</td>
</tr>
</tbody>
</table>
From Table 1, we can conclude that customers’ demand and lot size in each replenishment are proportional whereas lot size and selling price are complementary; if customers’ demand is high then the buyer’s selling price is low and vice-versa.

6.3 Comparison of the optimal results with those of Huang’s (2004) model

To compare the optimal results with those of Huang’s (2004) model, we take \(\alpha = 0, k = 50000, l = 0, x = 0, y = 0\).

Huang considered the expected number of defective items as 0.02 and hence we consider \(R = 0.98\). We further consider \(S_b = 100, S_v = 300, w = 30\). All the remaining parameters \(P, h_v, h_b, F, z\) (it is \(x\) in Huang’s model), and \(d\) are kept unchanged to match with Huang’s model. Comparing these two models, we have the following results:

For given \(n\), the average total cost of the integrated system is a function of \(Q\) only and this function is a convex function in \(Q > 0\). We determine the unique value of \(Q\) as

\[
Q^* = \sqrt[2]{\frac{2kP(nF + S_b + S_v)}{np^\alpha PRz[(n-1)b_v + h_b R] + nk(2h_b P(R-1)-(n-2)z h_v)}}
\]

and hence the average total cost of the integrated system becomes

\[
p^{-\alpha}R \left[ \frac{2k(nF + S_b + S_v)(np^\alpha PRz[(n-1)b_v + h_b R] + nk(2h_b P(R-1)-(n-2)z h_v))}{z^p} - nk((R-1)w - d) \right]
\]

With the chosen data set, the optimal results are obtained as \(n^*=7\) and \(Q^*=780.21\) and the corresponding average total cost of the supply chain is 66865.60. Huang (2004) obtained this cost as 67082.60. Hence our model outperforms Huang’s (2004) model in terms of average total cost of the whole supply chain.

Now, we consider the model from buyer’s perspective. The buyer’s annual cost is given by

\[
\frac{dp^{-\alpha}}{R} + \frac{Fc^{-\alpha}}{QR} + \frac{kp^{-\alpha}S_b}{nQR} + \frac{Qb_{bh}}{2} + \frac{kp^{-\alpha}Q(1-R)}{Rz}
\]

which can be shown to be a convex function in \(Q(>0)\). Therefore, there exists a unique value

\[
Q^{**} = \sqrt[2]{\frac{2k(nF + S_b)z}{nh_b(2k(R-1) + p^\alpha R^2 z)}}
\]

With the chosen data set, the optimal value of \(Q\) is obtained as \(Q^{**}=1603.90\). Substituting \(Q^{**}=1603.90\) and \(n = 1\) into the buyer’s annual cost and the vendor’s annual cost yield the values 33462.7 and 40666.8, respectively (Huang obtained these values as 33467.17 and 40881.64, respectively). Hence, there is a cost reduction of (33462.7 + 40666.8 - 66865.6) =19726.9 (in Huang’s model, this reduction is 7265.21) in the integrated approach, which is as impressive as Huang’s model.

7. Conclusions and Future Research Directions

In this paper, we consider a supply chain model with a single-vendor and a single-buyer where the market demand depends on selling price of the product. All items produced by the vendor may not be perfect. The buyer screens all the products collected by him/her from the vendor. We optimize the profit of the supply chain with respect to the number of shipments, shipment size, buyer’s selling price, the vendor’s setup cost and the discount cost of the defective products. We apply signomial Geometrical Programming technique to optimize the profit. We analyze the impacts of sales revenue, different costs and profits of the vendor, the buyer and the whole supply chain on the price elasticity to demand, lot size elasticity to unit production cost, unit screening cost, scaling constant for demand, scaling constant for unit production cost and reliability of the product.

From the numerical study, we observe that lot size dependent unit production cost is more sensitive than constant unit production cost. If lot size increases then unit production cost decreases, and hence profit of the vendor increases. In real situations, it is impossible to produce all perfect items. The profits of the vendor, the buyer and the whole supply chain increase if the vendor produce more reliable items. Also, we investigate that it is necessary for the vendor to produce items not more than certain percentage (in our numerical example, 14%) of defective items.
The scaling constant for the demand also acts an important role to increase the profit of the buyer, the vendor and the whole supply chain. The price elasticity to demand must maintain the interval which lies between 1 and 3. We construct our model similar to any other model, based on a set of assumptions. We have assumed market demand as deterministic, which has limited applications in the real world. So, one can consider stochastic demand instead of deterministic demand to extend this model for further research. We have assumed two-layer supply chain model with a single vendor and a single buyer. Further research can extend the model by considering multi-layer, multi-buyer and/or multi-vendor supply chain. The vendor may impose ‘terms and conditions’ when selling the defective items with discount price.

References


Biographies

**Bibhas C. Giri** is a Professor of the Department of Mathematics, Jadavpur University, Kolkata, India. He did his M.S. in Mathematics and Ph. D. in Operations Research both from Jadavpur University, Kolkata, India. His research interests include inventory/supply chain management, production planning and scheduling. Professor Giri has published more than 100 research papers in the journals of international repute. His papers appeared in journals such as Naval Research Logistics, International Journal of Production Research, OMEGA, Journal of the Operational Research Society, European Journal of Operational Research, International Journal of Production Economics and so on. He was a JSPS Post-doctoral Research Fellow at Hiroshima University, Japan during the period 2002-2004 and Humboldt Research Fellow at Mannheim University, Germany during the period 2007-2008.

**Biswarup Samanta** is a Junior Research Fellow in the Department of Mathematics, Jadavpur University, Kolkata, India. He did his B.S. and M.S. in Mathematics both from Vidyasagar University, India. Her doctoral work is related to reliability in supply chain management.